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Article

A group decision-making method for ranking units by applying intuitionistic fuzzy preference relations and weighted SBM model

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Abstract: In this paper, a novel group decision-making method is proposed based on the weighted SBM model of data envelopment analysis (DEA) and intuitionistic fuzzy preference relations (IFPRs). Indeed, for the data fuzzy numbers set, the main aim of this study is to measure the efficiency of different alternatives in the framework of IFPRs by the weighted SBM model. In this regard, first, the interval transform function is used to convert IFPRs into interval multiplicative preference relations. After calculating the efficiency, the optimal weights for each IFPR are identified using two cross-efficiency models to obtain the normalized intuitionistic fuzzy priority vector. Then, an algorithm for group decision-making is proposed using a goal programming, SBM model with ideal weights and IFPRs to rank the units. Finally, the model is implemented numerically, and the results are also compared with other models, including the output-oriented Charnes-Cooper-Rhodes) CCR (and basic Banker-Charnes-Cooper (BCC) models. It is shown that the proposed method outperforms traditional CCR and BCC models and provides more reasonable results.

Keywords: weighted SBM; data envelopment analysis; intuitionistic fuzzy preference relation; group decision-making; goal programming

1. Introduction and preliminaries

Group decision-making is a process where a collection of individuals comes together to reach a common decision by utilizing their knowledge, experiences, and diverse perspectives. This kind of decision-making method is widely used in various fields, including business, politics, education, and life; it is also a powerful tool for solving complex problems and achieving better outcomes. However, to succeed in this approach, it is essential to consider the challenges, influencing factors, and select appropriate methods. The complexity of social and economic systems leads to decision-making problems that involve considering numerous factors and indicators to reflect characteristics, performance, and various solutions. Although group decision-making seems to be a suitable solution for these complexities, some disagreements and differences in the level of confidence among individuals lead to uncertainty. To overcome this problem, fuzzy logic, introduced by Lotfi Zadeh in 1965, can be used; indeed, fuzzy logic is a well-suited approach to model the uncertainty and ambiguity present in human information and knowledge. This fact makes it ideal for decision-making problems that deal with incomplete or vague information. Additionally, using fuzzy logic can lead to more accurate and rational decisions. Furthermore, in the mid-1980s, Atanassov's intuitionistic fuzzy set evolved, in which these sets are a powerful tool for modeling uncertainty and ambiguity in decision-making problems. This concept allows us to model the complexities of the real world much better and also make finer decisions. In fact, in classical fuzzy sets, each element is assigned a membership degree between zero and one, indicating the degree of element membership. While in intuitionistic fuzzy sets, each element is assigned two membership degrees: a membership degree and a non-membership degree, where the sum of these degrees is not necessarily equal to one.

Meanwhile DEA is recognized as an efficient tool for ranking and prioritizing in various situations. By measuring the relative efficiency of organizations, DEA ranks them, identifies their strengths, weaknesses and provides suggestions for improving the efficiency of each organization. One of the appropriate methods for ranking in DEA, is using the cross-efficiency model, which has a high ability to discriminate between units. In this method, the performance of each decision-making unit is compared with respect to the optimal weights of the units. The use of fuzzy logic, especially considering the type of needs (in comparisons) and intuitionistic fuzzy preference relations, can reduce the complexity of information and provide more reliable decision-making results due to its closer proximity to reality. Therefore, using cross-efficiency and preference relations can be useful in performing better pairwise comparisons, especially in group decision-making. In this regard, the main purpose of this paper is to use intuitionistic fuzzy preference relations from fuzzy mathematics and the weighted SBM model of DEA for ranking and comparing. This paper is organized as follows: A literature review and some necessary backgrounds are presented in Section 2. Section 3 is devoted to some DEA preliminaries perspective on pairwise comparisons. Section 4 discusses a group decision-making algorithm based on fuzzy intuitive preference relation. In Section 5 a numerical example is presented and the results are compared to CCR and BCC Models. Finally, the last section is devoted to some concluding remarks.

2. Literature review

In the 1960s, by introducing the concept of membership degrees, Lotfi Zadeh paved the way for a more precise and mathematical representation of vague concepts. He introduced a new logic, that allows elements of a set to be belong to the set with a membership degree between 0 and 1, and called it fuzzy logic. Building upon fuzzy logic, the concept of fuzzy sets emerged and this concept has became a powerful tool for modeling complex and uncertain phenomena in various fields, including engineering, medicine, and economics.

After that, the concept of intuitionistic fuzzy sets (IFS) was introduced as an extension of the fuzzy sets, by defining three components: membership degree, non-membership degree, and hesitation degree. Atanassov also pointed out that in many cases, in addition to the membership degree, the non-membership degree is also important; in the other words, there may be hesitation about the membership of an element. By adding the hesitation degree component, IFS significantly enhanced the ability to model ambiguity. Indeed, grasping the concept of flexibility and application of intuitionistic fuzzy sets, to address, ambiguity and uncertainty, is much simpler compared to fuzzy sets. Atanassov and Gargov [1] also generalized the IFS and defined the concept of an interval-valued intuitionistic fuzzy set (IVIFS), which enhances greatly the representation ability of uncertainty.

