

The structure of the Hungarian insurance market and the invariant distribution of market shares

Norbert Kovács^{1,*}, Dávid Fülep², István Á. Harmati², Krisztián Koppány¹, Lóránt Dénes Dávid^{3,4,5,6}

¹ Department of International and Applied Economics, Széchenyi István University, HU-9026 Győr, Hungary

² Department of Mathematics and Computational Sciences, Széchenyi István University, HU-9026 Győr, Hungary

³ Department of Tourism and Hospitality, Faculty of Economics and Business, John von Neumann University, HU-6000 Kecskemét, Hungary

⁴ Institute of Rural Development and Sustainable Economy, Department of Sustainable Tourism, Hungarian University of Agriculture and Life Sciences (MATE), HU-2100 Gödöllő, Hungary

⁵ Savaria Department of Business Economics, Savaria University Centre, Faculty of Social Sciences, Eötvös Loránd University, HU-9700 Szombathely, Hungary

⁶ Department of International and Applied Economics, Széchenyi István University, HU-9026 Győr, Hungary

* Corresponding author: Norbert Kovacs, kovacs@sze.hu

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Abstract: The Hungarian economy exhibits a notable underinsurance phenomenon, with insurance penetration at a mere 2.8%, significantly lower than the European Union average of 8%. This situation indicates substantial growth potential within the Hungarian insurance market, particularly in the life and non-life insurance sectors, contingent upon the development of solvent demand and favorable demand-stimulating factors. Anticipated transformations in the structure of the Hungarian insurance market may arise due to both endogenous and exogenous influences, likely resulting in heightened market concentration and alterations in competitive dynamics. This study aims to conduct an analysis of the historical and expected future transformations of the Hungarian insurance market structure by utilizing publicly available data on gross premium income. The analysis employs traditional market structure indicators, such as market shares, concentration ratios, and the Herfindahl-Hirschman Index (HHI), while also examining market share transitions through the application of the Markov chain method. Markov transition probabilities offer a more accurate representation of historical market structure processes compared to conventional market structure indicators. Furthermore, the calculation of these transition probabilities facilitates the prediction of anticipated future changes in market shares. The stationary (ergodic) distribution of market shares, derived from the transition probability matrix, denotes a market share distribution toward which the market converges under stable conditions. This approach also enables the computation of an equilibrium market share distribution achievable in the future under specified conditions, driven by the internal mechanisms of the market. The analysis reveals an upward trend in the market shares of larger companies and an increase in market concentration across both the life and non-life insurance sectors in Hungary. Traditional methods of indirect measurement indicate a prospective rise in market concentration and a potential decline in competitive conditions. However, when considering stationarity, the invariant distributions estimated via the Markov chain methodology suggest a decrease in the market shares of the largest companies, accompanied by a leveling effect among leading firms. This indicates that, assuming unchanged conditions over the past decade, the intrinsic processes of the market could lead to a less concentrated market structure in both the life and non-life insurance sectors of the Hungarian insurance market. Removing the stationarity assumption presents new opportunities for determining the equilibrium state of the insurance market under specific conditions. Future research will venture further in this direction. The objective is to develop a model capable of indirectly measuring market power, which will provide essential insights for

competition authorities and management of market participants, even within asymmetric information contexts, regarding the anticipated trajectory of market structure transformation.

Keywords: market share; market concentration; competition; markov chains; insurance; equilibrium; ergodic distribution

1. Introduction

The study of public measurement of market power has a long history dating back about 150 years to the inception of the first competition law regulations and literature on the subject (Motta, 2004). There is extensive literature on this topic, with notable contributions from various authors (Arrow, 1971; Bikker and Leuvensteijn, 2005; Cummin 2001; Cummins and Weiss, 2000; Denny 1980; Douven et al., 2007; Hirschhorn-Geehan, 1977; Schmalensee and Willig, 1989; Slade 1986; Stigler and Sherwin, 1985; Tirole, 1988).

Measuring market power involves three main steps: identifying the affected market, calculating market power indicators, and interpreting and evaluating the results. The SSNIP (Small but Significant Non-transitory Increase in Price) test, recommended by competition authorities, is commonly used to determine the relevant market. However, challenges arise in implementing this test fully due to statistical and interpretive obstacles and the need for comprehensive data elements. Furthermore, reconciling the boundaries of the relevant market with public governmental and supervisory databases poses a significant challenge. These databases are crucial for calculating market power indicators but are often limited by national geographic boundaries, potentially differing from the relevant market limits.

Due to market characteristics, issues related to indirect measurement of market power are more pronounced in the insurance market. The SSNIP test cannot be straightforwardly applied to insurance markets, primarily because of the nature of insurance services and the complexities of defining substitute products. Additionally, there are limitations in interpreting demand—and supply-side substitutability.

Interpreting insurance companies' output presents its own set of challenges, particularly in achieving consensus regarding the conceptual interpretation of the output and the data used in its measurement. Most authors rely on gross fee income for analysis, and while comparing results based on multiple data types is unique, it is necessary for comprehensive analysis (Fenn et al., 2004; Motta, 2004; Schmalensee, 1977).

Considering these factors, an indirect measurement of market power was implemented for the Hungarian insurance market from 2011 to 2021, treating insurance sectors as relevant product markets and using gross premium income as output. This analysis aimed to provide insights into market concentration and structure. Calculating market shares helped identify dominant positions and define market structure categories, thus indirectly shedding light on competitive conditions.

Traditional concentration indicators used in the indirect measurement of market power should accurately depict its stability. The pure Markov chain model was employed to gain a deeper understanding of the restructuring processes and dominant positions, particularly on the supply side of the market. This model further develops indirect measurement of market power and offers insightful results in the form of life

and non-life branch transition matrices and invariant distributions of market shares (Kovács, 2014).

