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ORIGINAL ARTICLE

Housing need and search markets

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ABSTRACT

Purpose: The housing need (owning a home to live in) represents one of the primary needs of people. Housing markets are search markets and, thus, the relation between housing need and trading frictions deserves a lot of attention. **Design/methodology/approach:** To study the relation between housing need and trading frictions, this theoretical paper develops a search and matching model. To the best of our knowledge, this is the first paper that addresses the topic by the search and matching theory. **Findings:** The model finds that only in a case the supply side's profit maximisation condition coincides with the housing need satisfaction. In general, therefore, search markets are not able to satisfy the housing need. Housing policies that protect the weaker side (the demand side) of the market are thus needed. **Policy implications:** Housing policies can increase the "actual" and the "realised" housing demand, but a "potential" housing demand remains unsatisfied. Hence, labour market policies that increases both employment and wages are also needed. **Originality/value:** To the best of our knowledge, this is the first paper that analyses the housing market equilibrium with search frictions from the perspective of "non-well-off first-time buyers". **Research limitations/implications:** This model represents a first theoretical attempt to study the housing need by the search and matching framework. Future studies, therefore, should further develop the theoretical model by considering the close link between housing and labour markets.

KEYWORDS

housing need; market equilibrium; housing demand; search and matching theory

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1. Introduction

Search and matching models are widely used in the housing markets analysis (Albrecht, Anderson, et al., 2007; Albrecht et al., 2016; Caplin and Leahy, 2011; Cheng et al., 2015; Díaz and

Jerez, 2013; Gabrovski and Ortego-Marti, 2019; Genesove and Lu, 2012; Ioannides and Zabel, 2018; McDonald, 2000; Piazzesi and Schneider, 2009; Piazzesi et al., 2020; Rocheteau and Weill, 2011; Wheaton, 1990).[1] Housing markets are search markets (Piazzesi et al., 2020) and trading frictions affect house price dynamics (Caplin and Leahy, 2011). Indeed, search and matching frictions produce trading delays in the short run that may propagate in the long run (Díaz and Jerez, 2013). Furthermore, housing market outcomes depend on appreciation expectations (Hattapoglu and Hoxha, 2014), external factors, such as seasonality (Hattapoglu and Hoxha, 2021), and natural phenomena (Dogan et al., 2022).

Following this large literature, the present theoretical paper studies the relation between housing need and the market equilibrium with search frictions. The housing need, namely, owning a home to live in, represents one of the primary needs of people. The housing need analysis is crucial for social housing planning and housing policy evaluation (Barnett and Lowe, 1990; Bramley, 2012; Guerrieri, 2022; Kuminoff et al., 2013; Martínez, 2000; Ménard, 2009; Ytrehus, 2000). The housing need is satisfied by the purchase of the main dwelling (the "first" home or the habitual residence), namely the house where one will live most of the time. Consequently, buying a house could not satisfy the housing need, since the purchase of a house could represent a real estate investment. Therefore, the key actor in the market of the main dwelling is the "first-time" buyer. In this framework, there is a need to distinguish between "potential" housing demand, "actual" housing demand and "realised" housing demand. The "potential housing demand" represents the existing housing need; whereas the "actual demand" is the share of potential housing demand that enter the market, since it has the "monetary capacity" (income and/or initial endowments) to actively search for the main dwelling. Finally, the "realised" housing demand is the "actual demand" that can satisfy its housing need. However, due to search and matching frictions, only a share of actual demand turns into "realised" housing demand. Hence, the housing need remains unfulfilled for several "first-time" buyers.

To the best of our knowledge, this is the first paper that studies: (i) the housing need by the search and matching theory; (ii) the housing market equilibrium with search frictions from a demand side perspective. Precisely, (i) the paper defines the "actual" housing demand and the "realised" housing demand in terms of search and matching (purchasing) process of a house; (ii) the model evaluates the (standard) market equilibrium—characterised by the supply side's profit maximisation—in terms of the satisfaction of the housing need.