Intuitionistic Fuzzy Sets (IFS) have been widely applied to Multi-Attribute Decision Making (MADM) and Multi-Attribute Group Decision Making (MAGDM) problems. After that, Xu et al. [2], introduced the concept of IFPRs by integrating IFSs and preference relations. This concept provided a powerful tool for modeling decision-making preferences under uncertainty and vagueness. Wang et al. [3], defined intuitionistic fuzzy weight vectors concept and also addressed how to optimize the decision-makers weights to achieve an optimal solution. In the sequence Zhang et al. [4], also presented a study on deriving priority weights from intuitionistic multiplicative preference relations under group decision-making settings. In the sequence, we know that Analytic Hierarchy Process (AHP) [5] is a popular and effective tool for multi-criteria decision-making (MCDM). AHP facilitates pairwise comparisons of criteria and alternatives by breaking down complex problems into hierarchical structures. It is considered superior to other MCDM methods due to its ease of use and ability to calculate both criteria weights and alternative priorities.

Research on fuzzy preference relations encompasses various dimensions, including consistency, transitivity, weighting, and dealing with incompleteness. These studies provide valuable insights into effectively managing and utilizing fuzzy preference relations in decision-making processes. By considering these dimensions, fuzzy preference relations can be effectively applied to complex decision-making problems for achieving more accurate results. Meng et al. [6] proposed group decision making with intuitionistic triangular fuzzy preference relations (ITFPRs); they first introduced the concept of ITFPR and in the second stage, the concept of collective consistency for ITFPRs was presented. They also proposed a programming model ITFPRs for checking the consistency. It is worth to mention that Meng et al. [7] conducted research on the use of the advantages of linguistic intuitive fuzzy sets and preference relations, (LIFPRs) and studies their application in decision-making. Also, Ren et al. [8] published a paper on intuitive fuzzy sets to help decision-makers interpret and apply them in group decision-making. They show that supporting information, by intuitive fuzzy concepts, enabling decision-makers to express their evaluations and ensure effective scientific decision-making. Furthermore, Yang et al. [9] presented a new method for deriving intuitionistic fuzzy priority vectors (IFPVs) from intuitionistic fuzzy preference relations (IFPRs).

Besides, over the past four decades, we have witnessed the extending of DEA in the field of measuring and evaluating the performance of organizations. This approach has taken a new step towards measuring the efficiency and effectiveness of organizations and has opened a new window for improving their performance. One of the remarkable achievements of this approach is the invention of the cross-efficiency model for ranking efficient units. This model, with its high discrimination power, is able to accurately distinguish between efficient and inefficient units. For instance, Doly and Green [10] introduced two benevolent and aggressive cross-efficiency models, which were a turning point in this field; Liu et al. [11], took a new step towards ranking units by using the cross-efficiency method.

Liu et al. [12] proposed a novel method for group decision-making that utilizes both data envelopment analysis (DEA) and stochastic weight space to rank multiple alternatives. The primary advantage of this approach is that it reduces the complex adjustments and achieving complete consensus among decision-makers, empowering decision-makers to select their options with greater confidence. Moreover, Liu et al. [13], proposed a new group decision making (GDM) method with probabilistic hesitant fuzzy preference relations (PHFPRs) based on a modified probability calculation and cross-efficiency method in DEA. After that Liu et al. [14], also presented a novel group decision-making method with interval linguistic fuzzy preference relations (ILFPRs) to model decision-makers preferences with uncertainty; this method integrates cooperative game, ordinal consistency improvement algorithm, DEA cross-efficiency model. Also, Song et al. [15], proposed a group decision-making method based on a multiplicative DEA cross-efficiency and also stochastic acceptability analysis with hesitant fuzzy linguistic preference relations (HFLPRs), which can avoid distorted information results in decision making distortion and obtain more credible. Furthermore Fan et al. [16], introduced a novel method for evaluating the performance of (DMUs) and building upon the foundational theories of prospect theory and regret theory, Zhang et al. [17], introduced a novel approach to group decision-making that leverages multi-granularity probabilistic linguistic Z-numbers (PLZN); In the sequence, here, we rely on the significant capabilities of the weighted SBM model in DEA and the use of fuzzy intuitionistic preference relations to improve this method and present a new achievement in this field. It is worth noting that the weighted SBM model, in comparison with other basic DEA models (CCR and BCC), has better discrimination power in the field of the units inefficiency. This model, based on the strengthened auxiliary variables of the aggregate model, provides a more accurate analysis of units performance. In addition, the weighted SBM model is much robust under changing the measurement units; in other words, this model provides a fixed and reliable criterion for measuring efficiency that is not affected by changes in measurement units. Relying on these advantages, we apply this to rank efficient units based on intuitionistic fuzzy preference relations, to do group decision making.

3. Preliminaries and definitions

Decision-making is a specific process that includes forecasting, evaluating, comparing existing results and solutions, and making a definitive choice of a solution to achieve desired goals and etc., which is known as a multi-criteria problem. In the decision-making process, it is assumed that the set of possible decisions, *X*, is a finite set. There are some important concepts related to this set where are explained as follows.

Definition 1. *A multiplicative preference relation (MPR) on a set of options X is a matrix R where each element* r_{ij} *of R, represents the relative preference of option i over option j. R satisfies the following properties* [18]:

$$r_{ij} . r_{ji} = 1, r_{ii} = 1, r_{ij} \geq 0, i , j = 1, 2, ..., n ,$$

where $\boldsymbol{r_{ij}}$ represents the preference intensity of option $\boldsymbol{x_i}$ over $\boldsymbol{x_j}$ in *X*.

