

Article

Construction of low-carbon economic enterprise management mode based on grey digital model

Yongjun Han¹, Xiao Han^{2,*}, Vincent Wee Eng Kim³, Wen Wen^{4,5}

¹China Institute of Neighborhood Communication on Geographical Indication, Zhejiang Guangsha Vocational and Technical University of Construction, Jinhua 322103, Zhejiang, China

³ First University College, Bandar Utama, Selangor 47800, Malaysia.

⁴ School of Finance, Renmin University of China, Beijing 100872, China

⁵Legal and Compliance Department, China Securities, Beijing 100010, China

* Corresponding author: Xiao Han, 23085448@siswa.um.edu.my

CITATION

Han Y, Han X, Kim VWE, Wen W. (2025). Construction of low-carbon economic enterprise management mode based on grey digital model. Journal of Infrastructure, Policy and Development. 9(1): 11023. https://doi.org/10.24294/jipd11023

ARTICLE INFO

Received: 23 December 2024 Accepted: 9 January 2025 Available online: 15 January 2025

COPYRIGHT



Copyright © 2025 by author(s). Journal of Infrastructure, Policy and Development is published by EnPress Publisher, LLC. This work is licensed under the Creative Commons Attribution (CC BY) license. https://creativecommons.org/licenses/ by/4.0/ Abstract: Since the proposal of the low-carbon economy plan, all countries have deeply realized that the economic model of high energy and high emission poses a threat to human life. Therefore, in order to enable the economy to have a longer-term development and comply with international low-carbon policies, enterprises need to speed up the transformation from a high-carbon to a low-carbon economy. Unfortunately, due to the massive volume of data, developing a low-carbon economic enterprise management model might be challenging, and there is no way to get more precise forecast data. This study tackles the challenge of developing a low-carbon enterprise management mode based on the grey digital paradigm, with the aim of finding solutions to these issues. This paper adopts the method of grey digital model, analyzes the strategy of the enterprise to build the model, and makes a comparative experiment on the accuracy and performance of the model in this paper. The results show that the values of MAPE, MSE and MAE of the model in this paper are the lowest. And the r² of the model in this paper is also the highest. The MAPE value of the model in this paper is 0.275, the MSE is 0.001, and the MAE is 0.003. These three indicators are much lower than other models, indicating that the model has high prediction accuracy. r^2 is 0.9997, which is much higher than other models, indicating that the performance of this model is superior. With the support of this model, the efficiency of building an enterprise model has been effectively improved. As a result, developing an enterprise management model for the low-carbon economy based on the gray numerical model can offer businesses new perspectives into how to quicken the shift to the low-carbon economy.

Keywords: low carbon economy; enterprise management model; construction mode; grey digital model

1. Introduction

The idea of a low-carbon economy has become increasingly popular recently as a way to address a critical environmental issue the world is currently experiencing. Reducing carbon emissions and implementing sustainable, energy-efficient practices are hallmarks of a low-carbon economy, which aims to achieve a more economically and environmentally feasible model of development (Chu et al., 2022; Woon et al., 2023). Enterprises, being a primary catalyst for economic expansion and advancement, are essential to the shift to a low-carbon economy. For this reason, developing a low-carbon economic enterprise management mode is crucial to directing and enabling this shift.

² Faculty of Computer Science and Information Technology, University of Malaya, Kuala Lumpur 50603, Malaysia

The history of the low-carbon economic enterprise management mode maybe links to the industrial revolution, when environmental pollution and resource depletion gradually came to the forefront of public attention (Drahos, 2021; Sun, 2021). The recognition of the limitations of the traditional high-carbon economic model led to the emergence of a growing awareness of the need for more sustainable and environmentally-conscious business practices. This, in turn, spurred the development of the low-carbon economic enterprise management mode, which aims to achieve low-carbon, high-efficiency, and sustainable development through the transformation of the enterprise's internal management and operation strategies (Xu et al., 2022; Zhang et al., 2022).