2. Methodology for examining market structure

Examining the market structure is essential from the point of view of the competition supervision work, which, on the one hand, controls market competition and dominant positions of leaders and, on the other hand, supports the welfare effects of competition. On the other hand, it is essential to monitor the competitive environment from the market players' side and examine their position's stability.

From the point of view of economics, both approaches have a serious *raison d'être*, of course. On the one hand, the goal of economic policy and competition policy is to ensure the appropriate boundary conditions for the healthy functioning of the economy and, of course, the various industries. An essential part of this is the effect of the intensity of competition on quality and better customer service. On the other hand, we cannot forget that from a business and management point of view, examining the market environment and competitors within it is of particular importance. In other words, analysing and evaluating competition and its indirect background factors is a significant issue from both the state and the market/institutional side.

2.1. The meaning and significance of the market structure

The structure of the market is determined based on entry and exit barriers, the number of players on the supply and demand sides of the market, their market share, and changes in all of these. Entry and exit barriers can be divided into economic and legal barriers. Market shares are calculated based on market participants' total market turnover share on both the demand and supply sides, in money and/or volume.

This chapter presents the calculation methodology of traditional market structure analysis indicators, market share and concentration rates, and indices. We also discuss a novel interpretation that can show more about market competition by analysing the market structure based on the same data. By applying the stochastic model of Markov chains, we can more efficiently describe the state and transformation process of the characteristics of the market structure.

2.2. Market structure measurement and traditional methods

Numerous studies have examined various industries through a range of methodologies, systematically summarizing and applying these approaches within the existing literature (Jaehyung et al., 2020; Stepanova et al., 2024). This study is dedicated to the application of indirect methods for measuring market power and provides a comprehensive overview of the relevant scholarly works in this field. In the field of industrial organisation theory, key metrics such as the Herfindahl–Hirschman Index (HHI) and concentration ratios (CR) play a central role in assessing market concentration and identifying potential anti-competitive behaviour (Bikker and Haaf, 2002). The HHI is calculated by summing the squared market shares of individual firms, providing a measure of market concentration that indirectly reflects market power (Scherer and Ross, 1990). The CR(2) and CR(5) ratios measure the combined market share of the two or five most prominent firms, respectively, with increases in

these indicators signalling growing concentration and a potential reduction in competition (Hall and Tideman, 1967; Kwoka and White, 2004).

In the United States, competition law, specifically the Horizontal Merger Guidelines, sets official thresholds for the HHI. According to these guidelines, an HHI between 1000 and 1800 indicates moderate concentration, while an HHI above 1800 suggests high concentration, which could facilitate anti-competitive practices (U.S. Department of Justice and Federal Trade Commission, 2010). Additionally, the Sherman Act (1890) and the Clayton Act (1914) regulate monopolistic practices and control mergers through the application of concentration measures such as the HHI (Sherman Act; Clayton Act).

In the United Kingdom, competition law is governed by the Competition Act (1998) and the Enterprise Act (2002), which enable the assessment of market concentration and merger activity. The U.K. Competition and Markets Authority (CMA) 2021 Merger Guidelines provide detailed criteria for using HHI and C.R. metrics to evaluate merger competitive risks (UK Competition and Markets Authority (CMA), 2021).

Similarly, in Germany, the Gesetz gegen Wettbewerbsbeschränkungen (GWB) outlines concentration measures, including the HHI, in merger control proceedings. The German competition authority, the Bundeskartellamt, provides comprehensive guidelines on how HHI and CR measures should be applied in evaluating market concentration (Bundeskartellamt, 2012). In France, the Code de Commerce and the guidelines of the Autorité de la Concurrence govern the assessment of market concentration using HHI and CR ratios, although the thresholds applied in the French regulatory framework are more flexible (Autorité de la Concurrence, 2009; Code de Commerce, 2024).

At the European level, the European Commission's merger guidelines consider an HHI above 1800 to indicate high concentration, while an HHI between 1000 and 1800 reflects moderate concentration, warranting further investigation (European Commission, 2004). Rhoades (1993) underscores that while the HHI is widely accepted as a critical measure of market concentration, its application varies across regulatory frameworks. Cabral (2000) elaborates on how these metrics relate to market power and the identification of anti-competitive conduct, while Pindyck and Rubinfeld (2013) argue that both CR and HHI metrics are essential for detecting anti-competitive behaviour (Demsetz, 1982).

In conclusion, although the HHI and C.R. metrics are widely used in the literature and regulatory environments to measure market concentration, there must be a uniform consensus on the critical thresholds for competition-related concerns. While jurisdictions such as the United States and the European Union provide relatively clear guidelines, other countries, such as Germany and France, apply these measures more flexibly, depending on the specific characteristics of the market in question. There is no consensus on the threshold values of the indicators used to evaluate market power in international competition law practice. According to European competition law, the share of the largest company permanently above 25% already has a signal value and indicates dominance.

There are also several approaches to HHI thresholds. According to the most common approach in the literature, a value that remains permanently above 1800 basis

points in each market is considered critical. For the insurance market assessment of market concentration measured with the HHI, I will use the most common threshold values in the literature below, namely:

When the HHI exceeds 1800, the market concentration is deemed high. This value roughly corresponds to a CR(5) of 80 percent. In the context of competition law, such a high concentration allows market leaders to abuse their dominant position.

- if $1000 < \text{HHI} < 1800$, the market concentration is moderate. Then, regarding competition law, the strategic interactions of the companies require further investigation.
- if the HHI is < 1000 , the market concentration is low. $\text{HHI} = 1000$ equals CR(5) of approximately 60 per cent. Below this value, according to the competition authorities, there is no way to exercise dominance; therefore, the functioning of the market does not require intervention.