Definition 2. *A multiplicative preference relation (MPR) is considered as consistent if the preferences between options exhibit transitivity; that is, if option i is preferred to option j and option j is preferred to option k, then option i must also be preferred to option k. So, R satisfies the following properties* [19]*:*

$$r_{ik} \times r_{kj} = r_{ij} \text{ (i, j, k = 1, 2, …, n).}$$

Definition 3. *A normalized priority weight vector is the unique vector that satisfies all conditions specified in the definition. These conditions ensure that the vector, provides the best possible representation of the relative importance of the alternatives. So the vector w is a normalized priority weight vector of R iff* [20]:

$$r_{ij} = {w_i}/{w_j},\ i, j = 1, 2, \ldots, n,\ \mathrm{w}_i > 0,\ \sum\nolimits_{i=1}^{n} w_i = 1.$$

In order to express the decision maker uncertainty preferences, the interval of multiplicative preference relation (IMPR) R is introduced as follows:

$$R = (r_{ij})_{n\times n} = \begin{pmatrix} [1,1][r_{12}^-, r_{12}^+]\ldots[r_{1n}^-, r_{1n}^+] \\ [r_{21}^-, r_{21}^+][1,1]\ldots[r_{2n}^-, r_{2n}^+] \\ \ldots \\ [r_{n1}^-, r_{n1}^+][r_{n2}^-, r_{n2}^+]\ldots[1,1] \end{pmatrix},$$

that, $r_{ij}^-, r_{ij}^+ > 0$, $\boldsymbol{r_{ij}^-} < \boldsymbol{r_{ij}^+}$ and $\boldsymbol{r_{ij}^- r_{ij}^+ = 1}$, when for all *i*, *j*, $\boldsymbol{r_{ij}^- = r_{ij}^+}$ this relation is equivalent to a multiplicative preference relation.

Definition 4. *An interval multiplicative preference relation is called consistent where it satisfies the follow property* [18]:

$$r_{ik}^- r_{ik}^+ r_{kj}^- r_{kj}^+ = r_{ij}^- r_{ij}^+, (i, j, k = 1, 2, \ldots, n).$$

Based on to the above definition, an IMPR *R* is consistent iff:

$$\rho(r_{ik})\rho(r_{kj}) = \rho(r_{ij}), (i, j, k = 1, 2, \ldots, n).$$

that $\rho(r_{ij}) = \sqrt{r_{ij}^- \times r_{ij}^+}$.

Definition 5. *A specific set A, constitutes a refinement of the fuzzy set concept, where for each element within A, a degree of membership and a corresponding degree of non-membership are assigned. This dual valuation enables a more precise representation of ambiguity and uncertainty inherent in set membership. Therefore, we can define an intuitionistic fuzzy set A on the set X (which essentially is an upgraded version of a fuzzy set) as follows* [20]:

$$A = \left\{ \left\langle x, \mu_A(x), \nu_A(x) \right\rangle | x \in X \right\},$$

where $\mu_A: X \to [0,1]$ represents the membership degree and $\boldsymbol{\nu_A: X \to [0, 1]}$ is the non-membership degree of x in the set A, with the condition $0 \le \mu_A(x) + \nu_A(x) \le 1$.

In this regard, An intuitionistic fuzzy preference relation (IFPR) R on a set $\boldsymbol{X} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_n}\}$ is represented by a matrix $\boldsymbol{P} = (\boldsymbol{p_{ij}})_{\boldsymbol{n\times n}}$, where $\boldsymbol{p_{ij}} = \langle \boldsymbol{\mu_{ij}}, \boldsymbol{\nu_{ij}} \rangle$ is an intuitionistic fuzzy number, $\boldsymbol{\mu_{ij}}$ represents the degree of preference of $\boldsymbol{x_i}$ over $\boldsymbol{x_j}$, and $\boldsymbol{\nu_{ij}}$ represents the non-preference of $\boldsymbol{x_i}$ over $\boldsymbol{x_j}$ [21]. An intuitionistic fuzzy preference relation is said to be consistent if and only if

$$\mu_{ij}\mu_{jk}\mu_{ki} = \mu_{ik}\mu_{kj}\mu_{ji},\ (1-\nu_{ij})(1-\nu_{jk})(1-\nu_{ki}) = (1-\nu_{ik})(1-\nu_{kj})(1-\nu_{ji}).$$

We note that consistency in IFPRs is much important because:

Ensures that preference relations are logical and reliable,

Helps decision-makers to make more informed decisions,

Prevents contradictions from arising in the decision-making process.

Definition 6. *For an intuitionistic fuzzy number* $\boldsymbol{\alpha} = \langle \boldsymbol{\mu_\alpha}, \boldsymbol{\nu_\alpha} \rangle \in \boldsymbol{\psi}^+$, *the interval transform function* $\boldsymbol{\tau}: \boldsymbol{\psi}^+ \to \boldsymbol{\Omega}^+$ *is defined as follows:*

$$\tau(\alpha) = \tau(\langle \mu_\alpha, \nu_\alpha \rangle) = [\frac{\mu_\alpha}{1-\mu_\alpha}, \frac{1-\nu_\alpha}{\nu_\alpha}],$$

where $\psi^+ = \{\langle \mu_\alpha, \nu_\alpha \rangle : \mu_\alpha > 0, \nu_\alpha > 0, \mu_\alpha + \nu_\alpha \leq 1\}$ is a set of intuitionistic fuzzy numbers [11]. The interval transform function defined by Definition 6 can transform an intuitionistic fuzzy number into an interval value.

Example: According to Definition 6, if the interval transform function τ is applied as follows:

$$\tau(\alpha) = \tau((0.6, 0.3)) = [0.6/(1 - 0.6), (1 - 0.3)/0.3],$$

then we have:

$$\tau(\alpha) = [0.6/0.4, 0.7/0.3] = [1.5, 2.33].$$

Hence, the intuitionistic fuzzy number α = (0.6, 0.3) is transformed into the interval [1.5, 2.33].

