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Article

OWA operators in the insurance industry

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Abstract: In this paper, we examine a possible application of ordered weighted average (OWA for short) aggregation operators in the insurance industry. Aggregation operators are essential tools in decision-making when a single value is needed instead of a couple of features. Information aggregation necessarily leads to information loss, at least to a specific extent. Whether we concentrate on extreme values or middle terms, there can be cases when the most important piece of the puzzle is missing. Although the simple or weighted mean considers all the values there is a drawback: the values get the same weight regardless of their magnitude. One possible solution to this issue is the application of the so-called Ordered Weighted Averaging (OWA) operators. This is a broad class of aggregation methods, including the previously mentioned average as a special case. Moreover, using a proper parameter (the so-called orness) one can express the risk awareness of the decision-maker. Using real-life statistical data, we provide a simple model of the decision-making process of insurance companies. The model offers a decision-supporting tool for companies.

Keywords: aggregation; ordered weighted averaging; OWA; risk awareness

1. Introduction

The insurance market is full of decision-making situations. On one hand on the customer's side, and on the other hand on the company's side. Both sides try to act rationally, but of course there are many subjective factors or uncertainties, which are very difficult to quantify. The classical way of uncertainty modelling is based on probability theory, but due the presence of non-probabilistic uncertainties, some models apply fuzzy methods (Shapiro, 2004). The main aim is maximization of the profit and minimization of risks. It is somewhat similar to the well-known multi arm bandit problems (Kim and Lim, 2016), with correlated rewards (Gupta et al., 2021), since in real-life the economic situation determines the options of the participants. Decision-making for optimization and risk management is in the first line of empirical research (Chen and Ye, 2024; Van Tran et al., 2024) and model selection (Seong-Min, and Byung-Soo, 2024).

Insurance market sometimes modelled using game theoretical tools (Hamidoğlu, 2021), but it requires too many assumptions. Kacprzyk et al. (2019) highlight the problem of human-centric information aggregation, contrary to pure numeric information aggregation, since the later one sometimes leads to undesired results, although mathematically correct. The ordered weighted averaging (OWA) is a mathematically correct framework, moreover can be tailor-fitted to human centric applications. In this paper, we show a possible way of a simple application of OWA method in insurance industry. Assuming a limited budget (as it happens in real-life), where should the insurance company focus their efforts to acquire more new customers? To answer this question, we use statistical data combined with OWA operators.

2. OWA operators

Consider the situation when we have to make a choice between two or more alternatives, based on multiple criteria. In real life problems, if we have several criteria, we are faced with the following:

(i) satisfying a criteria is not considered a yes/no question, usually there are levels of satisfaction, like closely, more or less, roughly, medium, low etc.;
(ii) in most of the cases, we are satisfied with an approximate solution, i.e. not all of the criteria should be fully satisfied;
(iii) if a solution is excellent w.r.t. some criteria, then it may overwrite that it is only average w.r.t. some other criteria.

The insurance company wants to choose a region for the focus of their next campaign. All of the regions can be evaluated by several different criteria, like number of inhabitants, demographic trends, development etc. To be able to rank the alternatives, we aggregate the numerical values assigned to each criteria. Out of the many aggregation methods, we recommend a highly flexible one below.

Yager (1988) introduced a novel aggregation technique called ordered weighted average (OWA). In (Figuerola-Wischke et al., 2024), the authors provide an overview of OWA related publications, highlighting the most influential authors and institutions, theoretical and applied research areas, co-authorship networks etc.

Definition: An Ordered Weighted Average (OWA) operator of dimension n is a mapping $F: \mathbb{R}^n \rightarrow \mathbb{R}$ that has an associated weighting vector $W = (w_1, w_2, \ldots, w_n)$ of dimension n, such that $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$, and for a given dataset $A = \{a_1, a_2, \ldots, a_n\}$ it is given by the following formula:

$$F_W(A) = \sum_{i=1}^{n} w_i b_i \tag{1}$$

where b_i is the ith value of dataset A being ordered in non-increasing order.

