Pure and New Mathematics in AI 2024, 1(1), 6585.
https://doi.org/10.24294/pnmai6585

Article

New estimations on fixed-time stabilization of delayed neural networks with event-triggered control

Guodong Zhang

School of Mathematics and Statistics, South-Central Minzu University, Wuhan 430074, China; zgd2008@mail.scuec.edu.cn

CITATION

Zhang G. New estimations on fixed-time stabilization of delayed neural networks with event-triggered control. Pure and New Mathematics in AI. 2024; 1(1): 6585. https://doi.org/10.24294/pnmai6585

ARTICLE INFO

Received: 5 July 2024
Accepted: 25 October 2024
Available online: 14 November 2024

Abstract: New estimations on settling-time for fixed-time stabilization of nonlinear systems are derived. By using the new proposed results on fixed-time stable and designing proper effective event-triggered control (ETC), fixed-time stabilization (FTS) for a kind of delayed neural networks is investigated. The new estimations on settling-time for fixed-time stabilization can be used to discussed other systems, such as complex networks, multi-agent systems and so on. At last, example simulations are given to corroborate the effectiveness of the derived results.

Keywords: fixed-time stabilization; neural networks; distributed delays; event-triggered control

1. Introduction

Neural networks(NNs) have important applications in lots of fields such as information security [1] and forecasting [2], and these applications heavily depend on the dynamic behaviours of NNs [3–7]. Among them, stabilization has been given more discussions and many good reports have been found in these years. In 2010, Phat and Trinh [5] discussed exponential stabilization of NNs with various activation functions and mixed time-varying delays via feedback controllers. In 2019, Wang et al. [6] investigated finite-time stabilization(FS) of memristor-based inertial NNs with distributed delays with feedback controller. In 2020, Zhang and Zeng [5] studied global stabilization of second-order memristive NNs via feedback controller and non-reduced order method.

Unfortunately, the previous works [5–7] are all about asymptotic stabilization or FS for NNs. Convergence time of asymptotic stabilization will tend to infinite that do not satisfy instantaneous motion control. Because the settling-time function(STF) of FS depends on initial values of NNs, which settling-time is difficult to estimate if the initial values of NNs can not be got. In 2012, Polyakov [8] showed the fixed-time stability results, and the upper bound of STF for fixed-time stability is a fixed positive constant. In recent years, fixed-time stabilization(FTS) and synchronization of NNs has attracted the attention of scholars and many meaningful results have been achieved. In 2017, Hu et al. [9] studied fixed-time stability of coupled discontinuous NNs. In 2019, Chen et al. [10] obtained synchronization in fixed-time of memristive NNs. In 2023, Zhang and Cao [11] showed fixed-time synchronization of delayed fuzzy inertial discontinuous NNs with non-reduced order approach. Zhang et al. [12] given further results on fixed-time projective lag synchronization control of delayed hybrid inertial NNs via feedback controller.

Noteworthy, the above works [5–7,9–12] all use feedback control to investigate stabilization or synchronization. Feedback control usual causes high control costs and low efficiency. Unlike feedback control, event-triggered control(ETC) has both advantages in enhancing efficiency and saving costs due to its control signals update only if the preassigned triggering condition is triggered. In these years, many meaningful results on ETC of NNs are reported, e.g., see [13–16]. In 2021, by event-triggered impulsive control, Chen et al. [15] gave some effective results on synchronization of multiple NNs. In 2023, Zhang [16] derived some novel results on FTS of delayed discontinuous NNs with ETC.

However, the fixed-time lemma used in most of the previous works are given by Polyakov [8]. The upper bound of the settling-time showed in Polyakov [8] is more bigger and need re-estimate to get more accurate one. Therefore, the authors of [9, 10] extended the fixed-time lemma given in Polyakov [8]. And with the development of analysis, more accurate settling-time of fixed-time stable will be reached. So, in this article, we will further extend the fixed-time lemmas to get more accurate settling-time, then, the new extended lemmas on fixed-time stable are applied to discuss FTS of NNs with distributed delays. The new points of this article are:

(1) New estimations on settling-time of fixed-time stable are given, which are more accurate than the previous one showed in [8, 10], and some details are also listed to show the advantages of the proposed results.

(2) Some corresponding flexible algebraic criteria on FTS of NNs with distributed delays are given. Here, the time delays do not need differentiable and their derivatives less than one.

(3) Different from the feedback control used in [5–7, 9–12], an effective ETC is constructed in this article to realize FTS of the delayed NNs. We think that the proposed method and ETC can be used to discuss more complex systems, such as NNs with inertial items [11, 12] and NNs with state-based switching [14, 15].

The following structures are: Part 2 shows the preliminaries. Part 3 gives new results on FTS of delayed NNs. Part 4, simulations are reached. Finally, conclusions are given.

For convenience, **Table 1** is given following to show some mathematical notations.

Table 1. Mathematical notations of this article.