Definition 7. *There is a score function that assigns a desirability value to each element of an intuitionistic fuzzy set; this value is determined by subtracting the degree of non-membership from the degree of membership*. *For any* $\alpha = \langle \mu_\alpha, \nu_\alpha \rangle$, *this score functions S is defined as follows* [22]:

$$S(\alpha) = \mu_\alpha - \nu_\alpha.$$

Theorem 1. *Let* $p = (p_{ij})_{n\times n} = (< \mu_{ij}, \upsilon_{ij} >)_{n\times n}$ *be an IFPR; then* $H = ([h_{ij}^-, h_{ij}^+])_{n\times n} = (\tau(p_{ij}))_{n\times n}$ *is an IMPR. Moreover, if* p *is a consistent multiplicative IFPR, then* $H = ([h_{ij}^-, h_{ij}^+])_{n\times n} = (\tau(p_{ij}))_{n\times n}$ *is a consistent IMPR* [11].

DEA perspective on pairwise comparisons

The primary objective of this research is to assess the relative efficiency of each IFPR option using the DEA model. In the decision-making process under uncertainty, sometimes the decision-maker can select an IFPR option from a set of proposed alternatives. In this regard, in the decision-making process, each IFPR option can be considered as an independent DMU, If the decision-maker believes that alternative x_i is better than x_j for any $k \in \{1, 2, ..., n\}$, the certainty degree that alternative x_i is preferred to x_k is greater than the certainty degree that alternative x_j is preferred to x_k, i.e., $\mu_{ik} \geq \mu_{jk}$, and the certainty degree that alternative x_i is non-preferred to x_k is smaller than the certainty degree that alternative x_j is non-preferred to x_k, i.e., $\nu_{ik} \leq \nu_{jk}$; hence based on the provided definitions, the following mathematical relationship holds:

$$p_{ik} = \langle \mu_{ik}, \nu_{ik} \rangle \geq p_{jk} = \langle \mu_{jk}, \nu_{jk} \rangle, \ \frac{\mu_{ik}}{1-\mu_{ik}} \geq \frac{\mu_{jk}}{1-\mu_{jk}}, \ \frac{1-\nu_{ik}}{\nu_{ik}} \geq \frac{1-\nu_{jk}}{\nu_{jk}}, \ \tau(p_{ik}) \geq \tau(p_{jk}).$$

Therefore, each row of an IFPR is corresponded to a DMU, includes input and outputs and any column of an IFPR represents the outputs. So we consider a hypothetical fuzzy intuitive input as follows:

$$p_0 = (\langle 0/5, 0/5 \rangle),$$

thus the relationships between inputs and outputs can be shown as below **Table 1**.

Table 1. Relationships between inputs and outputs.

	Output 1	**Output 2**	**…**	**Output n**	**Dummy input**
DMU 1	$\tau(p_{11})$	$\tau(p_{12})$	…	$\tau(p_{1n})$	$\tau(p_0)$
DMU 2	$\tau(p_{21})$	$\tau(p_{22})$	…	$\tau(p_{2n})$	$\tau(p_0)$
…	…	…	…	…	…
DMU N	$\tau(p_{n1})$	$\tau(p_{n2})$	…	$\tau(p_{nn})$	$\tau(p_0)$

4. Optimal fuzzy intuitive weights based on cross-efficiency

We utilize the SBM model for better inefficiency detection and more accurate weight assignment [23]. Since SBM model deals with input surpluses and output shortfalls, it can identify inefficiencies more effectively than the CCR and BCC model. Although, SBM and CCR model are two common models in DEA, SBM considers both input slacks and output shortfalls simultaneously, while the CCR model only focuses on output shortfalls. Due to the consideration of input slacks, the SBM model tends to provide lower efficiency values compared to the CCR model.

Theorem 2. *The efficiency value in the optimal SBM solution is not greater than the efficiency value in the optimal CCR solution* [24].

4.1. SBM and weighted SBM models

The SBM and the weighted SBM (WSBM) model are two useful models in DEA used to evaluate the efficiency of DMUs [25]. The main difference between them lies in the consideration of inputs and outputs as:

- SBM model assumes that all inputs and outputs are equally important.
- WSBM model allows to assign different weights to inputs and outputs.

In this regard, WSBM model has some applicable advantages in which the main of them could be listed as follows:

- Flexibility: since WSBM allows user to assign different weights to inputs and outputs, it can help user to reflect reality more accurately.
- Realism: WSBM model the production process more accurately, since it allows to consider the relative importance of different factors.
- Interpretability: WSBM can help to interpret DEA results better, since one can see how different weights affect the results.

One of the primary limitations of traditional DEA methods is the inability to rank efficient DMUs, even researchers have proposed various methods for this ranking. In this study, we initially introduce the cross-efficiency method, which is used to rank efficient DMUs; where the performance of a DMU is compared against the optimal weights of other units. Based on the efficiency obtained from the WSBM model, now for IFPR, we determine two new cross-efficiency models that are inspired by the benevolent and aggressive DEA cross-efficiency models, as follows:

$$Max \sum_{j=1}^{n}\sum_{r=1}^{n} u_{rk} h_{jr}^{-},$$

S.to: $\nu_k .n = 1;$

$$\sum\nolimits_{r=1}^{n} u_{rk} h_{kr}^{+} - \alpha_{kk}^{*} \nu_k = 0;$$

$$\sum\nolimits_{r=1}^{n} u_{rk} h_{kr}^{+} - \nu_k \leq 0, \quad j \neq k, \quad j = 1, 2, ..., n; \tag{1}$$

$$\nu_k , u_{rk} \geq 0, \ k = 1, 2, ..., n.$$

$$Min \sum_{j=1}^{n}\sum_{r=1}^{n} u_{rk} h_{jr}^{-},$$

S.to: $\nu_k \times n = 1;$

$$\sum\nolimits_{r=1}^{n} u_{rk} h_{kr}^{+} - \alpha_{kk}^{*} \nu_k = 0;$$

$$\sum\nolimits_{r=1}^{n} u_{rk} h_{kr}^{+} - \nu_k \leq 0, \quad j = 1, 2, ..., n; \ j \neq k; \tag{2}$$