Example:

Consider the following set of data: $\{5, 1, 2, 10, 4\}$. Then the data in decreasing (non-increasing) order: (10, 5, 4, 2, 1). Of course we can choose various different weighting vectors for different purposes:

- if we apply the weighting vector $W = (1, 0, 0, 0, 0)$, then $F_W = 10$, the maximum value.
- if we apply the weighting vector $W = (0, 0, 0, 0, 1)$, then $F_W = 1$, the minimum value.
- if we apply the weighting vector $W = (1/5, 1/5, 1/5, 1/5, 1/5)$, then $F_W = 4.4$, the arithmetic average of the data.
- if we apply the weighting vector $W = (0, 0, 1, 0, 0)$, then $F_W = 4$, the middle value, i.e. the median of the data.
- if we apply the weighting vector $W = (0.7, 0.2, 0.1, 0, 0)$, then $F_W = 8.4$, a value closer to the maximum.
- if we apply the weighting vector $W = (0, 0, 0.1, 0.2, 0.7)$, then $F_W = 1.5$, a value closer to the minimum.

2.1. The orness

An important feature of the OWA operator is the measure of similarity to the maximum operator. In other words, it classifies OWA operators regarding to their location between the minimum (logical and) and maximum (logical or). It is the so-called orness, usually denoted by α or α(W):

$$orness(W) = \alpha = \sum_{i=1}^{n} w_i \frac{n-i}{n-1} \quad (2)$$

It is obvious that the orness value always lies between 0 and 1. The orness measures of the OWA operators in the previous example: 1, 0, 0.5, 0.5, 0.9 and 0.1, respectively. The closer the value to 1, the closer the OWA value to the maximum, and vice versa, the closer the orness to 0, the closer the OWA value to the minimum.

The notion of orness can be extended in several ways, by taking other coefficients of the weights in the definition of the orness. The paper (Kishor et al. 2013) considers an axiomatic approach and introduces a novel orness measure, for which the maximum Shannon entropy of the weight vector is higher than in the original case. Sometimes there is a desire or constraint for a given orness (i.e. risk taking willingness). There are parametric families of OWA operators with weights derived from the Beta probability density function with constant level of orness (Srivastava et al., 2023).

One of the advantages of the OWA operator is the compensation property. Namely, if one (or more) criteria are satisfied over the given threshold, then it can compensate a poor performance at other criteria. A real-life example: in football, an excellent left foot can not only compensate, but dominate a closer-to-average right foot (see for example Ferenc Puskás). The orness level mentioned above expresses this compensating level: if we consider the minimum operator, then there is a minimum level to be reached by all of the criteria (the orness level is α = 0). In this case, there is no compensation: a great performance at a specific property cannot overwrite the under-the-level performance on other properties. On the other hand, if consider the maximum operator, then we take into account only the best performance over all of the criteria (the orness level is α = 1). This is the case of total compensation: a great performance in at least one property overwrite the poor performances at other criteria. If the orness level is somewhere between 0 and 1, then great performance in some criteria may compensate weaker performance in other criteria, but not to all extent. Among other features, the compensation property makes the OWA operator very suitable for real-life decisions, involving the possible subjectivity of human decision-makers.

2.2. Weight determination

As it was highlighted in (Filev and Yager, 1998), one of the most important problem is to find the proper weights in the OWA operator. From the strict mathematical point of view, the equations describing the weight properties and orness value together form an underdetermined system of equations, thus it has infinite number of solution. Consequently, we need some other constraints, like considerations based on the experts' view or maximal dispersion etc. One possible way is to find the weight set with maximal entropy under a given level of orness. Maximal entropy ensures that we get the most information by taking into account as many non-zero weights as possible (Fullér and Majlender, 2001). Harmati et al. (2022) proved that the maximum entropy weights are continuous functions of the orness level. It means that a small change in the orness level does not cause a large difference in the weights, thus the output of the OWA operator behaves smoothly, without unexpected jumps. Other approach is to find the weight set with the most equal weights for a given orness. From the mathematical point of view, it is the minimization of the Euclidean distance from the arithmetic average operator (Fullér and Majlender, 2003). Carlsson and Fullér (2018) provide a survey of minimal variability and maximal entropy weight determination methods under a given level of orness. Minimal variability means that we use a weight vector as close to the arithmetic average as possible, maximal entropy means that we set to nonzero as many weights as possible. Thus, both approaches provide the best dispersion of weights, but according to different criteria.