In the case of CR(2), 50%, while in the case of CR(5), 80%, can be considered critical threshold values with an indicative value. If the value of the named concentration ratios is above this, then this indicates the dominance of the leading companies. I will take these threshold values into account during further investigations and evaluation.

I used the insurance companies' annual gross premium income data for the calculations. The gross premium income can be interpreted as the quasi-gross sales income of the insurance companies, which must cover the risk premium part, the safety supplement, and the contractor's premium part in the case of non-life insurance, the risk premium part and the contractor's premium part in the case of life insurances. The risk premium part is intended to cover the loss amounts, the contractor premium part is intended to cover the insurance companies' profits, and the safety supplement premium part is intended to cover damage fluctuations. The latter, at least in a theoretical approach, only exists in the case of non-life insurances because life insurances—at least traditional life insurances—are so-called well-behaved insurances; that is, their risk (probability of damage occurring) can be calculated more accurately with the help of actuarial models based on mortality tables than the probability non-life insurances that require more calculation methods.

2.3. Markov transition matrices

The Markov chain model makes it possible to examine the stability of the market structure in a specific time interval with the help of insurance companies' market uses based on their gross premium income. We know that the traditional indirect indicators of market power are market effects, CR(2), CR(5), and HHI. According to the literature, if their value is high and stable over a specific period, then the leading companies have significant market power, and unfair abuse of their dominant position harms consumers. The high market power of the market-leading companies is indicated by the value of CR(2) permanently above 60% and CR(5) above 80%. If you think about it more, this is especially true if 60% and 80% always come from the stable market factors of the same two or five companies. Therefore, if the composition and order of the leading companies do not change, the dominant positions should be stable. Since the concentration indicators do not name the individual actors, during the time

series analysis of the concentration, we only see the evolution of the concentration value: increase, stagnation, or even decrease, not what happened in the background. Using the Markov chain model, the market stability of the leading companies can be examined, and the probabilities of transitions in positions and between positions can be calculated. The results, in this way, supplement the knowledge obtained with the help of traditional indirect indicators with significant additional information.

In the first step, it is necessary to determine how much and in which direction the predefined size categories changed during the examined period, for which we also determine the value of market measurements and concentration. The order of the first five companies is essential because the CR(2) and CR(5) ratios were calculated in the concentration analysis. I record the changes in the ranking of the top five in a 6×7 “frequency” matrix as follows:

size category	1.	2.	3.	4.	5.	0.	$\sum_{i=1}^0. \sum_{j=1}^0. d_{ij}$
1.	$d_{1.1}$	$d_{1.2}$				$d_{1.0}$	$\sum_{j=1}^0. d_{1,j}$
2.		$d_{2.2}$					$\sum_{j=1}^0. d_{2,j}$
3.	$d_{3.1}$	$d_{3.2}$				$d_{3.0}$	$\sum_{j=1}^0. d_{3,j}$
4.							$\sum_{j=1}^0. d_{4,j}$
5.							$\sum_{j=1}^0. d_{5,j}$
0.	$d_{0.1}$	$d_{0.2}$				$d_{0.0}$	$\sum_{j=1}^0. d_{0,j}$

I recorded the positions in the frequency matrix’s first column and first row. A Category 0 (ranking) is also listed here. Category 0, that is, the category of entrants and exits. The values in the diagonal of the frequency matrix show the number of times in the examined time interval that a company maintained its position in the next examined year. Specifically, for example, $d_{1.1}$ denotes the number of cases when the company belonging to the first category remained in the first place in the following year, and $d_{2.2}$ the number of cases when the company belonging to the second category second remained in the second category. In all other cases, we can interpret the values by moving from left to right in the rows of the matrix. Specifically, for example, $d_{1.2}$ represents the number of cases when the company belonging to the first size category returned to the 2nd in the following period.

In contrast, $d_{3.0}$ Represents the cases when the company previously belonging to the third category exited the market. The values in the last summary column record the number of all movements starting from a given position and the number of stays in that position. Specifically, for example, $\sum_{j=1}^0. d_{1,j}$ shows the observed aggregate frequency of staying in the first place and moving from the first place to other positions in the examined time interval.

Transitioning to the second phase of the analysis, we estimate a $P n \times n$ transition probability matrix from the frequency matrix. This step is crucial as it allows us to understand the likelihood of a company moving from one position to another in the examined time interval.

	<i>size category</i>	1.	2.	3.	4.	5.	0.	$\sum_{i=1}^0, \sum_{j=1}^0, p_{ij}$
$P^{6 \times 6} \equiv$	1.	$p_{1.1.}$					$p_{1.0.}$	1
	2.							1
	3.			$p_{3.3.}$				1
	4.							1
	5.							1
	0.	$p_{0.1.}$					$p_{0.0.}$	1

The formula $p_{i,j} = \frac{d_{i,j}}{\sum_{j=1}^0, d_{i,j}}$ represents the probability of transitioning from the i -th category to the j -th category, known as the transition probability. Specifically, $p_{0,1} = \frac{d_{0,1}}{\sum_{j=1}^0, d_{0,j}}$ denotes the likelihood that a company not initially belonging to the first five categories will transition to one of these categories in the subsequent year. The values along the diagonal represent the probabilities of remaining in the same position. For instance, $p_{1,1} = \frac{d_{1,1}}{\sum_{j=1}^0, d_{1,j}}$ signifies the probability of a company maintaining its first-place ranking from one period to the next. These transition probabilities can be leveraged to assess the stability of maintaining specific positions and the transitions between them. Higher values along the diagonal of the transition probability matrix indicate more excellent stability in individual positions, implying that the leading companies' positions are more secure. Conversely, lower diagonal values increase off-diagonal cell values, reflecting better communication and potential instability among positions. Analyzing the stability of the top five companies' positions over a considerable period signifies the stability of market power, potentially suggesting insufficient market competition. The mobility coefficient of the transition probability matrix is crucial for assessing market position stability and indirectly measuring market power. In the third phase of the analysis, we determine the mobility coefficient. In our case, the formula for calculating the mobility coefficient, denoted as $\mu(\hat{P}) = \frac{n - \sum_{i=1}^0, p_{ii}}{n}$, where n represents the number of categories under examination, and p_{ii} are the diagonal values of the transition probability matrix, indicating the probabilities of remaining in position. The mobility coefficient ranges from 0, signifying perfect immobility, to 1, representing perfect mobility. The mobility coefficient close to 1 indicates a lower likelihood of remaining in each category, suggesting a higher probability of position changes. This coefficient can be tailored to quantify the combined mobility of the positions of the first five companies or specific subsets, such as the first two positions. It is a valuable tool for assessing market position stability, and it is unnecessary to consider all values when calculating the mobility coefficient. The mobility coefficient can effectively gauge the stability of individual market positions and the degree of market power.