Notation	**Mathematical description**
$\mathcal{P}$	Set $\{1, 2, 3, ...m\}$
$\mathbb{R}^m$	m-dimensional Euclidean space
$\Vert\mathbb{k}\Vert_1$	$1-$ norm of vector $\mathbb{k} \in \mathbb{R}^m$
τ	$\max_{l \in \mathcal{P}}\{\sigma_l, \varrho_l\}$
$\mathcal{L}_j$	$\max\{\vert\mathcal{L}_j^-\vert, \vert\mathcal{L}_j^+\vert\}$
$\mathcal{C}([-\tau, 0], \mathbb{R}^m)$	All continuous functions in Banach space
$\mathcal{C}^1(\mathbb{R}^m, \mathbb{R}_+)$	All nonnegative differentiable functions

2. Preliminaries

2.1. Model, assumption, definitions and Lemmas

The NNs with distributed delays is

$$\begin{aligned}\frac{\mathrm{d}x_l(t)}{\mathrm{d}t} = & -\alpha_l x_l(t) + \sum_{j=1}^{m} \beta_{lj}\Gamma_j(x_j(t)) + \sum_{j=1}^{m} \gamma_{lj} \\ & \times \Gamma_j(x_j(t-\sigma_j(t))) + \sum_{j=1}^{m} \delta_{lj} \\ & \times \int_{t-\varrho_j(t)}^{t} \Gamma_j(x_j(s))\mathrm{d}s,\ l \in \mathcal{P}, t \geqslant 0\end{aligned} \tag{1}$$

where $x_l(t)$ is the l-th neural state, $\alpha_l > 0$, $\beta_{lj}, \gamma_{lj}, \delta_{lj}$ are connect weights, $\Gamma_j(\cdot)$ is the bounded feedback function, and time delays $\sigma_j(t) > 0$, $\varrho_j(t) \geqslant 0$ which satisfy $\sigma_j(t) \leqslant \sigma_j$, $\varrho_j(t) \leqslant \varrho_j$.

Initional positions of NNs (1) are $x_l(s) = \digamma_l(s)$, and $\digamma_l(s) \in \mathcal{C}([-\tau, 0], \mathbb{R}), l \in \mathcal{P}$.

Assumption 1. *For* $\forall \mathcal{S}_1, \mathcal{S}_2 \in \mathbb{R}$, $\Gamma_j(\cdot)$ *in NNs (1) fulfils* $\Gamma_j(0) = 0$ *and*

$$\mathcal{L}_j^- \leq \frac{\Gamma_j(\mathcal{S}_1) - \Gamma_j(\mathcal{S}_2)}{\mathcal{S}_1 - \mathcal{S}_2} \leq \mathcal{L}_j^+, |\Gamma_j(\cdot)| \leqslant \mathcal{M}_j$$

where $\mathcal{S}_1 \neq \mathcal{S}_2$, $\mathcal{L}_j^-, \mathcal{L}_j^+$ *are constants,* $\mathcal{M}_j > 0, j \in \mathcal{P}$.

Definition 1. *Suppose* $\mathbb{T}(x(0))$ *is the STF, and exists a positive constant* $\mathbb{T}_{\max}$ *such that* $\mathbb{T}(x(0)) \leqslant \mathbb{T}_{\max}$ *and* $\lim_{t \to \mathbb{T}_{\max}} \|x(t)\|_1 = 0$*, then, NNs (1) is fixed-time stable, where* $\mathbb{T}_{\max}$ *is settling-time,* $x(t) = (x_1(t), x_2(t), ..., x_m(t))^T, x(0) \in \mathbb{R}^m$.

Lemma 1. *[8]: Suppose* $\mathbb{V}(x) : \mathbb{R}^m \to \mathbb{R}_+ \cup \{0\}$ *is the radially unbounded and positive definite function, and any solutions of NNs (1) fulfil*

$$\frac{\mathrm{d}\mathbb{V}(x(t))}{\mathrm{d}t} \leqslant -(p\mathbb{V}^\lambda(x(t)) + q\mathbb{V}^\mu(x(t)))^k \tag{2}$$

where $p, q, \lambda, k > 0, \lambda \geqslant 0, \lambda k < 1, \mu k > 1$*, then, NNs (1) is fixed-time stable and setting-time is*

$$\mathbb{T}_{\max}^1 = \frac{1}{p^k(1 - \lambda k)} + \frac{1}{q^k(\mu k - 1)} \tag{3}$$

Lemma 2. *[10]: Suppose* $\mathbb{V}(x) : \mathbb{R}^m \to \mathbb{R}_+ \cup \{0\}$ *is the radially unbounded and positive definite function, and any solutions of NNs (1) fulfil*

$$\frac{\mathrm{d}\mathbb{V}(x(t))}{\mathrm{d}t} \leqslant -p\mathbb{V}^\lambda(x(t)) - q\mathbb{V}^\mu(x(t)) - c \tag{4}$$

where $p, q, c > 0, 0 < \lambda < 1, \mu > 1$*, then, NNs (1) is fixed-time stable and setting-time is*

$$\begin{aligned} \mathbb{T}_{\max}^2 = & \frac{1}{p^{\frac{1}{\lambda}}(1 - \lambda)}[(p^{\frac{1}{\lambda}} + c^{\frac{1}{\lambda}})^{1-\lambda} - c^{\frac{1-\lambda}{\lambda}}] \\ & + \frac{2^{\mu-1}}{q^{\frac{1}{\mu}}(\mu - 1)}(q^{\frac{1}{\mu}} + c^{\frac{1}{\mu}})^{1-\mu} \end{aligned} \tag{5}$$

Lemma 3. *[17]: Suppose* $s_1, s_2, \ldots, s_m \geqslant 0, 0 < z_1 \leqslant 1, z_2 > 1$*, then*

$$\sum_{l=1}^m s_l^{z_1} \geqslant \Big(\sum_{l=1}^m s_l\Big)^{z_1}, \sum_{l=1}^m s_l^{z_2} \geqslant m^{1-z_2}\Big(\sum_{l=1}^m s_l\Big)^{z_2} \tag{6}$$