$$\nu_k , u_{rk} \geq 0, \ k = 1, 2, ..., n.$$

In models (1) and (2) α_{kk}^{*} is the efficiency score obtained from the WSBM model**.** To calculate the upper and lower bounds of cross- efficiency, one may obtain optimal weights $u^{*} = (u_{1k}^{*}, u_{2k}^{*}, \dots, u_{nk}^{*})^{T}$ and υ_k^{*} for output and input by solving the above models, respectively; hence, using these optimal weights, one can calculate the following amounts [11]:

$$\theta_{jk}^{-} = \sum_{r=1}^{n} u_{rk}^{*} h_{jr}^{-} \Big/ \upsilon_k^{*}, \theta_{jk}^{+} = \sum_{r=1}^{n} u_{rk}^{*} h_{jr}^{+} \Big/ \upsilon_k^{*}, j = 1,2, \dots, n \tag{3}$$

where the upper and lower bounds of cross-efficiency are shown by θ_{jk}^{+} and θ_{jk}^{-}. Therefore, based on Liu et al [11]. we determine:

$$\Theta = (\theta_{jk})_{n\times n} = \begin{pmatrix} [\theta_{11}^{-}, \theta_{11}^{+}][\theta_{12}^{-}, \theta_{12}^{+}] \dots [\theta_{1n}^{-}, \theta_{1n}^{+}] \\ [\theta_{21}^{-}, \theta_{21}^{+}][\theta_{22}^{-}, \theta_{22}^{+}] \dots [\theta_{n2}^{-}, \theta_{n2}^{+}] \\ \dots \\ [\theta_{n1}^{-}, \theta_{n1}^{+}][\theta_{n2}^{-}, \theta_{n2}^{+}] \dots [\theta_{nn}^{-}, \theta_{nn}^{+}] \end{pmatrix} \tag{4}$$

For inconsistent IFPRs, we propose a novel method for obtaining the normalized fuzzy intuitive priority vector of IFPR. Now, based on the following relationship, we are able to calculate the average cross-efficiency value as well:

$$\theta_j = [\theta_j^-, \theta_j^+] = [\frac{1}{n}\sum_{k=1}^{n} \theta_{jk}^-, \frac{1}{n}\sum_{k=1}^{n} \theta_{jk}^+], j = 1,2,\dots,n. \quad (5)$$

Regarding this average cross-efficiency value, the fuzzy intuitive priority weight vector can be demonstrated as:

$$\omega_j^{\mu} = \frac{\theta_j^-}{\psi}, \omega_j^{v} = \frac{1}{\psi}\sum_{r=1, r\neq j}^{n} \theta_r^+, j = 1,2,\dots,n, \quad (6)$$

where $\psi = \sum_{r=1}^{n} \theta_r^+ + \frac{1}{n-2} max\{\theta_k^+ - \theta_k^-\}$; then, the following important result can be presented in theorem 3.

Theorem 3. *For each IFPR, the obtained fuzzy intuitive priority weight vector from the above relationship, is normalized* [11].

4.2. A group decision-making algorithm based on IFPR

Let D be the set of decision makers. Regarding the above discussions a new group decision-making process based on DEA cross-efficiency by SBM weight-dependent model and using fuzzy intuitive preference relations is performed as an algorithm as follows:

Step 1: Convert $P^{(L)} = \left(\langle \mu_{ij}^{(L)}, v_{ij}^{(L)} \rangle\right)_{n\times n}$ to $H^{(l)} = \left([h_{ij}^{(l),-}, h_{ij}^{(l),+}]\right)_{n\times n}$ using the interval transform function (6).

Step 2: Obtain the optimal weight vector $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)$ by solving the following goal programming model [26]:

$$Min\ J = \sum_{i=1}^{n}\sum_{j=i+1}^{n}\sum_{l=1}^{m}[\varepsilon_{ij}^{(l),+}, \varepsilon_{ij}^{(l),-}, \gamma_{ij}^{(l),+}, \gamma_{ij}^{(l),-}],$$

S.to: $h_{ij}^{(l),-} - \sum_{l=1}^{m} \lambda \ln h_{ij}^{(l),-} - \varepsilon_{ij}^{(l),+} + \varepsilon_{ij}^{(l),-} = 0, i < j, i,j = 1,2,\dots,n\ l = 1,2,\dots,m;$

$$h_{ij}^{(l),+} - \sum_{l=1}^{m} \lambda \ln h_{ij}^{(l),+} - \gamma_{ij}^{(l),+} + \gamma_{ij}^{(l),-} = 0, i < j, i,j = 1,2,\dots.n, l = 1,2,\dots,m \quad (7)$$

$$\sum_{l=1}^{m} \lambda_l = 1;$$

$$\lambda_l, \varepsilon_{ij}^{(l),+}, \varepsilon_{ij}^{(l),-}, \gamma_{ij}^{(l),+}, \gamma_{ij}^{(l),-} \geq 0, i < j, i,j = 1,2,\dots,n, l = 1,2,\dots,m.$$

For better expressing the concept, consider the goal function J; this function shows group consensus and can measure the agreement between individual preference relations and the synthetic preference relation. In this regard, the optimal value J^* and $\varepsilon_{ij}^{(l),+}, \varepsilon_{ij}^{(l),-}, \gamma_{ij}^{(l),+}, \gamma_{ij}^{(l),-}$ are deviation variables in model (7) [26].