The Markov chain model is widely used in insurance markets to assess market stability and predict transitions between competitive positions. Klugman et al. (2012) discuss the model's application in insurance pricing, highlighting its utility in tracking market position changes over time. This approach complements traditional metrics such as CR(2), CR(5), and the Herfindahl-Hirschman Index (HHI) by providing additional insights into the likelihood of a firm maintaining or losing its market position. Marker (1998) specifically applied Markov chains to policy retention,

demonstrating its effectiveness in capturing the persistence of market leaders and transitions between them.

Norberg (2002) further explores the model's flexibility, emphasizing its capacity to adapt to various market structures, including those exhibiting stability or volatility. Pantelous and Passalidou (2015) extend this by applying the model to competitive insurance markets, where it helps quantify firms' movement across market segments and assess the stability of dominant firms. Wu (2017) integrates Markov chains with game theory, illustrating how the model can forecast shifts in market power and competitive dynamics. Overall, the Markov chain model offers a robust framework for analyzing market dynamics and providing deeper insights into the stability of market leaders beyond traditional concentration measures.

2.4. The importance of the ergodic or otherwise invariant distribution

The model's other potential application is predicting the future distribution of insurance companies across specific size categories, thereby forecasting market structure transformation processes. The stationary or invariant distribution, which is a stable distribution towards which the stochastic process converges, is validated for all Markov chains (Stokey et al., 1989). Any probability distribution $\bar{v} \in \Delta^n$ for which $P(\xi_n = j) = P(\xi_0 = j), \forall j \in X, \forall n \in N$ exists are called invariant or stationary distributions.

An invariant distribution can be defined as follows: let P be an $n \times n$ matrix. In this case, the number k is regarded as an eigenvalue of matrix P if there is a non-zero $\bar{v} \in R^n$ eigenvector for which the equation $P \cdot \bar{v} = k \cdot \bar{v}$ holds true, where k is the eigenvalue of matrix P , and \bar{v} is the corresponding eigenvector. The eigenvalue and eigenvector of the matrix P can be derived by solving $[P - k \cdot E_n] \cdot \bar{v} = 0$ the homogeneous linear equation, where E_n is the n -th order unity matrix.

It is acknowledged that the relation $P \cdot \bar{v} = \bar{v}$ holds significance for the invariant distribution. Therefore, in determining the invariant distribution, the focus is on identifying the distribution vector \bar{v} , which represents the eigenvector associated with the eigenvalue of 1 in the P matrix. In essence, the non-triviality of the homogeneous system of linear equations $P \cdot \bar{v} = \bar{v}$ results in the search for non-zero solutions. In other words, we are looking for a non-trivial, i.e., $\bar{v} \neq 0$, solution of the homogeneous system of linear equations $[P - E_n] \cdot v = 0$. Such solutions exist if, and only if, the determinant of the coefficient matrix is 0, denoted as $|P - E_n| = 0$ (Sydsaeter and Hammond, 2006, p. 449). Consequently, the stationary distribution, derived as a non-trivial solution of the homogeneous linear equation system, serves as an equilibrium distribution vector to which the system will converge over time.

By substituting the estimated transition probability matrix for the \hat{P} matrix in the homogeneous linear equation system $[P - E_n] \cdot v = 0$ and leveraging the Excel program for resolution (sections 36–39), we obtain the invariant distributions presented in the tables. In preparing the forecast, we assumed stationarity as a given, implying that the transition probabilities remain constant over time.

3. The Hungarian insurance market structure 2012–2021—The results

In this chapter, we conduct an analysis of the structural characteristics of the Hungarian insurance market from 2012 to 2021, employing the indirect measurement methods outlined in the preceding section in conjunction with Markov chain methodology. A comprehensive examination of the market structure within the Hungarian insurance sector has been performed. The data utilized for this analysis—annual gross premium income for individual companies—was sourced from the Hungarian Insurance Yearbook, which is published annually by the Association of Hungarian Insurance Companies. Initially, we employed traditional market structure indicators, including market shares, concentration ratios, and the Herfindahl-Hirschman Index (HHI), to conduct our analysis based on this data. Subsequently, we applied the Markov chain method to further investigate the market structure. The application of Markov transition matrices provides two significant advantages. Firstly, it allows for the assessment of the stability of individual market positions, market shares, and concentration levels. Secondly, it aids in estimating the invariant distribution, which represents an equilibrium state of the market structure toward which the system converges under stationary conditions. In essence, the underlying mechanisms of the market propel the market structure toward this equilibrium state under specified conditions. The results of this analysis are presented for the Hungarian life insurance market, followed by an examination of the non-life insurance market.

3.1. Life insurance business

Regarding the life insurance sector, we will first present the evolution of the market shares of the leading companies and draw conclusions based on these trends.