2.2. New estimations on settling-time for Lemmas 1 and 2

Lemma 4. *Suppose* $\mathbb{V}(x) : \mathbb{R}^m \to \mathbb{R}_+ \cup \{0\}$ *is regular, and the radially unbounded and positive definite function, and for almost all solutions of NNs (1) fulfil (2), then, NNs (1) is fixed-time stable and setting-time is*

$$\mathbb{T}_{\max}^3 = \frac{1}{p^k(1 - \lambda k)} + \frac{(p^{\frac{1}{\mu}} + q^{\frac{1}{\mu}})^{1-k\mu}}{2^{k-k\mu}(\mu k - 1)q^{\frac{1}{\mu}}} \tag{7}$$

Proof. One can get STF is

$$\mathbb{T}_{\max}(x(0)) = \int_0^{\mathbb{V}(x(0))} \frac{1}{(p\mathbb{V}^\lambda + q\mathbb{V}^\mu)^k} \mathrm{d}\mathbb{V} \tag{8}$$

And from Equation (8), we know

$$\begin{aligned}\mathbb{T}_{\max}(x(0)) \leqslant &\int_0^{+\infty} \frac{1}{(p\mathbb{V}^\lambda + q\mathbb{V}^\mu)^k}\mathrm{d}\mathbb{V} \\ =&\int_0^{1} \frac{1}{(p\mathbb{V}^\lambda + q\mathbb{V}^\mu)^k}\mathrm{d}\mathbb{V} \\ &+\int_1^{+\infty} \frac{1}{(p\mathbb{V}^\lambda + q\mathbb{V}^\mu)^k}\mathrm{d}\mathbb{V} \\ \leqslant&\int_0^{1} \frac{1}{p^k\mathbb{V}^{k\lambda}}\mathrm{d}\mathbb{V} + \int_1^{+\infty} \frac{1}{(p + q\mathbb{V}^\mu)^k}\mathrm{d}\mathbb{V}\end{aligned} \tag{9}$$

By using Lemma 3, one has $p + q\mathbb{V}^\mu \geqslant 2^{1-\mu}(p^{\frac{1}{\mu}} + q^{\frac{1}{\mu}}\mathbb{V})^\mu$, from Equation (9), we get

$$\begin{aligned}\mathbb{T}_{\max}(x(0)) \leqslant &\int_0^{1} \frac{1}{p^k\mathbb{V}^{k\lambda}}\mathrm{d}\mathbb{V} \\ &+\int_1^{+\infty} \frac{1}{2^{k-k\mu}(p^{\frac{1}{\mu}} + q^{\frac{1}{\mu}}\mathbb{V})^{k\mu}}\mathrm{d}\mathbb{V} \\ =&\frac{1}{p^k(1-\lambda k)} + \frac{(p^{\frac{1}{\mu}} + q^{\frac{1}{\mu}})^{1-k\mu}}{2^{k-k\mu}(\mu k - 1)q^{\frac{1}{\mu}}}\end{aligned} \tag{10}$$

From Equation (10), one gets the Lemma 3 holds. This proof is end. □

Lemma 5. *Suppose* $\mathbb{V}(x) : \mathbb{R}^m \to \mathbb{R}_+ \cup \{0\}$ *is regular, and the radially unbounded and positive definite function,* $k = 1$ *and for almost all solutions of NNs (1) fulfil (2), then, NNs (1) is fixed-time stable and setting-time is*

$$\mathbb{T}_{\max}^{3*} = \frac{1}{p(1-\lambda)} + \frac{(p^{\frac{1}{\mu}} + q^{\frac{1}{\mu}})^{1-\mu}}{2^{1-\mu}(\mu - 1)q^{\frac{1}{\mu}}} \tag{11}$$

Remark 1. *If* $k \geqslant 1, p \geqslant q$*, one can get* $\mathbb{T}_{\max}^3 < \mathbb{T}_{\max}^1$.

Lemma 6. *Suppose* $\mathbb{V}(x) : \mathbb{R}^m \to \mathbb{R}_+ \cup \{0\}$ *is regular, and the radially unbounded and positive definite function, and for almost all solutions of NNs (1) fulfil (4), then, NNs (1) is fixed-time stable and setting-time is*

$$\begin{aligned}\mathbb{T}_{\max}^4 =&\frac{1}{p^{\frac{1}{\lambda}}(1-\lambda)}[(p^{\frac{1}{\lambda}} + c^{\frac{1}{\lambda}})^{1-\lambda} - c^{\frac{1-\lambda}{\lambda}}] \\ &+ \frac{[(p+c)^{\frac{1}{\mu}} + q^{\frac{1}{\mu}}]^{1-\mu}}{2^{1-\mu}(\mu-1)q^{\frac{1}{\mu}}}\end{aligned} \tag{12}$$

Proof. One can get STF is

$$\mathbb{T}_{\max}(x(0)) = \int_0^{\mathbb{V}(x(0))} \frac{\mathrm{d}\mathbb{V}}{p\mathbb{V}^\lambda + q\mathbb{V}^\mu + c} \tag{13}$$

From Equation (13), we have

$$\begin{aligned}\mathbb{T}_{\max}(x(0)) &\leqslant \int_0^{+\infty}\frac{\mathrm{d}\mathbb{V}}{p\mathbb{V}^{\lambda}+q\mathbb{V}^{\mu}+c}\\ &\leqslant \int_0^{1}\frac{1}{p\mathbb{V}^{\lambda}+c}\mathrm{d}\mathbb{V}\\ &\quad+\int_1^{+\infty}\frac{1}{q\mathbb{V}^{\mu}+p+c}\mathrm{d}\mathbb{V}\end{aligned} \tag{14}$$