The key importance of the low-carbon economic enterprise management mode lies in its ability to provide a green and sustainable development path for enterprises, enabling them to reduce their environmental impact and improve their competitiveness (Li, 2021; Yang et al., 2022). By optimizing resource allocation, improving energy use businesses can reduce their carbon emissions and lessen their detrimental effects on the environment by increasing efficiency and the usage of sustainable energy. (Aftab et al., 2023; Challoumis, 2024). This not only helps businesses lower operating expenses but also improve their position in the market, but also establishes a positive social image and enhances consumer trust (Islam, et al., 2021; Tiep Le et al., 2023).

Moreover, low-carbon economic enterprise management mode is essential to promoting the green and low-carbon development in the entire society (Zhang et al., 2022). As an important component of the social and economic fabric, the low-carbon transformation of enterprises can have an adverse impact, encouraging the growth of the whole industrial chain in a more efficient and ecologically friendly manner, so supporting the global economy's sustainable development. (Ibn-Mohammed et al., 2021).

The practical use of low-carbon economic company management models has encountered several obstacles, even though their significance has been widely acknowledged. One of the main concerns is the difficulty in accurately capturing and analyzing the vast amount of data required to inform the decision-making process (Sarker, 2021). Traditional management models often struggle to effectively process and interpret the complex and dynamic data associated with low-carbon initiatives, limiting their ability to provide reliable forecasts and optimize enterprise operations.

In this context, the application of the grey digital model offers a promising solution to the challenges faced in the construction of low-carbon economic enterprise management modes. The grey digital model is a powerful tool that can effectively handle incomplete, imprecise, or limited data sets, extracting valuable insights and patterns to support decision-making (Jayatilake, 2021). By integrating multiple data sources and leveraging advanced digital technologies, the grey digital model can provide more accurate forecasts and comprehensive solutions for the optimization of low-carbon economic enterprise management (Luo et al., 2022).

The unique feature of this research is how the grey digital model was utilized to build low-carbon economic enterprise management modes, a relatively unexplored area in the existing literature. Through the efficient collection and analysis of relevant data, the grey digital model can offer deeper insights and more accurate forecasts to support the transformation of enterprises towards low-carbon, sustainable operations (Luo et al., 2024; Salvi et al., 2022). This approach not only addresses the challenges faced in the implementation of low-carbon enterprise management models but also adds to the body of knowledge regarding the integration of digital technologies and sophisticated analytical methodologies within the framework of sustainable practices in business application.

The goal of the current study is to give a thorough grasp of how low-carbon economic firm management modes are built using the gray digital model. It begins with an overview of the various types of enterprise management models, highlighting the significance of the low-carbon economic enterprise management mode in the context of global environmental challenges. The study then delves into the connotation and characteristics of the low-carbon economy, setting the stage for the in-depth exploration of the grey digital model and its application in the construction of low-carbon economic enterprise management modes.

The core of the study focuses on the detailed examination of the grey digital model, including its underlying theory, methodologies, and capabilities in handling incomplete and uncertain data. The study then presents a comprehensive analysis of how the grey digital model can be leveraged to address the challenges faced in the construction of low-carbon economic enterprise management modes, highlighting the model's ability to provide accurate forecasts, optimize resource allocation, and guide the transformation towards more sustainable business practices.

Furthermore, the study reviews the existing research on the development of low-carbon economic enterprise management modes, critically evaluating the strengths and limitations of alternative approaches, such as Life Cycle Assessment, carbon footprint accounting, and eco-efficiency assessment. By contextualizing the grey digital model within this broader landscape of management strategies, the study underscores the unique advantages and potential of this novel approach in driving the low-carbon transformation of enterprises.

This study offers a comprehensive and innovative perspective on the construction of low-carbon economic enterprise management modes, leveraging the power of the grey digital model to address the complex challenges faced in the transition towards a more sustainable and environmentally-conscious business landscape. The findings of this research will offer insightful analysis and useful recommendations for businesses, decision-makers, and scholars, supporting the continuous endeavors to encourage the growth of a low-carbon economy and advance the long-term viability of international financial institutions.