3.1.1. Market share of leading companies

Based on the calculations conducted using the Hungarian Insurers’ Association (MABISZ) data (see **Figure 1**), we observe that concentration in the life insurance sector has slightly increased; however, it remains below the critical value.

Year	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	Minich:
Number of companies	20	19	20	20	18	18	15	14	15	15	
Market share of the largest company	16,53%	16,40%	16,49%	17,85%	17,25%	19,23%	16,93%	19,44%	17,98%	17,68%	
Market share of the second-largest company	10,71%	11,67%	13,98%	11,16%	13,12%	12,64%	15,33%	15,92%	10,81%	16,03%	
Market share of the third-largest company	10,47%	11,44%	10,76%	9,70%	9,93%	10,55%	10,59%	9,32%	10,35%	9,69%	
Market share of the fourth-largest company	9,48%	10,67%	9,22%	9,55%	9,87%	9,39%	9,76%	9,32%	9,91%	8,87%	
Market share of the fifth-largest company	9,04%	9,02%	9,16%	8,86%	8,60%	8,03%	9,58%	9,30%	9,83%	8,63%	
Combined market share of the three largest companies - CR(3)	27,25%	28,07%	30,47%	29,01%	30,37%	31,87%	32,26%	35,36%	28,79%	33,71%	
Combined market share of the four largest companies - CR(4)	37,72%	39,50%	41,23%	38,72%	40,30%	42,43%	42,85%	44,68%	39,15%	43,39%	
Combined market share of the five largest companies - CR(5)	47,20%	50,17%	50,45%	48,27%	50,17%	51,82%	52,61%	54,00%	49,05%	52,26%	
HHI Index	56,24%	59,19%	59,61%	57,13%	58,77%	59,84%	62,19%	63,30%	58,89%	60,90%	
HHI critical value	883	928	935	898	924	969	1 053	1 122	985	1 039	
HHI competitive market threshold	1800	1800	1800	1800	1800	1800	1800	1800	1800	1800	
HHI difference	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200	
HHI diferencia	-917	-872	-865	-902	-876	-831	-747	-678	-815	-761	

Figure 1. Visual representation of the market shares of leading companies based on own calculations, with data sourced from MABISZ.

Market Leaders' Shares: The most prominent company's market share fluctuated slightly over the period, with a peak of 19.44% in 2019 and a low of 16.40% in 2013. Similarly, the second and third largest companies saw varying market shares, with notable peaks in 2018 (15.33% for the second largest) and 2020 (10.35% for the third largest). These fluctuations suggest shifts in competitive dynamics.

CR(2) and CR(5) Ratios: The concentration ratios, CR(2) and CR(5) measure the combined market shares of the two and five most prominent companies, respectively. CR(2) values increased from 27.25% in 2012 to 33.71% in 2021, suggesting increasing dominance of the top two companies over time. CR(5) showed a similar trend, peaking at 63.30% in 2019, indicating a concentration of market power among the five largest firms.

HHI Index: The Herfindahl-Hirschman Index (HHI) measures overall market concentration. While the HHI remained below the critical value of 1800 throughout the period, it rose from 883 in 2012 to 1039 in 2021, with a notable peak of 1122 in 2019. The gradual increase in the HHI indicates a trend toward higher market concentration, although it still suggests a moderately concentrated market overall.

HHI Differentials: The HHI difference from the competitive threshold (1200) reveals that while market concentration increased, it remained below the threshold for highly competitive markets. The closest approach to this threshold occurred in 2019, when the HHI difference was -678 .

Trends: The mini charts in the table visually reflect the trends in market share, showing relatively stable market leadership among the top companies with occasional shifts in rankings and concentration levels.

In conclusion, the table reveals a trend of increasing market concentration among the largest companies in the Hungarian insurance market, particularly around 2019. However, the market remains moderately competitive based on HHI values, though some consolidation appears to occur over time.

3.1.2. Markov transition matrix

The Markov transition matrix for the life insurance sector of the Hungarian insurance market (see **Figure 2**) can be interpreted as follows:

The mobility coefficient, a significant 48.31%, clearly indicates the high level of mobility within the market shares during the analysed period. This underscores the dynamic nature of competition in the life insurance sector, with frequent changes in company rankings.

The market share categories are divided into intervals, where:

- Category 1: 0%–2.5% market share,
- Category 2: 2.5%–5%,
- Category 3: 5%–10%,
- Category 4: 10%–15%,
- Category 5: 15%–20% market share.

Interpretation of Transitions:

- Category 1: The value of 0.98 ($p(1, 1)$) in the first row indicates that companies with the smallest market share have a 98% probability of remaining in the same category in the next period. Only 2% of them move up to Category 2.

- Category 2: Companies in Category 2 have a 75% chance of staying in this category, 8% chance of moving up to Category 3, and 2% chance of returning to Category 1.
- Category 3: Companies in Category 3 have an 82% chance of remaining in this category, with 5% potentially moving down to Category 2 and 8% moving up to Category 4.
- Category 4: Companies in Category 4 have a 63% chance of staying, while 8% move up to Category 3, and 5% fall back to Category 5.
- Category 5: Companies in Category 5 have a 92% chance of maintaining their market share, while 8% move down.

General Observations:

High Stability in the Smallest and Largest Categories: The analysis reveals that Categories 1 and 5 showcase remarkable stability, with impressive retention rates of 98% and 92%, respectively. This suggests that the leading players in the market and the smaller, less prominent firms experience little fluctuation, maintaining their positions over time with limited competition for mobility.

Increased Mobility in Middle Categories: In contrast, companies positioned within the middle categories (2, 3, and 4) exhibit a dynamic and fluid landscape. Notably, Category 3 stands out for its vibrant activity, where firms frequently ascend or descend the rankings. This variance indicates that competition is particularly fierce among these middle-tier companies, as they jockey for better positions in the marketplace.