By using Lemma 3, one has $p+c+q\mathbb{V}^{\mu}\geqslant 2^{1-\mu}[(p+c)^{\frac{1}{\mu}}+q^{\frac{1}{\mu}}\mathbb{V}]^{\mu}, p\mathbb{V}^{\lambda}+c\geqslant(p^{\frac{1}{\lambda}}\mathbb{V}+c^{\frac{1}{\lambda}})^{\lambda}$, from Equation (14), we get

$$\begin{aligned}\mathbb{T}_{\max}(x(0)) &\leqslant \int_0^{1}\frac{1}{(p^{\frac{1}{\lambda}}\mathbb{V}+c^{\frac{1}{\lambda}})^{\lambda}}\mathrm{d}\mathbb{V}\\ &\quad+\int_1^{+\infty}\frac{1}{2^{1-\mu}[(p+c)^{\frac{1}{\mu}}+q^{\frac{1}{\mu}}\mathbb{V}]^{\mu}}\mathrm{d}\mathbb{V}\\ &\leqslant \int_0^{1}\frac{1}{(p^{\frac{1}{\lambda}}\mathbb{V}+c^{\frac{1}{\lambda}})^{\lambda}}\mathrm{d}\mathbb{V}\\ &\quad+\frac{[(p+c)^{\frac{1}{\mu}}+q^{\frac{1}{\mu}}]^{1-\mu}}{2^{1-\mu}(\mu-1)q^{\frac{1}{\mu}}}\\ &=\frac{1}{p^{\frac{1}{\lambda}}(1-\lambda)}[(p^{\frac{1}{\lambda}}+c^{\frac{1}{\lambda}})^{1-\lambda}-c^{\frac{1-\lambda}{\lambda}}]\\ &\quad+\frac{[(p+c)^{\frac{1}{\mu}}+q^{\frac{1}{\mu}}]^{1-\mu}}{2^{1-\mu}(\mu-1)q^{\frac{1}{\mu}}}\end{aligned} \tag{15}$$

From Equation (15), one has the Lemma 6 holds. This proof is finished.□

Remark 2. *From Equations (5) and (12), one can easily find that settling-time* $\mathbb{T}^4_{\max}<\mathbb{T}^2_{\max}$*, that is, Lemma 6 of this paper is more accurate than Lemma 2.*

3. Main results

In this part, as an application based on new estimation result given in Lemma 6, we show some results on FTS of NNs (1). If the parameters of NNs choose properly, the NNs (1) will occur oscillation and even get chaotic behaviours, under these cases, the following control model of NNs (1) is considered

$$\begin{aligned}\frac{\mathrm{d}x_l(t)}{\mathrm{d}t}=&-\alpha_l x_l(t)+\sum_{j=1}^{m}\beta_{lj}\Gamma_j(x_j(t))+\sum_{j=1}^{m}\gamma_{lj}\\ &\times\Gamma_j(x_j(t-\sigma_j(t)))+\sum_{j=1}^{m}\delta_{lj}\\ &\times\int_{t-\varrho_j(t)}^{t}\Gamma_j(x_j(s))\mathrm{d}s+u_l(t),\ l\in\mathcal{P}\end{aligned} \tag{16}$$

where, the controller $u_l(t)$ $(l\in\mathcal{P})$ in Equation (16) is

$$\begin{aligned}u_l(t)=&-\xi_l x_l(t_i)-\Big(\varpi_l|x_l^{\lambda}(t_i)|+\rho_l|x_l^{\mu}(t_i)|+\omega_l\Big)\\ &\times\mathrm{sign}(x_l(t_i)),\ t\in[t_i,t_{i+1}), i=0,1,2,...\end{aligned} \tag{17}$$

in which $\xi_l,\varpi_l,\rho_l,\omega_l>0$ and $0<\lambda<1, \mu>1$.

For $t\in[t_i,t_{i+1})$, let $U_l(t)=-\xi_l x_l(t)-\Big(\varpi_l|x_l^{\lambda}(t)|+\rho_l|x_l^{\mu}(t)|+\omega_l\Big)\mathrm{sign}(x_l(t))$, measure error is $e_l(t) = U_l(t) - u_l(t)$, and event-triggering is defined as follows:

$$t_{i+1} = \Big\{ t | t > t_i, |e_l(t)| \geqslant \theta_l |x_l(t)| + \vartheta_l \varpi_l |x_l^{\lambda}(t)| + \kappa_l \rho_l \\ \times |x_l^{\mu}(t)| + \varsigma_l (1 - \varepsilon_l)^t \Big\} \tag{18}$$

where $\vartheta_l, \kappa_l, \varepsilon_l \in (0,1), \theta_p, \varsigma_p > 0$ and the $i^{\,th}$ triggering instant is t_i $(i = 0, 1, 2, ..., l \in \mathcal{P})$.