The rest of this theoretical paper is organised as follows. The next section describes the market of the main dwelling, while Section 3 shows that, in general, search markets are not able to satisfy the housing need. Section 4 provides the policy implications of the model. Finally, Section 5 concludes the work.

2. The model

2.1. "First-time" buyers and the market of the main dwelling

The market of the main dwelling focuses on "first-time" buyers, namely people who want to fulfil the housing need, namely buying the house where they will live most of the time (the "first" home).

[1] The search and matching theory were initially developed for studying the labour market (Diamond, 2011; Mortensen, 2011; Pissarides, 2000; Pissarides, 2011).

The definition of "first-time" buyers also includes people who buy their main dwelling after selling their first house. Hence, people that sell the only house they have, become "first-time buyers". In general, moreover, all "first-time" buyers need a mortgage to buy a home, since only wealthy families can afford to buy a house without a mortgage loan. Consequently, the model considers "non-well-off first-time buyers".

The market of the main dwelling is a search and matching market and, thus, there is a "matching function" that describes the number of (satisfied) housing needs (h) as a function of both the number of vacant homes on the market (v) and "first-time" buyers (b), viz.:[2]

$$h = f(v, b) = (v^{\alpha} \cdot b^{1-\alpha}) = b \cdot \left(\frac{v}{b}\right)^{\alpha} \tag{1}$$

where $0 < \alpha < 1$ is the "vacant homes" elasticity. It is assumed that $b > v > 0$, since the number of satisfied housing needs (h) cannot (realistically) exceed the number of first-time buyers. This implies that the indicator of search frictions must satisfy the following condition:

$$0 < \theta \equiv \left(\frac{v}{b}\right) < 1 \tag{2}$$

In economic terms, $\theta < 1$ means that the market of the main dwelling is characterised by wider frictions on the housing demand side, namely, it is hard to satisfy the housing need.

2.2. Value functions

Search frictions affect the trading in the housing market. Precisely, the probability of selling a house is given by:

$$\left(\frac{h}{v}\right) = v^{\alpha-1} \cdot b^{1-\alpha} \equiv \theta^{\alpha-1}$$

with $\frac{\partial(\theta^{\alpha-1})}{\partial\theta} < 0$; whereas the probability of finding a home for sale is the following:

$$\left(\frac{h}{b}\right) = v^{\alpha} \cdot b^{-\alpha} \equiv \theta^{\alpha}$$

with $\frac{\partial(\theta^{\alpha})}{\partial\theta} > 0$. In the model, there are no homeless. Hence, for the sake of simplicity, "first-time" buyers are tenants, namely those who have only temporarily satisfied the housing need.[3] The rental market is a monopolistic competition market, where the landlord sets the rent by applying a mark-up on costs (Arnott and Igarashi, 2000). As a result, the present values (discounted at the rate i) of the two sides of the market are the following:

$$i \cdot V = -(\tau + cr) + \theta^{\alpha-1} \cdot [P - V] \tag{3}$$

[2] The matching function captures the implications of the costly trading process without the need to make the heterogeneities and other features that give rise to it explicit (Pissarides, 2000). In short, all "frictions" that delay or make difficult the trade are implicitly included in the matching function. Empirically, the Cobb-Douglas function (1), with constant returns to scale, seems to describe the search and matching process quite well (Petrongolo and Pissarides, 2001).

[3] The so-called "intrinsic preference for home ownership", indeed, makes homeownership the best housing tenure choice, since everyone would pay more to own a house, rather than paying less only for tenancy (Heston and Nakamura, 2009; Linneman and Voith, 1991).

$$i \cdot B = (y - R - s) + \theta^{\alpha} \cdot \left\{ \left[\left(\frac{M}{LTV} - \tau \right) + (y - m) \right] - P \right\} \tag{4}$$

where:

V is the value of a vacant (or unused) house put for sale by the seller.

B is the "first-time" buyer's value.

y, R, s, τ, m and cr are, respectively, the flows of income, rent, search cost (monetary or time spent, including the real estate agency commission), property taxation, mortgage payment and cost of maintaining and repairing a house.