Step 3: Prepare IFPRs by all *m* decision-makers of *D*, which aggregated into combined interval preference relation; that $H^{(l)} = \left([h_{ij}^{(l),-}, h_{ij}^{(l),+}]\right)_{n\times n}$ and $H^{(s)} = \left([h_{ij}^{(s),-}, h_{ij}^{(s),+}]\right)_{n\times n}$ are transformed into combined preference relations according to the following formula:

$$h_{ij}^{s,-} = \prod_{l=1}^{m} (h_{ij}^{(l),-})^{\lambda_l}, h_{ij}^{s,+} = \prod_{l=1}^{m} (h_{ij}^{(l),+})^{\lambda_l}, i, j = 1, 2, ..., n. \tag{8}$$

Step 4: For each IMPR, obtain normalized fuzzy intuitive priority weight vector based on the IFPR ranking method in subsection 4.1, which is shown by $\omega_i = \langle \omega_i^{\mu}, \omega_i^{\nu} \rangle$.

Step 5: Rank in descending order, the normalized fuzzy intuitive priority weight vector ω_i by using the score function.

Step 6: Rank alternative x_i according to the ranking of ω_i.

5. Numerical simulation

In the context of economic knowledge, market competition is not only competition between companies, but also competition between supply chain alliances as well. Indeed supply chain is a complex system interconnected by production and organizations centers; the most evaluation for supply chain, includes level of customer service, financial status, level of innovation and flexibility of production. The evaluation process is somewhat subjective because experts may not be familiar enough with all the situations of supply chains that they evaluated. Therefore, it is appropriate for decision makers to use IFPR for their evaluation information. In the following example, five decision makers who provide their IFPRs on four supply chains, is considered. After pairwise comparisons, suppose that the decision makers provide the following five IFPRs (See for example Liu et al. [11]).

$$P^{(1)} = \begin{pmatrix} \langle 0.50, 0.50 \rangle \langle 0.50, 0.20 \rangle \langle 0.70, 0.10 \rangle \langle 0.50, 0.30 \rangle \\ \langle 0.20, 0.50 \rangle \langle 0.50, 0.50 \rangle \langle 0.60, 0.20 \rangle \langle 0.30, 0.60 \rangle \\ \langle 0.10, 0.70 \rangle \langle 0.20, 0.60 \rangle \langle 0.50, 0.50 \rangle \langle 0.30, 0.60 \rangle \\ \langle 0.30, 0.50 \rangle \langle 0.60, 0.30 \rangle \langle 0.60, 0.30 \rangle \langle 0.50, 0.50 \rangle \end{pmatrix};$$

$$P^{(2)} = \begin{pmatrix} \langle 0.50, 0.50 \rangle \langle 0.60, 0.20 \rangle \langle 0.60, 0.20 \rangle \langle 0.60, 0.30 \rangle \\ \langle 0.20, 0.60 \rangle \langle 0.50, 0.50 \rangle \langle 0.50, 0.20 \rangle \langle 0.30, 0.50 \rangle \\ \langle 0.20, 0.60 \rangle \langle 0.20, 0.50 \rangle \langle 0.50, 0.50 \rangle \langle 0.20, 0.60 \rangle \\ \langle 0.30, 0.60 \rangle \langle 0.50, 0.30 \rangle \langle 0.60, 0.20 \rangle \langle 0.50, 0.50 \rangle \end{pmatrix};$$

$$P^{(3)} = \begin{pmatrix} \langle 0.50, 0.50 \rangle \langle 0.55, 0.25 \rangle \langle 0.65, 0.20 \rangle \langle 0.35, 0.55 \rangle \\ \langle 0.25, 0.55 \rangle \langle 0.50, 0.50 \rangle \langle 0.40, 0.25 \rangle \langle 0.55, 0.30 \rangle \\ \langle 0.20, 0.65 \rangle \langle 0.25, 0.40 \rangle \langle 0.50, 0.50 \rangle \langle 0.60, 0.20 \rangle \\ \langle 0.55, 0.35 \rangle \langle 0.30, 0.55 \rangle \langle 0.20, 0.60 \rangle \langle 0.50, 0.50 \rangle \end{pmatrix};$$

$$P^{(4)} = \begin{pmatrix} \langle 0.50, 0.50\rangle\langle 0.60, 0.30\rangle\langle 0.70, 0.20\rangle\langle 0.60, 0.30\rangle \\ \langle 0.30, 0.60\rangle\langle 0.50, 0.50\rangle\langle 0.50, 0.40\rangle\langle 0.30, 0.40\rangle \\ \langle 0.20, 0.70\rangle\langle 0.40, 0.50\rangle\langle 0.50, 0.50\rangle\langle 0.30, 0.60\rangle \\ \langle 0.30, 0.60\rangle\langle 0.40, 0.30\rangle\langle 0.60, 0.30\rangle\langle 0.50, 0.50\rangle \end{pmatrix};$$

$$P^{(5)} = \begin{pmatrix} \langle 0.50, 0.50\rangle\langle 0.60, 0.30\rangle\langle 0.75, 0.15\rangle\langle 0.60, 0.20\rangle \\ \langle 0.30, 0.60\rangle\langle 0.50, 0.50\rangle\langle 0.45, 0.20\rangle\langle 0.60, 0.20\rangle \\ \langle 0.15, 0.75\rangle\langle 0.20, 0.45\rangle\langle 0.50, 0.50\rangle\langle 0.40, 0.40\rangle \\ \langle 0.20, 0.60\rangle\langle 0.20, 0.60\rangle\langle 0.40, 0.40\rangle\langle 0.50, 0.50\rangle \end{pmatrix}.$$

More than simply implementing, in this example, we intend to have a numerical comparison between CCR and BCC and weighted SBM method. To solve this example, we follow the steps according to the presented algorithm in the previous section.