Let

$$\psi_l = \alpha_l + \xi_l - \theta_l - \sum\nolimits_{j=1}^{m} \mathcal{L}_l |\beta_{jl}|, \\ \varphi_l = \omega_l - \varsigma_l - \sum_{j=1}^{m} \Big(\mathcal{M}_j (|\gamma_{lj}| + \varrho_j |\delta_{lj}|) \Big), l \in \mathcal{P} \tag{19}$$

Theorem 1. *With Assumption 1 and ETC (17)–(18), if* $\psi_l > 0, \varphi_l > 0, c = \sum_{l=1}^{m} \varphi_l$ $(l \in \mathcal{P})$ *hold, then, NNs (1) is FTS, and settling-time is* $\mathbb{T}^4_{\max}$*. endtheorem*

Proof. Consider

$$\mathbb{V}(t) = \sum\nolimits_{l=1}^{m} |x_l(t)| \tag{20}$$

For $t \in [t_i, t_{i+1}), i = 0, 1, 2, ...,$ we get the derivative of $\mathbb{V}(t)$ with solutions of Equation (16), then,

$$\begin{aligned} \frac{\mathrm{d}\mathbb{V}(t)}{\mathrm{d}t} &= \sum\nolimits_{l=1}^{m} \mathrm{sign}(x_l(t)) \cdot \frac{\mathrm{d}x_l(t)}{\mathrm{d}t} \\ &= \sum\nolimits_{l=1}^{m} \mathrm{sign}(x_l(t)) \Big[- \alpha_l x_l(t) + \sum_{j=1}^{m} \beta_{lj} \Gamma_j(x_j(t)) \\ &\quad + \sum_{j=1}^{m} \gamma_{lj} \Gamma_j(x_j(t - \sigma_j(t))) + \sum_{j=1}^{m} \delta_{lj} \\ &\quad \times \int_{t-\varrho_j(t)}^{t} \Gamma_j(x_j(s)) \mathrm{d}s + u_l(t) \Big] \\ &\leqslant \sum\nolimits_{l=1}^{m} \Big[- \alpha_l |x_l(t)| + \sum_{j=1}^{m} |\beta_{lj} \Gamma_j(x_j(t))| \\ &\quad + \sum_{j=1}^{m} |\gamma_{lj} \Gamma_j(x_j(t - \sigma_j(t)))| + \sum_{j=1}^{m} |\delta_{lj}| \\ &\quad \times \int_{t-\varrho_j(t)}^{t} |\Gamma_j(x_j(s))| \mathrm{d}s + \mathrm{sign}(x_l(t)) u_l(t) \Big] \end{aligned} \tag{21}$$

From Assumption 1, one has

$$|\Gamma_j(x_j(t))| \leqslant \mathcal{L}_j |x_j(t)|, |\Gamma_j(x_j(t - \sigma_j(t)))| \leqslant \mathcal{M}_j \tag{22}$$

Then, one knows

$$\begin{aligned}\frac{\mathrm{d}\mathbb{V}(t)}{\mathrm{d}t} \leqslant &\sum\nolimits_{l=1}^{m}\Big[-\alpha_l|x_l(t)|+\sum_{j=1}^{m}\mathcal{L}_j|\beta_{lj}||x_j(t)| \\ &+\sum_{j=1}^{m}\mathcal{M}_j|\gamma_{lj}|+\sum_{j=1}^{m}\varrho_j\mathcal{M}_j|\delta_{lj}| \\ &+\mathrm{sign}(x_l(t))u_l(t)\Big] \\ \leqslant &\sum\nolimits_{l=1}^{m}\Big[-\alpha_l|x_l(t)|+\sum_{j=1}^{m}\mathcal{L}_j|\beta_{lj}||x_j(t)| \\ &+\sum_{j=1}^{m}\mathcal{M}_j|\gamma_{lj}|+\sum_{j=1}^{m}\varrho_j\mathcal{M}_j|\delta_{lj}| \\ &+\mathrm{sign}(x_l(t))(U_l(t)-e_l(t))\Big]\end{aligned} \tag{23}$$

From ETC (17)–(18) and (23), then

$$\begin{aligned}\frac{\mathrm{d}\mathbb{V}(t)}{\mathrm{d}t} \leqslant &\sum_{l=1}^{m}\Big[-\Big(\alpha_l+\xi_l-\theta_l-\sum\nolimits_{j=1}^{m}\mathcal{L}_l|\beta_{jl}|\Big)|x_l(t)| \\ &-\Big[\omega_l-\varsigma_l-\sum_{j=1}^{m}\Big(\mathcal{M}_j(|\gamma_{lj}|+\varrho_j|\delta_{lj}|)\Big)\Big] \\ &+|e_l(t)|-\theta_l|x_l(t)|-\vartheta_l\varpi_l|x_l^{\lambda}(t)|-\kappa_l\rho_l|x_l^{\mu}(t)| \\ &-\varsigma_l(1-\varepsilon_l)^t-(1-\vartheta_l)\varpi_l|x_l^{\lambda}(t)| \\ &-(1-\kappa_l)\rho_l|x_l^{\mu}(t)|\Big]\end{aligned} \tag{24}$$

Under the conditions of Theorem 1 and the triggering condition (18), we have

$$\frac{\mathrm{d}\mathbb{V}(t)}{\mathrm{d}t} \leqslant \sum_{l=1}^{m}\Big(-(1-\vartheta_l)\varpi_l|x_l^{\lambda}(t)|-(1-\kappa_l)\rho_l|x_l^{\mu}(t)|\Big)-c$$

Now, let $p=\min_{1\leqslant l\leqslant m}\{(1-\vartheta_l)\varpi_l\}, q_1=\min_{1\leqslant l\leqslant m}\{(1-\kappa_l)\rho_l\}$. Then,

$$\frac{\mathrm{d}\mathbb{V}(t)}{\mathrm{d}t} \leqslant -p\sum_{l=1}^{m}|x_l^{\lambda}(t)|-q_1\sum_{l=1}^{m}|x_l^{\mu}(t)| \tag{25}$$