$\frac{M}{LTV}$ is the appraisal value, where $0 < LTV < 1$ is the Loan-To-Value ratio (the banking policy parameter) and M is the value of loan granted.[4]

P is the selling price.

The endogenous variables in Equations (3) and (4) are search frictions (θ) and selling price (P).

2.3. Housing need and "monetary capacity"

The probability of finding a home for sale (θ^{α}) in Equation (4) does not necessarily coincide with the probability of buying a house, since "first-time" buyers need a mortgage to buy a home. The bank grants the loan only if the "first-time" buyer has sufficient economic and financial resources (the "monetary capacity") to repay the mortgage. Precisely, the flow of income y represents the "monetary capacity" of "first-time" buyers and may derive from salaried employment or the self-employment income (in the case of consumer households) or the entrepreneurial income (in the case of producer households). Equation (4), therefore, is the "actual" housing demand, namely the "potential" housing demand that has the "monetary capacity" (y) to actively search for the main dwelling, viz.:

$$y > R + S$$

the "actual" housing demand results in the "realised" housing demand when—at the rate θ^{α}—the "monetary capacity" (y) of a "first-time" buyer can repay the mortgage, viz.:

$$y > m + \tau$$

The sale price (P), instead, is paid by the mortgage loan (M).

3. Equilibrium

3.1. Housing supply

The so-called "free-entry" or "zero-profit" condition, usually used in search and matching models (Pissarides, 2000), needs to be reformulated. Concisely, in the housing market, the value of a house for sell can never be zero, namely $V > 0$ (instead of $V = 0$). In the long run equilibrium, the seller must be indifferent between selling and renting a house. Hence, the housing-supply-side equilibrium condition is the following:

[4] Thus, $\frac{M}{LTV}$ is always higher than M since $0 < LTV < 1$.

$$V = \frac{R}{i}$$

$$\xrightarrow{yields} R = -(\tau + cr) + \theta^{\alpha-1} \cdot \left[P - \frac{R}{i}\right]$$

where $\frac{R}{i}$ is the capitalised annuity.[5] Hence, it is straightforward to get search frictions in the housing-supply-side ($\theta = \theta_v^*$), viz.:

$$\theta_v^* = \left[\frac{R + (\tau + cr)}{\left(P - \frac{R}{i}\right)}\right]^{\frac{1}{\alpha-1}} \tag{5}$$

with $\theta_v^* > 0$ and $\frac{d\theta_v^*}{dP} < 0$, if $P > \frac{R}{i}$.

3.2. Housing demand

In a search and matching framework, the equality between the "actual" housing demand and the "realised" housing demand implies that the search and purchasing process of a home is successful and, thus, $y > m + \tau$. Hence, the housing-demand-side equilibrium condition is the following:

$$\underbrace{\underbrace{(y - R - s)}_{>0} + \theta^{\alpha} \cdot \left\{\left[\left(\frac{M}{LTV} - \tau\right) + (y - m)\right] - P\right\}}_{\text{"actual" housing demand}} = \underbrace{\left[\frac{M}{LTV} + \underbrace{(y - m - \tau)}_{>0}\right]}_{\text{"realised" housing demand}}$$

Search frictions in the housing-demand-side ($\theta = \theta_b^*$) are, thus, the following:

$$\theta_b^* = \left[\frac{\frac{M}{LTV} + (R + s) - (\tau + m)}{\left(\frac{M}{LTV} - P\right) + (y - m - \tau)}\right]^{\frac{1}{\alpha}} \tag{6}$$

with $\theta_b^* > 0$ and $\frac{d\theta_b^*}{dP} > 0$, if the numerator and the denominator of (6) are both positive, viz.:

$$\frac{M}{LTV} > (\tau + m) - (R + s)$$

$$\frac{M}{LTV} > P - (y - m - \tau)$$

Indeed, $\frac{M}{LTV} > P - (y - m - \tau)$ also implies $\frac{M}{LTV} > (\tau + m) - (R + s)$, since a present value (P) is always higher than an algebraic sum of flows, i.e., $P > y - (R + s)$.