2. Method of low carbon economy enterprise management mode

2.1. Overview of business management models

Enterprises employ various management models to guide their production activities and achieve business goals. According to the literature, there are five main types of modern enterprise management models (Klymchuk, 2021). The affectionate model relies on the cohesive power of family ties, which can be effective in the early stages but may hinder growth in the long run (Zou et al., 2021). The friendship model is based on the strong bonds between the founding entrepreneurs, but it may face challenges as the enterprise develops and profits are made (Zhang, 2020). The warmth model emphasizes the human element in management, but it can be problematic if it overshadows the need for clear definitions of interests and responsibilities (Xue, 2018). The randomization model can take the form of dictatorial decision-making or excessive government intervention, both of which can be detrimental to the enterprise's long-term development (Klymchuk, 2021; Liu, 2020). The institutionalization model relies on established rules and regulations, which provides structure but may need to be balanced with elements of warmth, friendship, and controlled randomization for optimal management (Hariram et al., 2023; Zou et al., 2021).

There are five main management modes of modern enterprises, as shown in **Figure 1**.

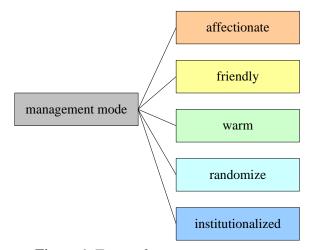


Figure 1. Types of management patterns.

The evolution of enterprise management models can be described in five stages (Hariram et al., 2023; Zou et al., 2021): management (direct coordination and supervision of subordinates by superiors), supervision (monitoring of managers' compliance with laws and regulations), monitoring (observation of the invested enterprises without direct participation), control (control of subsidiaries by the parent company or major shareholders), and governance (establishment of a comprehensive corporate governance structure to align the interests of management and the company) (Zhang, 2020). The importance of modern enterprise management models lies in their ability to attract diverse capital investments, optimize resource allocation, establish a scientific and standardized internal management system, and facilitate the participation of foreign capital (Klymchuk, 2021).

There are also five evolutions of enterprise management models. Specifically, it can be shown in **Figure 2**.

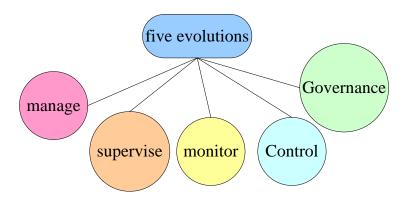


Figure 2. Five evolutions.

Modern enterprise management mode is very important for an enterprise, as shown in **Figure 3**.

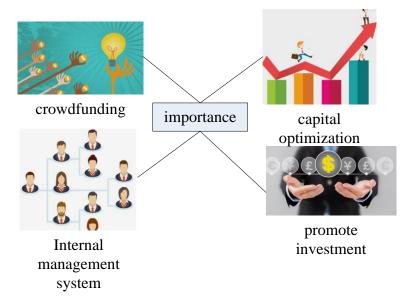


Figure 3. The importance of modern business management models.

2.2. Connotation of a low-carbon economy

A low-carbon economy is a model of economic growth that places a strong emphasis on lowering carbon emissions and consumption through institutional change, technical advancement, and the creation of new energy sources. (Liu, 2020). Compared to the traditional high-carbon, high-consumption, and high-emission economic model, a low-carbon economy has three main characteristics (Hariram et al., 2023; Zou et al., 2021). First, it addresses the urgent need for energy conservation and emission reduction, promoting sustainable social and economic development. Second, it introduces carbon emission restrictions and evaluation systems, aiming to drive sustainable development by addressing the environmental issues caused by the previous industrialization model (Zhang, 2020). Third, the shift to a low-carbon economy is a long process that calls for ongoing advancements in new energy technology as well as heightened public acceptance of low-carbon practices. (Klymchuk, 2021; Liu, 2020). The low-carbon economy represents a shift towards the harmonious coexistence of human activities and the natural environment, addressing the pressing need for sustainable development (Hariram et al., 2023; Zou et al., 2021).

In an economy with low carbon emissions, technical innovation and sustainable development are the foundation, institutional reform, development of new energy, and improved energy utilization. It reduces carbon consumption and reduces carbon dioxide emissions. It enables economic development to coexist in harmony with the earth's ecology. It mainly emphasizes the utilization of low-carbon energy and actively develops low-carbon technologies to replace the previous high consumption and high pollution model. Compared with the traditional economic model, it mainly has three characteristics, as shown in **Figure 4**.