From the Lemma 2, one gets

$$-p\sum_{l=1}^{m}|x_l^{\lambda}(t)| \leqslant -p\Big(\sum_{l=1}^{m}|x_l(t)|\Big)^{\lambda} \tag{26}$$

and

$$-q_1\sum_{l=1}^{m}|x_l^{\mu}(t)| \leqslant -q_1m^{1-\mu}\Big(\sum_{l=1}^{m}|x_l(t)|\Big)^{\mu} \tag{27}$$

Let $q=q_1m^{1-\mu}$. From Equations (25)–(27), one knows

$$\frac{\mathrm{d}\mathbb{V}(t)}{\mathrm{d}t} \leqslant -p\mathbb{V}^{\lambda}(t)-q\mathbb{V}^{\mu}(t)-c \tag{28}$$

Now, from Lemma 6, we get that NNs (1) realize FTS with ETC (17)–(18). And settling-time is $\mathbb{T}^4_{\max}$. This proof is end. □

Theorem 2. *No Zeno-behaviour with ETC (17)–(18).*

Proof. For $t \in [t_i, t_{i+1}), i = 0, 1, 2, ...,$

$$\begin{aligned} \frac{\mathrm{d}|e_l(t)|}{\mathrm{d}t} \leqslant& |\frac{\mathrm{d}U_l(t)}{\mathrm{d}t}| \\ \leqslant& (\xi_l + \lambda\varpi_l|x_l^{\lambda-1}(t)| + \mu\rho_l|x_l^{\mu-1}(t)|)|\dot{e}_l(t)| \end{aligned} \tag{29}$$

For system (16), one has

$$\begin{aligned} |\frac{\mathrm{d}e_l(t)}{\mathrm{d}t}| \leqslant& \Big(\alpha_l|x_l(t)| + \sum_{j=1}^{m}\mathcal{L}_j|\beta_{lj}||x_j(t)| \\ &+ \sum_{j=1}^{m}\mathcal{M}_j|\gamma_{lj}| + \sum_{j=1}^{m}\varrho_j\mathcal{M}_j|\delta_{lj}| + |u_l(t)|\Big) \end{aligned} \tag{30}$$

Because $\frac{\mathrm{d}\mathbb{V}(t)}{\mathrm{d}t} < 0$, therefore, $|x_j(t)| \leqslant \mathbb{V}(0)$, then,

$$\begin{aligned} |\frac{\mathrm{d}e_l(t)}{\mathrm{d}t}| \leqslant& \Big((\alpha_l + \sum_{j=1}^{m}\mathcal{L}_j|\beta_{lj}|)\mathbb{V}(0) + \sum_{j=1}^{m}\mathcal{M}_j|\gamma_{lj}| \\ &+ \sum_{j=1}^{m}\varrho_j\mathcal{M}_j|\delta_{lj}| + |u_l(t)|\Big) = \gimel_l(t_i) \end{aligned} \tag{31}$$

Let $\daleth_l = \max_{t\in[t_i,\, t_{i+1})}(\xi_l + \lambda\varpi_l|x_l^{\lambda-1}(t)| + \mu\rho_l|x_l^{\mu-1}(t)|)$, and from Equation (29), we have

$$\frac{\mathrm{d}|e_l(t)|}{\mathrm{d}t} \leqslant \daleth_l\gimel_l(t_i) \tag{32}$$

Noted that $|e_l(t_i)| = 0$, then

$$|e_l(t)| \leqslant \int_{t_i}^{t}\daleth_l\gimel_l(t_i)\mathrm{d}s = \daleth_l\gimel_l(t_i)(t - t_i) \tag{33}$$

By ETC (18), we know

$$\begin{aligned} |e_l(t_{i+1})| \geqslant& \theta_l|x_l(t_{i+1})| + \vartheta_l\varpi_l|x_l^{\lambda}(t_{i+1})| + \kappa_l\rho_l \\ &\times |x_l^{\mu}(t_{i+1})| + \varsigma_l(1-\varepsilon_l)^{t_{i+1}} \\ \geqslant& \varsigma_l(1-\varepsilon_l)^{t_{i+1}} > 0 \end{aligned} \tag{34}$$

From Equations (33)–(34), we get

$$t_{i+1} - t_i \geqslant \frac{\varsigma_l(1-\varepsilon_l)^{t_{i+1}}}{\daleth_l\gimel_l(t_i)} > 0 \tag{35}$$

This proof is end. □

4. Example and comparison

Example 1. *NNs (1) with two-dimensional is*

$$\begin{aligned}\frac{\mathrm{d}x_l(t)}{\mathrm{d}t} =& -\alpha_l x_l(t) + \sum_{j=1}^{2}\beta_{lj}\Gamma_j(x_j(t)) + \sum_{j=1}^{2}\gamma_{lj} \\ &\times \Gamma_j(x_j(t-\sigma_j(t))) + \sum_{j=1}^{2}\delta_{lj} \\ &\times \int_{t-\varrho_j(t)}^{t} \Gamma_j(x_j(s))\mathrm{d}s,\ l=1,2, t \geqslant 0\end{aligned} \tag{36}$$

in which $\Gamma_j(x_j) = \frac{1}{2}(|x_j+1| - |x_j-1|)$ *and* $\alpha_1 = \alpha_2 = 1, \beta_{11} = 2, \beta_{12} = -1, \gamma_{11} = 1, \gamma_{12} = -1, \delta_{11} = -0.1, \delta_{12} = -0.1, \beta_{21} = 2, \beta_{22} = -0.5, \gamma_{21} = 0.5, \gamma_{22} = -0.5, \delta_{21} = 0.1, \delta_{22} = 0.1, \sigma_j(t) = \varrho_j(t) = \frac{\exp(t)}{\exp(t)+1}$.