The mobility coefficient		48,31%				
Category code		1	2	3	4	5
1		0,98	0,20	0,00	0,00	0,00
2		0,02	0,75	0,08	0,00	0,00
3		0,00	0,05	0,82	0,32	0,08
4		0,00	0,00	0,08	0,63	0,00
5		0,00	0,00	0,02	0,05	0,92

Figure 2. Visual representation of the Markov transition matrix for the life insurance sector based on own calculations, with data sourced from MABISZ.

In conclusion, the Markov transition matrix reveals a clear trend: companies with the most significant market shares tend to remain highly stable. In contrast, smaller companies experience more frequent changes in market ranking. This conclusion reinforces the main findings of the analysis.

3.2. Non-life insurance business

For the non-life insurance sector, we will begin by examining the changes in market shares of the leading companies and then provide an analysis of the findings.

3.2.1. Market share of leading companies

Based on the calculations conducted using the Hungarian Insurers’ Association (MABISZ) data (see **Figure 3**), we observe that concentration in the non-life insurance sector has slightly increased; however, it remains below the critical value.

Year	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	Minichart
Number of companies	24	22	22	21	20	21	19	18	18	19	
Market share of the largest company	25.50%	22.46%	21.97%	21.53%	22.28%	22.86%	22.68%	22.30%	22.62%	22.84%	
Market share of the second-largest company	19.52%	18.95%	18.69%	18.63%	17.67%	17.10%	17.25%	17.17%	17.49%	18.50%	
Market share of the third-largest company	12.28%	12.78%	12.91%	13.11%	12.92%	14.02%	14.16%	13.85%	14.01%	13.75%	
Market share of the fourth-largest company	10.71%	10.52%	10.44%	10.37%	9.88%	10.09%	9.98%	9.64%	10.08%	10.26%	
Market share of the fifth-largest company	8.36%	8.31%	8.10%	7.30%	7.06%	7.34%	7.88%	8.16%	7.96%	7.39%	
Combined market share of the two largest companies - CR(2)	45.03%	41.41%	40.67%	40.16%	39.95%	39.96%	39.93%	39.46%	40.11%	41.34%	
Combined market share of the three largest companies - CR(3)	57.31%	54.18%	53.58%	53.28%	52.87%	53.99%	54.09%	53.31%	54.12%	55.09%	
Combined market share of the four largest companies - CR(4)	68.02%	64.70%	64.02%	63.65%	62.75%	64.07%	64.07%	62.95%	64.19%	65.34%	
Combined market share of the five largest companies - CR(5)	76.38%	73.02%	72.11%	70.95%	69.81%	71.42%	71.96%	71.12%	72.16%	72.73%	
HHI Index	1431	1299	1256	1244	1240	1284	1295	1274	1304	1336	
HHI critical value	1800	1800	1800	1800	1800	1800	1800	1800	1800	1800	
HHI competitive market threshold	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200	
HHI difference	-369	-501	-544	-556	-560	-516	-505	-626	-496	-464	

Figure 3. Visual representation of the market shares of leading companies based on own calculations, with data sourced from MABISZ.

Based on the calculations conducted using the Hungarian Insurers’ Association (MABISZ) data, we observe that concentration in the life insurance sector has slightly increased; however, it remains below the critical value.

The number of companies participating in the non-life insurance market slightly declines, decreasing from 24 in 2012 to 19 by 2021. This reduction could indicate market consolidation, with fewer competitors operating in this segment by the end of the period.

The largest company’s market share remained relatively stable throughout the period, starting at 25.50% in 2012, dropping slightly to 22.46% in 2013, and then fluctuating around 22% until reaching 22.84% in 2021. This stability suggests that the most prominent firm maintained its dominant position over the decade.

The second-largest company’s share also saw slight fluctuations, decreasing from 19.52% in 2012 to 17.10% in 2017 and increasing again to 18.50% in 2021.

Market shares for the third, fourth, and fifth largest companies remained relatively stable, although the third-largest company peaked at 14.16% in 2018.

- CR(2): The combined market share of the two largest companies decreased slightly from 45.03% in 2012 to 41.34% in 2021, showing a slight dispersion of market power among smaller players.
- CR(5): The top five companies’ combined market share also gradually declined, from 76.38% in 2012 to 72.73% in 2021. This trend suggests a relative decrease in market concentration at the top, with smaller firms gaining ground.
- HHI Index: The Herfindahl-Hirschman Index (HHI) consistently declined from 1431 in 2012 to 1336 in 2021. This index remained below the critical 1800 threshold throughout the period, indicating that while market concentration was moderate, it stayed well within the bounds of a competitive market. The lowest HHI was 1240 in 2016, with a slight increase after that, suggesting modest fluctuations in market concentration.
- HHI Difference: The HHI difference, which shows the deviation from the competitive market threshold of 1200, remained negative throughout the period, with the highest negative value being –560 in 2016. However, the gap narrowed to –464 in 2021, indicating a slow but steady increase in market concentration, although it remains far from a critically concentrated market.

The mini charts visually indicate stable market shares for the top companies, with some fluctuations between 2012 and 2021. The concentration ratios and HHI data suggest that while the non-life insurance market remains moderately concentrated, concentration has gradually reduced, particularly in the top two and five companies.

In summary, the data reveals a stable but slightly decreasing market concentration in the non-life insurance market, with the top firms maintaining dominance. However, there is some evidence of a shift toward a less concentrated market over the years.

3.2.2. Markov transition matrix

The Markov transition matrix (see **Figure 4**) indicates that the non-life insurance sector exhibits high stability, particularly among the largest companies. This predictability, with limited upward mobility and most movement between categories observed in the middle and lower market segments, suggests a relatively static but confident competitive environment in the non-life insurance market.