Renaud et al. (2008) suggest an alternative way of weight determination using paramteric identification, with application in food industry. The minimax method for the determination of the weights was introduced in (Wang and Parkan, 2005). Xu (2005) discusses some of the OWA weight determination techniques, without completeness.

2.3. Generalizations of the OWA operator

There are many options for the generalization of the OWA operator. The original OWA operator assigns the same importance to the information pieces (coordinates, channels). A possible way to include non-equal importance is introduced in (Yager, 1998). In (Beliakov and James, 2011) the ordering is considered by a so-called inducing variable, giving more flexibility to the aggregation. This variable can totally independent of x or it can be a function of x, like $f(x)$, importance of the information source etc. A multi-person and multi-criteria decision-making problem is solved in (Casanovas et al., 2020) applying the induced probabilistic OWA distance operator. A unified model of the weighted average and the induced OWA operator was introduced in (Merigó, 2011).

In (Figuerola-Wischke et al., 2023), the authors present a novel method for optimizing forecasts of the average pension by using the OWA operator and its extensions, like the induced ordered weighted averaging (IOWA) operator, the generalized ordered weighted averaging (GOWA) operator, the induced generalized ordered weighted averaging (IGOWA) operator, and particular forms of the probabilistic ordered weighted averaging (POWA) operator and the quasi-arithmetic ordered weighted averaging (Quasi-OWA) operator. The model takes into consideration the inflation or deflation, thus gives a more trustable estimation of the average pension.

Other generalization of the OWA operator is the weighted OWA (WOWA), where the additional weights express the importance of different pieces of information (Torra, 2000). Zheng et al. (2023) propose a method for OWA aggregation of attribute values given in a linguistic form. Additionally, OWA weights can be derived from linguistic quantifiers and vice versa (Yager, 1996).

2.4. Some other applications of the OWA operator

Due to their flexibility, OWA operators are widely used in decision-making. Without seeking for completeness, we mention a few applications, emphasizing their high variability.

In (Belles-Sampera et al., 2013), the authors investigate the connection between distortion risk measures and ordered weighted averaging operators and show that the distortion risk measures can be derived from OWA operators. Jiang and Tu (2023) discuss the risk management of shantytown renovation, using order weighted averaging technique. The authors of (Benati and Conde, 2024) apply a version of OWA (called robust OWA) in order to find an optimal portfolio for the different attitudes towards risk of a decision maker. Using OWA, Bueno et al. (2019) rank the tourist sites of a city, and the ranking is used by a recommender system. Casanovas et al. (2016) provide the aggregation of alternatives using their Minkowski distance from an optimal result. The basic alternatives are Quota Share Reinsurance, Excess of Loss Reinsurance Risk and Stop Loss Reinsurance. The criteria for optimal reinsurance are Maximum Gain, minimum variance and Low Probability of Ruin. The study (Casanovas et al., 2020) develops an application for group decision making in insurance management.

The paper (Cheng et al., 2023) offers a novel method for multi-criteria group decision-making, using OWA aggregation operator and Z-numbers. A Z-number is a pair of ordered values, where the first component is a restriction on the values of the real-valued uncertain variable, while the second parameter measures the reliability of the first component using linguistic variables (for example: (at about 30 min, very sure)).

The articles (Figuerola-Wischke and Gil-Lafuente, 2024) propose a model to forecast the real average retirement benefit in the United States taking into account the price changes. The model applies OWA operators and order-inducing variables. Paper (Figuerola-Wischke et al., 2023) discusses the role of OWA operators in pension.

Ma et al. (2024) introduce a novel approach to consensus modelling for social network group decision making. The traditional maximum expert consensus model applies OWA for parameter determination.

Pachêco Gomes and Wolf (2024) apply OWA operator with a linguistic quantifier in driving style recognition.