Initial values of NNs (30) are $x_1(s) = 0.5, x_2(s) = -0.6, \forall s \in [-1, 0)$. *And the states* $x_1(t), x_2(t)$ *of NNs (36) without control input are showed in* **Figure 1**.

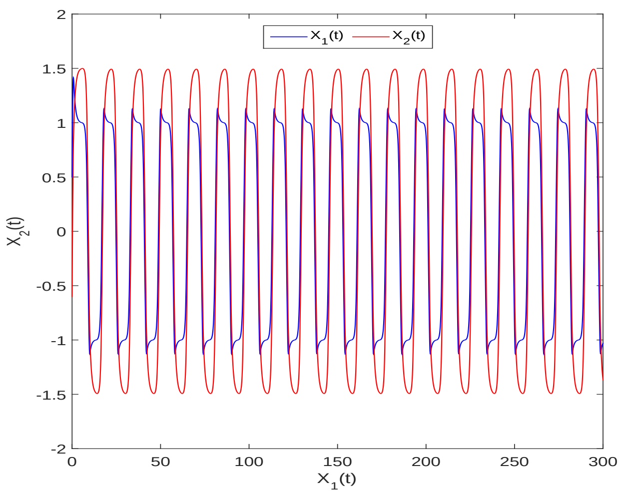

Figure 1. State trajectories $x_1(t), x_2(t)$ without ETC (17)–(18).

From Equation (30), we know $\mathcal{L}_j = \mathcal{M}_j = \varrho_j = 1$. Now, let $\xi_1 = 20, \xi_2 = 15, \varpi_1 = \varpi_2 = 2, \omega_1 = 3, \omega_2 = 2, \rho_1 = \rho_2 = 1, \vartheta_1 = \vartheta_2 = \kappa_1 = \kappa_2 = \varsigma_1 = \varsigma_2 = 0.1, \theta_1 = \theta_1 = 1, \lambda = 0.7, \mu = 1.2$, then, $\psi_1 = 16, \psi_2 = 13.5, \varphi_1 = \varphi_2 = 0.6, p = 1.8, q = 0.6364$ and $\mathbf{T}^4_{\max} = 0.8429 < \mathbf{T}^2_{\max} = 0.8734, \mathbf{T}^3_{\max} = 8.4635 < \mathbf{T}^1_{\max} = 9.2456$.

Now, all conditions of Theorem 1 hold, by using Theorem 1, we get Equation (36) is FTS with ETC (17)–(18), randomly choose 30 initial values, **Figure 2** show that state trajectories $x_1(t), x_2(t)$ of Equation (3) are FTS with ETC (17)–(18). Transmission intervals $t_{i+1} - t_i$ of ECT (17)–(18) are showed in **Figure 3**. Measure errors $|U_1(t) - u_1(t)|, |U_2(t) - u_2(t)|$ and their thresholds in triggering condition (18) are given in **Figure 4**. From **Figures 2–4**, we can find that the results on FTS of NNs (1) derived in this paper are very effective. And **Table 2** shows that the fixed-time stable Lemma 4 and Lemma 6 of this paper are more accurate than the previous works [8] and [10].

Table 2. Comparisons of settling-time with Lemma 6 of this article.

Settling-time on FTS	**Values**
$\mathbf{T}^1_{\max}$ [8]	9.2456
$\mathbf{T}^3_{\max}$ used in Lemma 4	8.4635
$\mathbf{T}^2_{\max}$ [10]	0.8734
$\mathbf{T}^4_{\max}$ in Lemma 6 of this paper	0.8429

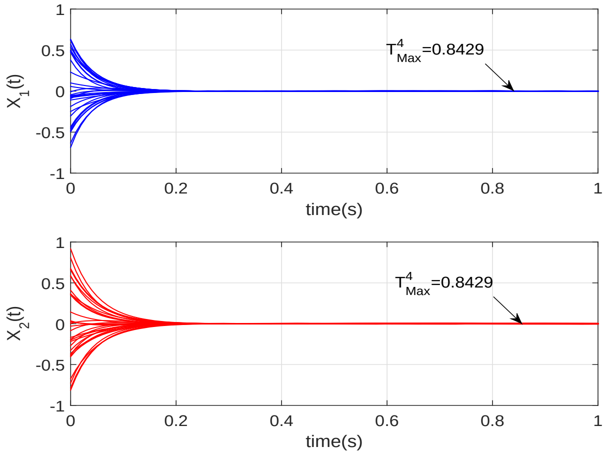

Figure 2. State trajectories $x_1(t), x_2(t)$ with ETC (17)–(18).

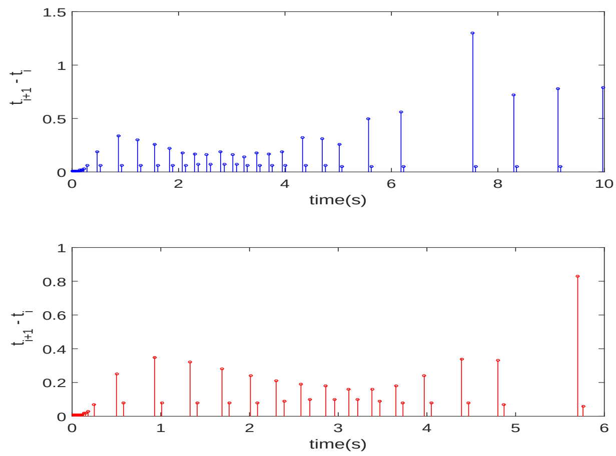

Figure 3. Transmission intervals $t_{i+1} - t_i$ of ECT (17)–(18).