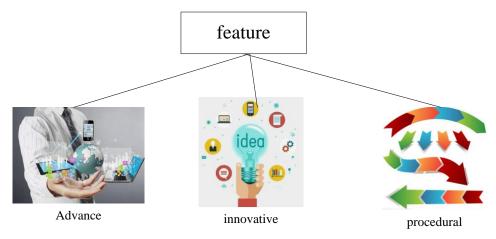


Figure 4. Characteristics of a low-carbon economy.

From **Figure 4**, we can get information that a low-carbon economy has three characteristics.

2.3. Grey mathematical model

The grey mathematical model is a strong instrument for handling incomplete, imprecise, or limited data sets, as it can effectively extract valuable knowledge and patterns from such information (Magno et al., 2024; Yousuf et al., 2021). The key concepts and methods of the grey mathematical model include the whitening of grey numbers, whitening weight functions, and grey prediction models (Yousuf et al., 2021; Magno et al., 2024).

Whitening of grey numbers means that grey numbers that fluctuate around a certain basic value are prone to whitening. If S is used as the grey number of the base value, its whitening value is:

$$\alpha(S) = S \tag{1}$$

If it is a general interval grey number, it can express the whitening value as:

$$\alpha = \beta S + (1 - \beta)N, \beta \in [0, 1]$$
⁽²⁾

A typical weight function for whitening is shown in **Figure 5**:

1

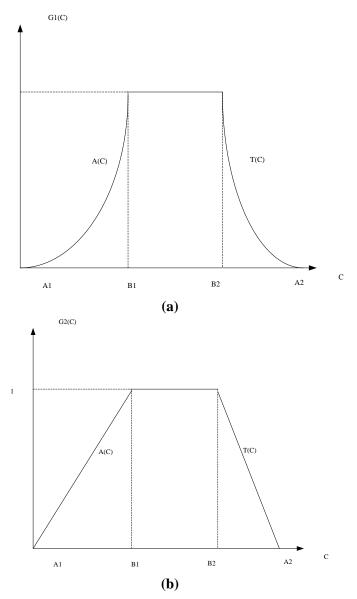


Figure 5. Typical whitening weight function (a) G1(C); (b) G1(C).

A(C) is a left increasing function, T(C) is a right increasing function, and the whitening weight function can be expressed as:

$$G_{1}(C) \begin{cases} A(C), C \in [S_{1}, N_{1}] \\ 1, C \in [N_{1}, N_{2}] \\ T(C), C \in [N_{2}, S_{2}] \end{cases}$$
(3)

When the left-increasing function and the right-increasing function are simplified to a straight line, its function becomes:

$$G_{2}(C) \begin{cases} A(C) = \frac{C - C_{1}}{C_{2} - C_{1}}, C \in [S_{1}, N_{1}] \\ 1, C \in [N_{1}, N_{2}] \\ T(C) = \frac{C_{4} - C}{C_{4} - C_{3}}, C \in [N_{2}, S_{2}] \end{cases}$$
(4)

The following article will introduce the grey prediction model. These models

are suitable for a generalized system whose environment is relatively invariant.

The first prediction model is GM (1, 1). When C is the sequence, I_0 is the next time unit of the O level.

$$F^{(0)}(L_0) = C(L_0) - C(L_0 - 1_0) \neq 0$$
(5)

If C is a sequence with differential formulas, we get:

$$F^{(0)}(L_0) = LIM(C(L_0) - C(L_0 - 1_0))$$
(6)

The second is the GM (1, M) model. $SX_1^{(1)}$ is the adjacent mean generation sequence of $C_1^{(0)}$, then we get:

$$C_1^{(0)}(L) + SX_1^{(1)}(L) = \sum_{O=2}^M N_O C_O^{(1)}(L)$$
(7)

Its least-multiplied-two value needs to satisfy:

$$\hat{S} = (N^Y N)^{-1} N^Y U \tag{8}$$

$$\hat{S} = [S, N_2, N_3, \dots, N_M]^Y = (N^Y N)^{-1} N^Y U$$
(9)

The third model is the GM (0, N) model, $C_1^{(0)}$ is the system feature sequence, then $C_1^{(1)}$ can be expressed as:

$$C_1^{(1)}(L) = \sum_{O=2}^{M} N_O C_O^{(1)}(L) + S$$
⁽¹⁰⁾

Its least squares estimation needs to satisfy:

$$\hat{S} = (N^{Y}N)^{-1}N^{Y}U$$
(11)