Mobility Coefficient (17.97%): This relatively low mobility coefficient suggests that the market shares in the non-life insurance sector are relatively stable, with limited movement between different market share categories. Companies tend to maintain their positions over time.

Category Definitions: The matrix shows seven categories, which likely correspond to different ranges of market shares, although these ranges are not explicitly provided. The rows represent the companies' initial category, while the columns represent their possible category in the next period.

Transition Probabilities:

- **Category 1:** Companies in the first category (probably the most prominent firms) have a 99% chance of staying in the same category, with only 1% of them moving to Category 2. This indicates high stability among the largest companies in the market.
- **Category 2:** Firms in Category 2 have a 71% chance of staying in the same category, but there is a 24% probability of moving to Category 3. This suggests moderate stability, with a possibility of downward movement.
- **Category 3:** Firms in Category 3 show a high level of stability (87% remain in the same category). However, 11% can move to Category 4, while 7% may move to Category 2.
- **Category 4:** Companies in Category 4 have an 89% chance of remaining, with a 10% probability of moving down to Category 5.
- **Category 5:** Firms in Category 5 show high stability (90%), with only 10% moving to Category 6.
- **Category 6:** Firms in Category 6 have a 90% chance of staying, but 33% might move to Category 7, suggesting a downward mobility trend for some firms.
- **Category 7:** Firms in Category 7 are relatively stable, with 67% remaining in the same category, indicating a concentration of companies with smaller market shares.
- **Categories 1, 3, 4, and 5** show the highest levels of stability, with firms rarely moving between categories.

- Category 2 and Category 6 show the most significant mobility, with firms in Category 2 more likely to move down to Category 3 and firms in Category 6 more likely to move down to Category 7.

High Stability Among Major and Minor Enterprises: The leading firms within Category 1 of the market, much like those in the life insurance industry, demonstrate an extraordinary level of consistency and resilience. Remarkably, these top-tier companies maintain a staggering 99% retention rate of their market share. This statistic underscores the relatively low level of competitive dynamics among the elite players in the market, indicating that once firms achieve a dominant position, they are adept at holding onto it, thus minimizing upheaval at this level.

Increased Mobility in Intermediate and Lower Categories: In contrast, Categories 2 and 6 reveal a much more dynamic environment. Companies within these classifications exhibit a significantly higher degree of mobility, indicating that they frequently change their market standings. This trend is particularly evident in downward movements, where firms may fall out of their current positions due to various strategic missteps, market shifts, or increased competition.

Small Enterprises (Category 7): The enterprises classified under Category 7, which are likely to represent the smallest market shares, show a probability of 67% of remaining within the same category over a specified timeframe. This statistic suggests that while there is some level of stability, there is also a notable occurrence of fluctuations within this lower segment of the market. The smaller firms face unique challenges that can lead to shifts in their market status, whether through growth opportunities or the risk of declining competitiveness.

The mobility coefficient	17,97%						
Category	1	2	3	4	5	6	7
1	0,99	0,06	0,00	0,00	0,00	0,00	0,00
2	0,01	0,71	0,07	0,00	0,00	0,00	0,00
3	0,00	0,24	0,87	0,11	0,00	0,00	0,00
4	0,00	0,00	0,07	0,89	0,10	0,00	0,00
5	0,00	0,00	0,00	0,00	0,90	0,10	0,00
6	0,00	0,00	0,00	0,00	0,00	0,90	0,33
7	0,00	0,00	0,00	0,00	0,00	0,00	0,67

Figure 4. Visual representation of the Markov transition matrix for the non-life insurance sector based on own calculations, with data sourced from MABISZ.

4. The invariant distribution of market shares in the Hungarian insurance market

The invariant distribution in a Markov chain signifies the probabilities of a system’s long-term stable state. This implies that, irrespective of state transitions, the system remains unchanged. The distribution reflects the likelihood of the system occupying each state when equilibrium is reached, indicating how frequently each state is visited over time. Grasping this distribution is essential for analyzing the steady-state behavior of dynamic systems, providing insights into long-term trends and the stability of market shares or other significant indicators. In competitive markets, it aids in assessing how consistently companies retain their positions within specific market share categories over time.

In this context, we can determine the invariant distribution of market shares for companies in the Hungarian insurance market by using data on gross insurance

premium income alongside the Markov transition matrix. This distribution can be interpreted as the long-term stable state of the market structure under stationary conditions, or as the market share allocation toward which the internal mechanisms of the market naturally steer the system. Consequently, the invariant distribution acts as an equilibrium distribution under the specified conditions among the numerous potential future distributions of market shares. The market’s internal mechanisms effectively drive the structure toward this equilibrium.

Our calculation results are presented accordingly. Based on traditional methods for analysing market shares and concentration, an increase in the market shares of leading companies and a rise in concentration seem to be realistic scenarios. However, the Markov method—which emphasizes the market’s internal mechanisms, the stability of market shares, and the volatility of positions among leading companies—provides a more optimistic perspective on the characteristics of the market structure. As shown in the table below (see **Figure 5**), the invariant distribution for the life insurance sector reveals that 78% of companies are in the lowest category (0.0%), meaning that most companies hold tiny market shares. Only 9% of companies occupy the next category (2.5%), 8% in the 5% market share category, and the remaining 5% is distributed among the 10% and 15% categories.

Categories	v
0,0%	78%
2,5%	9%
5,0%	8%
10,0%	2%
15,0%	3%
20,0%	0%
25,0%	0%
Sum	100%

Figure 5. Invariant distribution for the life insurance sector based on own calculations, with data sourced from MABISZ.

Comparing this with the Markov transition matrix for the life insurance sector:

The probability of remaining in the smallest market share category (0.98 in the matrix) aligns with the 78% share of companies in the lowest category in the invariant distribution. This suggests that most companies stay in the smallest market share bracket over time.