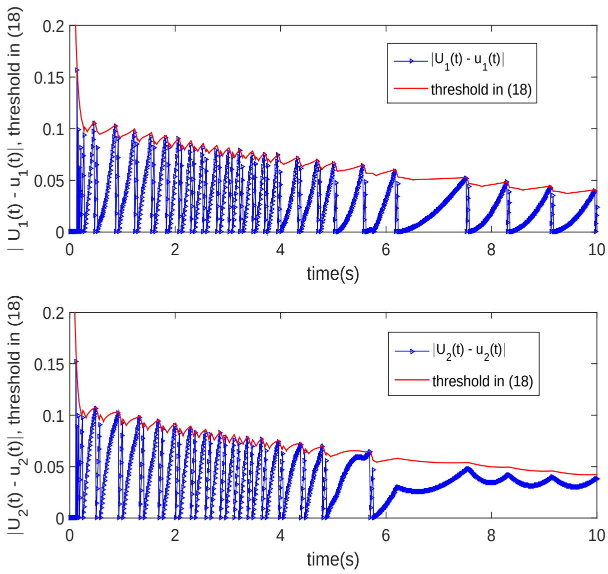

Figure 4. Measure errors and their thresholds in triggering condition (18).

5. Conclusions

By using the new estimations of settling-time for FTS and designing an effective ETC, new criteria on FTS for delayed neural networks was investigated. And example simulations showed the effectiveness of the derived results. As we know, NNs with complex-valued has better performance when dealing with 3D or 4D data than real network systems, therefore, basing on the new estimations proposed in this paper, the FTS of NNs with complex-valued will be investigated in our future works.

Funding: This work is supported by National Science Foundation of China No. 61976228 and the Fundamental Research Funds of South-Central Minzu University (CZQ24020).

Conflict of interest: The author declares no conflict of interest.

References

1. Wen S, Zeng Z, Huang T, et al. Lag synchronization of switched neural networks via neural activation function and applications in image encryption. IEEE Trans. Neural Netw. Learn. Syst. 2015; 26(7): 1493–1502.
2. Sangiorgio M, Dercole F, Guariso G. Forecasting of noisy chaotic systems with deep neural networks. Chaos, Solitons & Fractals. 2021; 153(111570).
3. Li P, Gao R, Xu C, et al. Exploring the impact of delay on Hopf bifurcation of a type of BAM neural network models concerning three nonidentical delays. Neural Process. Lett. 2023; 55: 11595–11635.
4. Xu C, Zhao Y, Lin J, et al. Bifurcation investigation and control scheme of fractional neural networks owning multiple delays. Comput. Appl. Math. 2024; 43(186).
5. Phat VN, Trinh H. Exponential stabilization of neural networks with various activation functions and mixed time-varying delays. IEEE Trans. Neural Netw. 2010; 21(7): 1180–1184.
6. Wang L, Zeng Z, Zong X, Ge MF. Finite-time stabilization of memristor-based inertial neural networks with discontinuous activations and distributed delays. J. Franklin Inst. 2019; 356: 3628–3643 .
7. Zhang GD, Zeng ZG. Stabilization of second-order memristive neural networks with mixed time delays via nonreduced order. IEEE Trans. Neural Netw. Learn. Syst. 2020; 31(2): 700–706.
8. Polyakov A. Nonlinear feedback design for fixed-time stabilization of linear control systems. IEEE Trans. Autom. Control. 2012; 57: 2106–2110.

9. Hu C, Yu J, Chen Z, et al. Fixed-time stability of dynamical systems and fixed-time synchronization of coupled discontinuous neural networks. Neural Netw. 2017; 89; 74–83.
10. Chen C, Li L, Peng H, et al. A new fixed-time stability theorem and its application to the synchronization control of memristive neural networks. Neurcomput. 2019; 349: 290–300, .
11. Zhang GD, Cao J. New results on fixed/predefined-time synchronization of delayed fuzzy inertial discontinuous neural networks: non-reduced order approach. Appl. Math. Comput. 2023; 440(127671).
12. Zhang GD, Cao J, Kashkynbayev A. Further results on fixed/preassigned-time projective lag synchronization control of hybrid inertial neural networks with time delays. J. Franklin Inst. 2023; 360: 9950–9973.
13. Wen S, Zeng Z, Chen MZQ, Huang T. Synchronization of switched neural networks with communication delays via the event-triggered control. IEEE Trans. Neural Netw. Learn. Syst. 2017; 28(10): 2334–2343.
14. Guo Z, Gong S, Wen S, Huang T. Event-based synchronization control for memristive neural networks with time-varying delay. IEEE Trans. Cybern. 2019; 49(9): 3268–3277, .
15. Chen J, Chen B, Zeng Z. Synchronization in multiple neural networks with delay and disconnected switching topology via event-triggered impulsive control strategy. IEEE Trans. Ind. Electron. 2021; 63(3): 2491–2500.
16. Zhang GD. Novel results on event-triggered-based fixed-time synchronization and stabilization of discontinuous neural networks with distributed delays. Franklin Open. 2023; 4(100032).
17. Khalil HK, Grizzle JW. Nonlinear Systems. Prentice-Hall Publishing; 2002.