In (Vizuete-Luciano et al., 2015), a large variety of OWA operators (distance OWA) are used for decision making in the assignment process considering a parameterized family of aggregation operators from the minimum to the maximum distance.

The paper (Xie et al., 2024) introduces a new approach to quantify the influence of decision-makers' risk attitudes on the group decision-making process, using a version of OWA, the so-called ordered weighted utility distance operator.

3. Decision-making in the insurance market

There are several different decision-making situations in the insurance industry. Just to mention the two most important: from the customer's and from the company's point of view.

At the customer level:

- Which contraction (offer) to accept? It is sometimes subjective or at least contains some subjective elements, like personality of the broker, cheap, but nice representative gifts. Also counts the opinion of relatives, neighbors, co-workers, who may recommend this or that product (and in most of the cases, they are definitely not experts of the field!).

At the company's side there are can be personalized and not personized decisions:

- Where should the company start the next campaign?
- What kind of products should be involved in the next campaign?
- Which product to offer?

At first, these are not really personalized offers, since there is not enough data about the customer and his/her preferences. If a potential customer shows interest, i.e. responds the call, visits the office, fills the questionnaire, then the company may get more data, like monthly income, debts, average spending, personal preferences etc. Based on these information, they can offer a tailor-fitted product or construction (or, to be honest, they make the customer believe that he gets a completely unique, personalized offer). This step requires a complex combination of numerical data, psychology and sometimes a bit of manipulation (like: Oh, you have two children? Responsible parents used to choose this and that…). At this point, a customer makes a decision, and the decision-making process contains subjective factors also.

In this paper, we concentrate on the company's side, especially the case where we are not faced with a specific customer. We consider the case, when the goal of the insurance organization is network development. Thus, the main task is to find out where should the company put the most efforts to acquire new clients or at least new contractions with current clients. As usually, the decision depends on the data available. In our case, there are two main databases:

- Historical data of the current and previous customers (age, gender, accommodation etc.);
- Regional data available from the Hungarian Central Statistical Office (https://www.ksh.hu/).

The two databases naturally contain different level of information. Namely, the database of the insurance company contains details about individuals, but the set of individuals is not a representative sample of a region. On the other side, the database available from the Hungarian Central Statistical Office provides no details about individuals, only aggregated data on different subgroups, but the dataset is representative regarding a given subgroup.

From the first dataset, we may estimate the popularity of a product, depending on various personal variables. In the second dataset, we have a huge amount of data describing the livability of a settlement, town, county or region. These are related to demography, development, schools, employment rate, healthcare, closeness to highways and bigger cities, internet access etc. Some variables without seek for completeness:

- Demography related data (number of): inhabitants, children below 2 years, children below 14 years, deaths, migration and immigration.
- Accomodation, welfare: nurseries, flats, average price of flats, comfort level.

- Economics: employment rate, gross income, education level, cars, enterprises.
- Infrastructure: condition of roads, total length of roads, distance from bigger cities, highways, internet access, waste management.

The insurance company can reach these data, so they have a more or less clear picture about the regions in small, middle and large scale. Since the potentially targeted new customers are currently not clients of the insurance company, we do not have any individual, personal data about them. Consequently, we can rely only on the aggregated regional data provided by the statistical office.

We may use the data point-by-point, but statisticians created complex, normalized values (between 0 and 1) for each of the categories above. These normalized values are comfortable tools to compare different towns or regions. For illustration, we chose seven Hungarian towns of similar size, roughly 26–27,000 inhabitants (see **Table 1**).

Table 1. Complex, normalized metrics for seven Hungarian towns of similar population size (ca. 26,000 inhabitants).

Town	demography	accomodation	economics	infrastructure	average
Siófok	0.74948	0.782505	0.70064	0.78048	0.753278
Szentes	0.67244	0.444330	0.68380	0.65701	0.614394
Kazincbarcika	0.74571	0.462234	0.68509	0.80809	0.675279
Kiskunhalas	0.63943	0.46799	0.61713	0.52804	0.56315
Gyöngyös	0.71835	0.53659	0.67460	0.66064	0.64754
Jászberény	0.65514	0.58900	0.71049	0.58228	0.634228
Orosháza	0.63931	0.44545	0.64129	0.67100	0.599260

Similar size means similar expenses for the possible campaign. Let us assume that the company decides to start a campaign at the four best locations. In order to find them, we have to define an ordering of the alternatives. We may sort them according to the scores of demography, accomodation, economics, infrastructure and average (see **Table 2**). As we can see, we get many different rankings.