Step1: We use the interval transform function (Definition 6) $\tau(p_{ij}^{(l)}) = \left[h_{ij}^{(l),-}, h_{ij}^{(l),+}\right] = \left[\frac{\mu_{ij}^{(l)}}{1-\mu_{ij}^{(l)}}, \frac{1-\upsilon_{ij}^{(l)}}{\upsilon_{ij}^{(l)}}\right]$ to achieve:

$$\mathrm{H}^{(1)} = \begin{bmatrix} [1.0,1.0][1.0,4.0][2.33,9.0][1.0,2.33] \\ [0.25,1.0][1.0,1.0]\ [1.50,4.0]\ [0.4286,0.66] \\ [0.11,0.4286][0.25,0.66,][1.0,1.0][0.4286,0.66] \\ [0.4286,1.0]\ [1.50,2.33][1.50,2.33][1.0,1.0] \end{bmatrix},$$

$$\mathrm{H}^{(2)} = \begin{bmatrix} [1.00,1.00][1.50,4.00][1.50,4.00][1.50,2.33] \\ [0.25,0.66][1.00,1.00][1.00,4.00][0.4286,1.00] \\ [0.25,0.66][0.25,1.00][1.00,1.00][0.25,0.66] \\ [0.4286,0.66][1.00,2.33][1.50,4.00][1.00,1.00] \end{bmatrix},$$

$$\mathrm{H}^{(3)} = \begin{bmatrix} [1.00,1.00][1.22,3.00][1.8571,4.00][0.5358,0.8182] \\ [0.33,0.8182][1.00,1.00][0.66,3.00][1.22,2.33] \\ [0.25,0.5358][0.33,1.50][1.00,1.00][1.50,4.00] \\ [1.22,1.8571][0.4286,0.8182][0.25,0.66][1.00,1.00] \end{bmatrix},$$

$$\mathrm{H}^{(4)} = \begin{bmatrix} [1.00,1.00][1.50,2.33][2.33,4.00][1.50,2.33] \\ [0.4286,0.66][1.00,1.00][1.00,1.50][0.4286,1.50] \\ [0.25,0.4286][0.66,1.00][1.00,1.00][0.4286,0.66] \\ [0.4286,0.66][0.66,2.33][1.50,2.33][1.00,1.00] \end{bmatrix},$$

$$\mathrm{H}^{(5)} = \begin{bmatrix} [1.00,1.00][1.50,2.33][3.00,5.66][1.50,4.00] \\ [0.4286,0.66][1.00,1.00][0.8182,4.00][1.50,4.00] \\ [0.1756,0.33][0.25,1.22][1.00,1.00][0.66,1.50] \\ [0.25,0.66][0.25,0.66][0.66,1.50][1.00,1.00] \end{bmatrix}.$$

Step 2: By using the GAMS software Distribution 24.8.2 to solve the Goal programming model (7), we obtain the optimal weight vector for the five decision makers as follows:

1) 0.08829066, 2) 0.36881986, 3) 0.09415188, 4) 0.26568974, 5) 0.18304786.

Step 3: Now, we introduce the combined interval preference relation by (8) as follows:

$$H^S = \begin{pmatrix} [1.0000, 10000][1.4196, 3.0567][2.0316, 4.5796][1.3142, 2.3333] \\ [0.3271, 0.7044][1.0000, 1.0000][0.9615, 3.0000][0.5949, 1.5000] \\ [0.2184, 0.4922][0.3333, 1.0399][1.0000, 10000][0.4286, 0.9154] \\ [0.4286, 0.7609][0.6667, 1.6809][1.0924, 2.3333][1.0000, 1.0000] \end{pmatrix}.$$

Step 4: First, we use the DEA-Solver software to calculate the efficiencies under the weighted SBM, CCR and BCC models (DEA-Solver 13 was utilized for efficiency analyses in this study). In **Table 2**, it can be observed that as we expect the SBM model identifies inefficiencies better than CCR and BCC model, which leads to a more logical ranking in the end.

Table 2. Efficiency based on BCC, CCR, and WSBM

Variables	Efficiency CCR-O	Efficiency BCC-O	Efficiency WSBM
A	1	1	1
B	0.5568	0.5568	0.5118
C	0.3577	0.3577	0.3217
D	0.5711	0.5711	0.542

It is worth mentioning that generally WSBM model exhibited a higher CPU time (mean = 1.5 s) compared to the CCR (mean = 1 s) and BCC (mean = 1 s) models, due to its non-radial nature and slack-based calculations. Moreover, the algorithm demonstrates scalability, with CPU time increasing linearly with problem size.

In **Table 3**, by applying the efficiency score obtained from the WSBM model, one can solve the cross-efficiency model (1) and obtain the optimal weights for outputs and input as follow:

Table 3. Optimal weights

$u_{11}^* = 0.25$	$u_{12}^* = 0.1223$	$u_{13}^* = 0.0943$	$u_{14}^* = 0.0853$	$v_1^* = 0.25$
$u_{21}^* = 0.0$	$u_{22}^* = 0.0417$	$u_{23}^* = 0.0$	$u_{24}^* = 0.0$	$v_2^* = 0.25$
$u_{31}^* = 0.0$	$u_{32}^* = 0.0$	$u_{33}^* = 0.0339$	$u_{34}^* = 0.0$	$v_3^* = 0.25$
$u_{41}^* = 0.0$	$u_{42}^* = 0.0$	$u_{43}^* = 0.0$	$u_{44}^* = 0.0705$	$v_4^* = 0.25$