Mobility between categories is limited, as indicated by the low mobility coefficient (48.31%). Companies in higher categories (such as 10% or 15%) are unlikely to move upwards or downwards, as the transition probabilities between these categories are small.

Categories	ν
0,0%	59%
2,5%	6%
5,0%	22%
10,0%	13%
15,0%	0%
20,0%	0%
25,0%	0%
Sum	100%

Figure 6. Invariant distribution for the non-life insurance sector based on own calculations, with data sourced from MABISZ.

For the non-life insurance sector, the invariant distribution shows a more balanced spread across categories (see **Figure 6**):

- 59% of companies are in the smallest market share category (0.0%).
- 22% of companies are in the 5% market share category.
- 13% are in the 10% market share category, and 6% in the 2.5% category.
- The non-life sector has a lower mobility coefficient (17.97%), indicating even less movement between market share categories compared to the life insurance sector:

Companies in the smallest category (0.0%) are also highly stable, as shown by the high probability of 0.99 remaining in the same position. This corresponds with this category’s 59% market share in the invariant distribution.

The next largest share of companies (22%) are in the 5% market share category, which also aligns with the high stability for this category in the transition matrix (0.87 probability of remaining in the same category).

Concentration in Small Market Players: The life and non-life sectors show an intense concentration of companies in the most miniature market share categories (0.0%), but this concentration is more pronounced in the life insurance sector (78% vs. 59% in the non-life industry). This suggests that competition among the smallest companies is more intense in the life insurance market, with fewer companies breaking into higher market share categories.

Limited Upward Mobility: The dynamics within both sectors reveal a notable constraint on upward mobility, as illustrated by the low transition probabilities observed between higher market share categories. For instance, companies categorized within the 5% market share bracket (referred to as Category 3) predominantly remain anchored in this tier, exhibiting only minimal upward movement into the 10% market share category. This trend is consistent with the invariant distribution, which indicates that a mere fraction of companies manage to ascend into the higher echelons of market share (beyond the 5% threshold).

Stability in Market Leadership: In both sectors, organizations that successfully secure higher market share categories—such as 10% or 15%—demonstrate a significant degree of stability. This is evidenced by the marked transition probabilities that cluster along the diagonal of the Markov matrix, signifying that these companies tend to maintain their positions over time. This consistency is underscored in the invariant distribution, where a relatively small percentage of companies (only about

3%) within the life insurance sector reach and hold positions in the top market share categories.

The analysis of invariant distributions and Markov matrices paints a clear picture of enduring stability and limited mobility within the life and non-life insurance sectors, particularly evident in both the lower and upper market share categories. The life insurance landscape is characterized by a concentration of smaller players that dominate the market, whereas the non-life insurance sector shows a somewhat more balanced distribution of companies yet remains primarily governed by smaller firms. In summary, both sectors exhibit a striking lack of movement between categories, with those firms that achieve higher market shares demonstrating a strong tendency to maintain their established positions over time.

5. Summary

The article presents a comprehensive analysis of the structural characteristics of the Hungarian insurance market from 2012 to 2021. This market structure analysis employs traditional indicators such as market shares, concentration ratios, and the Herfindahl-Hirschman Index (HHI), applied to both the life and non-life insurance sectors, utilizing gross premium income data published by the Association of Hungarian Insurance Companies. However, the study extends beyond conventional analytical methods. The incorporation of the Markov chain method allows for a nuanced examination of the internal dynamics of the market structure, facilitating more insightful conclusions. Specifically, the calculation of the Markov transition matrix aids in evaluating the stability of market shares, while also permitting the determination of the invariant distribution of market shares.

The invariant distribution signifies the stable state of a dynamic system toward which the system converges. In this context, the invariant distribution of market shares, derived from gross premium income, reflects the equilibrium state of the Hungarian insurance market's structure under stationary conditions. This may also be interpreted as a potential future state, indicating an equilibrium under specified conditions.

The geographical focus of this study is confined to the Hungarian insurance market, with the temporal scope defined by the availability of Hungarian Insurance Yearbook data at the time of research and composition of the article. The analysis encapsulates the period from 2012 to 2023, offering a nearly decade-long perspective conducive to a thorough understanding of the market structure's characteristics and underlying dynamics.

During the examined period, the structure of the Hungarian insurance market has evolved to become more conducive to competition. The key findings are noteworthy; there is a marginal increase in market concentration in both the life and non-life insurance sectors. Nonetheless, this concentration remains comfortably below the critical HHI threshold of 1800, which signifies a healthy level of competition.

The Markov transition matrices reveal significant stability among leading companies, with minimal fluctuations between market share categories, particularly at the extremes. The mobility coefficients, recorded at 48.31% for life insurance and 17.97% for non-life insurance, emphasize the pronounced stability within the non-life insurance sector, characterized by minimal changes in company rankings.

The market share of the largest companies continues to decline, consistent with trends observed in prior periods, although their positions exhibit stability in both the life and non-life sectors. The non-life sector shows a higher concentration, with leading companies commanding larger market shares and demonstrating greater stability. The invariant distributions indicate a potential further decline in market shares based on gross premium income, alongside a reduction in the stability of leading companies' positions under static conditions.

Despite the valuable insights provided by this study, it is crucial to acknowledge its limitations, including the reliance on historical data and traditional market power indicators, which may not fully encapsulate the complexities of market competition. Furthermore, the exclusive focus on the Hungarian market restricts the generalizability of the findings to other regions.

Future research endeavors will involve an analysis of contract portfolio data and a consideration for the removal of the stationarity condition. Additionally, a significant research direction will be the extension of the analysis to the Central and Eastern European region. Alongside the removal of the stationarity condition, methodological advancements will include the development of an analytical model designed to more accurately reveal the internal structural dynamics of the market, which is essential for predicting future market states.

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