Table 2. Ranking of the towns by different criteria.

Town	demography	accomodation	economics	infrastructure	average
Siófok	1	1	2	2	1
Szentes	4	7	4	5	5
Kazincbarcika	2	5	3	1	2
Kiskunhalas	6	4	7	7	7
Gyöngyös	3	3	5	4	3
Jászberény	5	2	1	6	4
Orosháza	7	6	6	3	6

Or we can rank them according to some statistical metrics (see **Table 3**) derived from the scores. Again, there are different orderings for different metrics.

Table 3. Ranking of the towns by different statistical metrics.

Town	average	median	minimum	maximum
Siófok	1	1	1	2
Szentes	5	4	7	5
Kazincbarcika	2	2	5	1
Kiskunhalas	7	7	4	7
Gyöngyös	3	3	3	3
Jászberény	4	6	2	4
Orosháza	6	5	6	6

The previous rankings are based on the aggregation of the scores given to the four attributes and special cases of the OWA based rankings. As we discussed previously, one of the main challenges in OWA applications is the determination of weights. There are only two constraints: (i) the weights are nonnegative and their sum is one; and (ii) the orness level is the predefined value, thus there is no unique solution. For this illustrative example, we choose the maximum entropy weights (it means we set as many weights as possible to nonzero). For the derivation of the weights see (Fullér and Majlender, 2001) or (Harmati et al., 2022).

Figure 1 shows the aggregated values for the seven towns in function of the orness level. At a given level of orness, the ranking of the towns is just the 'from top to bottom' order of the curves. We can observe that the ranks depend on the value of the orness. Remember that orness level is closely related to risk taking willingness. In other words, if we consider the risk taking willingness as a factor, then according to its specific value, we may get different rankings of the alternatives.

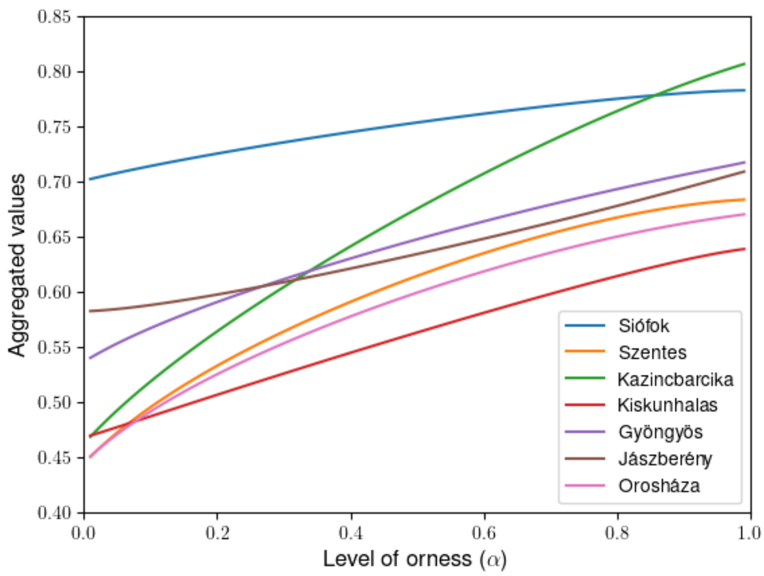

Figure 1. Aggregated values of the four attributes for seven Hungarian towns in function of the orness level.

4. Discussion

In this paper, we introduced an application of the OWA operator.The application aims the selection of the best locations for starting the next campaign of the company. In the example, we considered four normalized attributes derived from various statistical data. The OWA aggregation of them provided the base of the ranking.

As future directions, we mention some possible improvements. We may use more attributes, for example the data available from the insurance company about the previous customers or regional data about the popularity of the company and its products. Additionally, we may include subjective factors of the decision-makers or extend the model to group decision-making.

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