3.3. Selling price and search frictions

In search markets, the selling price is the outcome of a bargaining between buyer and seller. Concisely, the sale price must maximise the gain (of both parties) arising from the exchange:

[5] For the sake of simplicity, we assume that the income growth rate is zero and, thus, the capitalization rate is equal to the discount rate. Indeed, Sevelka (2004) shows that the capitalization rate can be approximated by the difference between the discount rate and the income growth rate.

$$\underbrace{max}_{P}\ \Phi \equiv \left\{\left[\left(\frac{M}{LTV}-P\right)+(y-m-\tau)\right]^{\beta}\cdot\left[P-\frac{R}{i}\right]^{1-\beta}\right\}$$

where $0 < \beta < 1$ is the bargaining power of "first-time" buyers. From the previous maximisation, it is straightforward to get the equilibrium selling price ($P = P^*$):[6]

$$P = \left[\frac{M}{LTV}+(y-m-\tau)\right]\cdot(1-\beta)+\beta\cdot\frac{R}{i} \tag{7}$$

Accordingly, P^* ranges between a minimum, i.e., $\frac{R}{i}$, when the bargaining power of "first-time" buyers tends to maximum ($\beta \approx 1$); and, a maximum, i.e., $\left[\frac{M}{LTV}+(y-m-\tau)\right]$, when the bargaining power of "first-time" buyers tends to minimum ($\beta \approx 0$). Note that the range:

$$\frac{M}{LTV}+(y-m-\tau) > P^* > \frac{R}{i}$$

implies that $\theta_v^* > 0$ and $\frac{d\theta_v^*}{dP} < 0$ in Equation (5), and $\theta_b^* > 0$ and $\frac{d\theta_b^*}{dP} > 0$ in Equation (6). Regarding search frictions, it is straightforward to get the starting point of Equations (5) and (6):

$$\lim_{P\to\frac{R}{i}} \theta_v^* \to \infty$$

$$\lim_{P\to\frac{R}{i}} \theta_b^* = \left[\frac{\frac{M}{LTV}+(R+s)-(\tau+m)}{\left(\frac{M}{LTV}-\frac{R}{i}\right)+(y-m-\tau)}\right]^{\overline{\alpha}}$$

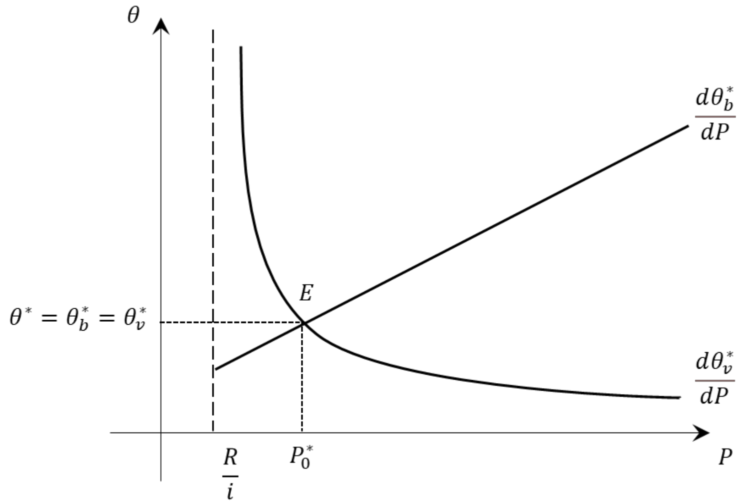

Figure 1. Equilibria in the market of the main dwelling.

As a result, only in one case the supply side's profit maximisation condition coincides with the housing-demand-side equilibrium condition (see **Figure 1**). In general, therefore, search markets are not able to satisfy the housing need.

4. Policy implications

An increase in the satisfaction of the housing need requires an increase in the number of vacant

[6] See the Appendix.

homes in the market (v). By increasing the indicator of search frictions (θ), indeed, an increase in the number of vacant homes improves the market conditions in the housing demand side. An increase in the number of vacant homes in the market, however, requires an increase in the value of a vacant (or unused) house put for sale (V), viz.:

$$V = \frac{-(\tau + cr) + \theta^{\alpha-1} \cdot P}{i + \theta^{\alpha-1}} \tag{3'}$$

By reducing property taxation (τ) and/or costs of maintaining and repairing a house (cr), therefore, it is possible to increase both the value and the number of vacant homes.