Now, we calculate the cross-efficiency matrix (4) using MATLAB R2024a software as follows:

$$\Theta = \begin{bmatrix} [1.0000,1.0000] & [0.7266,1.0000] & [0.6536,1.0000] & [0.7123,1.0000] \\ [0.3271,0.7044] & [0.3271,0.5118] & [0.2542,0.6737] & [0.2796,0.6639] \\ [0.2184,0.4922] & [0.1626,0.4146] & [0.2184,0.3217] & [0.1955,0.4264] \\ [0.4286,0.7609] & [0.3211,0.6232] & [0.3103,0.6044] & [0.4286,0.5420] \end{bmatrix}.$$

Then, the average cross-efficiency values by (5) are determined as:

$$\theta_1 = [0.773125,1.0000],\ \theta_2 = [0.297,0.63845],$$

$$\theta_3 = [0.198725,0.413725],\ \theta_4 = [0.37215,0.632625].$$

Based on the above values, the fuzzy intuitive priority weight vector (6) can be calculated as follows:

$$\omega_1 = <0.2707, 0.5900>,\ \omega_2 = <0.1040, 0.7166>,\ \omega_3 = <0.0695, 0.7953>,\ \omega_4 = <0.1303, 0.7186>.$$

Step 5: According to the definition of the score function, the rank of normalized fuzzy intuitive priority vector is $\omega_1 > \omega_4 > \omega_2 > \omega_3$. We remind that it is the same ranking using the CCR and BCC model.

Step 6: The final ranking of 4 supply chain options is A1 > A4 > A2 > A3. Therefore, it is concluded that A1 is the best supply chain.

In **Table 4**, based on the obtained results of this example, application of the BCC, CCR, and WSBM DEA models is compered**.** As can be seen, the CCR and BCC models for the mentioned fuzzy situation determine the same efficiency and ranking. But overall, the proposed ranking method, shows that using the weighted SBM model provides more accurate efficiency than both. Furthermore, due to the more precise inefficiency detection of the weighted SBM model, more accurate weights also are obtained when using the cross-efficiency. Finally, obtained the normalized intuitionistic fuzzy weight vector was using the proposed method Causes that WSBM model penalizes units with weaknesses in specific input/output improvements more heavily due to its focus on reducing absolute inefficiencies (rather than relative ones). In contrast, CCR/BCC models may overlook such inefficiencies.

Table 4. Comparison of models BCC, CCR, and WSBM.

Models	**CCR**	**BCC**	**WSBM**
Fuzzy intuitive priority weight vector	ω1 = <0.2798,0.5975>, ω2 = <0.1036,0.7164>, ω3 = <0.0694,0.7929>, ω4 = <0.1318,0.7133>.	ω1 = <0.2798,0.5975>, ω2 = <0.1036,0.7164>, ω3 = <0.0694,0.7929>, ω4 = <0.1318,0.7133>.	ω_1 = <0.2707,0.5900>, ω_2 = <0.1040,0.7166>, ω_3 = <0.0695,0.7953>, ω_4 = <0.1303,0.7186>.
Ranking of the four-supply chain	A1 > A4 > A2 > A3	A1 > A4 > A2 > A3	A1 > A4 > A2 > A3

Therefore, due to its more accurate detection of inefficiencies, it can be used in other financial sectors, such as banks. Banks due to the nature of their activities, banks are exposed to significant risks in financial markets, such as credit, liquidity, and operational risks. Therefore, the fragile and high-risk nature of the banking sector necessitates periodic monitoring, measurement, and evaluation of their' performance. Additionally, a reliable and robust performance evaluation system helps banks objectively assess their operational results, make faster and more effective decisions, and enables them to achieve sustainable competitive advantages and create long-term value for stakeholders [27]. Thus, one of the sequence research could be applying such a proposed algorithm with the WSBM model to evaluate banks.

6. Conclusion

To purpose a new group decision making based on cross-efficiency in DEA with fuzzy intuitive preference relations, an interval conversion function is first defined.

This function establishes an important relationship between IFPRs and IMPRs that can ensure the consistency of the relationship. Then, the SBM weighted model based on the interval transform function is determined and applied to obtain the ranking vector of consistent IFPRs, so that each alternative is considered as a DMU. It is shown that this model can identify inefficiencies better than the CCR and BCC model for intuitionistic fuzzy data. Based on the interval conversion function, two DEA cross-efficiency models are also constructed to obtain efficiency values and evaluate alternatives as well; after that, normalized fuzzy intuitive preference weight vector for each IFPR is illustrated based on the obtained cross-efficiency values. In the last step, a new way of group decision making based on cross-efficiency with IFPRs is proposed, which does not require consistency and can act as a normalized fuzzy intuitive preference weight vector. Specially, the proposed method is more applicable and can provide more reasonable results than some known methods when the IFPRs provided by decision makers, have poor consistency. This study compared the performance of DEA models CCR, BCC, and WSBM in evaluating efficiency in this process. While both CCR and BCC yielded identical efficiency scores and rankings, the WSBM model is able potentially for fairer ranking of DMUs. This finding emphasizes the importance of the distinguishing technical and managerial efficiency for a comprehensive understanding of performance. In addition, the proposed method can be integrated with group decision making with hesitant fuzzy preference relations (HFPRs) and interval linguistic fuzzy preference relations (ILFPRs) [28] and intuitionistic triangular fuzzy preference relations [29], in future studies. Also the integration of the CODAS method with the fuzzy data [30], intuitionistic fuzzy data, and the WSBM model can create a powerful approach for efficiency evaluation and ranking of options in future studies under uncertainty and multi-criteria conditions.

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