From the housing demand side, a reduction in taxation (for example in the form of tax reliefs for the purchase of the first home) increases the "actual" housing demand, as well as the "realised" housing demand, viz.:

$$B = \frac{(y - R - s) + \theta^{\alpha} \cdot \left[\underbrace{\frac{M}{LTV} + (y - m - \tau) - P}_{} \right]}{i} \tag{4'}$$

Further housing policies that increase the "first-time" buyers' value (B) are: (i) a rise in the Loan-To-Value (LTV) ratio; (ii) a reduction in the burden of the mortgage payment (m) that, in turn, implies the reduction of the mortgage interest rate. The changes in the mortgage rate, indeed, affect the perceived value and affordability of purchasing a house for the buyers.

Note that all these housing policies can increase the "actual" and, thus the "realised" housing demand, but a "potential" housing demand remains, however, unsatisfied. In this case, labour market policies that increases both employment and wages are needed. According to the model, the "monetary capacity" (y) to actively search for the main dwelling should increase. Without an enough flow of income (y), in fact, the housing need will remain unfulfilled for many "first-time" buyers.

Finally, people who live with their parents can satisfy the housing need by a family legacy. However, if wages and employment do not grow, in the future fewer and fewer people could satisfy the housing need by a family legacy.

Concisely, the close link between housing and labour markets is crucial in the housing need analysis.

5. Conclusions

Owning a home where one will live most of the time is likely the primary need of human being. Since the housing markets are search markets, the housing need should be analysed by search and matching theory. Furthermore, the housing need analysis should focus on "first-time" buyers.

Accordingly, this theoretical paper introduces the housing need analysis into a search and matching model and evaluates the standard market equilibrium (characterised by the supply side's profit maximisation) in terms of "realised" housing demand.

Two are the main findings of this work. First, the standard market equilibrium with search and matching frictions is not, in general, consistent with the satisfaction of the housing need. Therefore, housing policies should protect the weaker side (the demand side) of the market. Second, housing policies can increase the "actual" and, thus the "realised" housing demand, but labour market policies are needed to increase the "potential" housing demand.

This theoretical paper is a first attempt to study the housing need by the search and matching theory. Future research should develop theoretical models that explicitly consider the close link between housing and labour markets.

Data availability statement

Data not available (theoretical research).

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Conflict of interest

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Appendix

Formally, the solution to the bargaining problem is the following:

$$\frac{d\Phi}{dP} = \beta \cdot \left[\left(\frac{M}{LTV} - P\right) + (y - m - \tau)\right]^{\beta-1} \cdot \frac{d\left[\left(\frac{M}{LTV} - P\right) + (y - m - \tau)\right]}{dP} \cdot \left[P - \frac{R}{i}\right]^{1-\beta} + \left[\left(\frac{M}{LTV} - P\right) + (y - m - \tau)\right]^{\beta} \cdot (1-\beta) \cdot \left[P - \frac{R}{i}\right]^{-\beta} \cdot \frac{d\left[P - \frac{R}{i}\right]}{dP} = 0$$

That yields:

$$\frac{\left[\left(\frac{M}{LTV} - P\right) + (y - m - \tau)\right]^{\beta}}{\left[\left(\frac{M}{LTV} - P\right) + (y - m - \tau)\right]^{\beta-1}} \cdot (1-\beta) = \beta \cdot \frac{\left[P - \frac{R}{i}\right]^{1-\beta}}{\left[P - \frac{R}{i}\right]^{-\beta}}$$

$$\left[\left(\frac{M}{LTV} - P\right) + (y - m - \tau)\right] \cdot (1-\beta) = \beta \cdot \left[P - \frac{R}{i}\right]$$

Finally, it is straightforward to get Equation (7):

$$P = \left[\frac{M}{LTV} + (y - m - \tau)\right] \cdot (1-\beta) + \beta \cdot \frac{R}{i}$$