Grey relational clustering is also part of the grey model. Assuming that there are M observations, it can be obtained:

$$C_1 = (C_1(1), C_1(2), \dots, C_1(M))$$
(12)

$$\tau_o = \left(\sum_{K=1}^N G_K^1(\mathcal{C}_{OK}) \cdot \gamma_K^1, \sum_{K=1}^N G_K^2\right)$$
(13)

O is the object and L is the grey class, then its fixed-weight clustering function is:

$$\tau_O^L = \sum_{K=1}^N G_K^L(\mathcal{C}_{OK}) \cdot \mu_K \tag{14}$$

Then the coefficients can be obtained by calculation:

$$\tau_O^K = \sum_{K=1}^N G_K^L \left(\mathcal{C}_{OK} \right) \cdot \mu_K \tag{15}$$

The following model can be built to intervalize the feature data:

$$N_{K} = min\{C_{1}(K), C_{2}(K), \dots, C_{N}(K)\}$$
(16)

Making the formula satisfy the following conditions:

$$U_{O}(K) = \frac{C_{O}(K) - N_{K}}{N_{K} - N_{K}}$$
(17)

If we use the whitening function to represent the clustering metrics, we can get **Figure 6**.

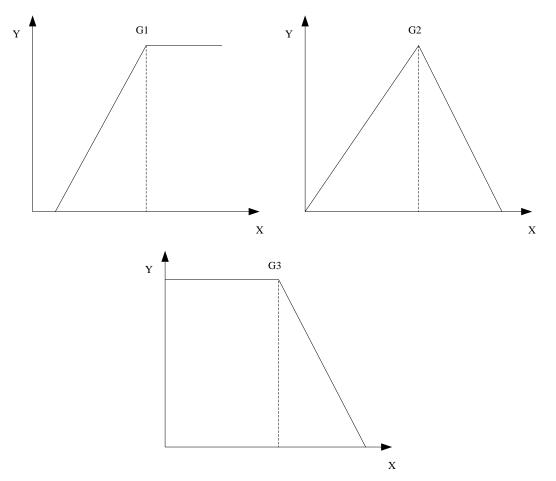


Figure 6. Three types of whitening functions.

Its analytical formula is as follows:

$$G_{K1}(F_{OK}) = \frac{F_{OK} - \gamma_{K1}(1)}{\mu_{K1}(2) - \gamma_{K1}(1)}$$
(18)

The whitening of grey numbers involves transforming fluctuating grey numbers around a certain basic value into a single value, which can be expressed as (Li et al., 2021):

$$\alpha\left(S\right) = S \tag{19}$$

$$\alpha = \beta S + (1 - \beta) N, \beta \in [0, 1]$$
(20)

where S is the grey number of the base value, and N is the general interval grey number.

The whitening weight function can be represented by the following general

form (Li et al., 2021):

 $G_{1}(C) \in \{matrix\}$

 $A(C), C \in S_{1}, N_{1} \in S_{1}, N_{1} \in \mathbb{N}$

1,C\epsilon\left\lbrack N_{1},N_{2} \right\rbrack (21)

 $\left(matrix \right) \right)$

When the left-increasing function and the right-increasing function are simplified to straight lines, the function becomes (Okujeni et al., 2023):

 $\label{eq:constraint} 1, C\epsilon\left\brack N_{1}, N_{2} \right\brack \ (22) \\ T(C) = \frac \{C_{4} - C\} \{C_{4} - C_{3}\}, C \in \left\brack N_{2}, S_{2} \\ \right\brack \ (22)$

\end{matrix} \right.\ \$

The grey prediction models are suitable for generalized systems with relatively invariant environments. The three main models are (Magno et al., 2024; Yousuf et al., 2021):

GM(1,1) model:

$$F^{(0)}(L_{(0)}) = C(L_{(0)}) - C(L_{(0)}-1_{(0)}) \neq 0$$
(23)

$$F^{((O))}(L_{(O)}) = LIM(C(L_{(O)})-C(L_{(O)}-1_{(O)}))$$
(24)

GM(1,M) model:

 $C_{1}^{(0)}(L) + \{SX\}_{1}^{(1)}(L) = \sup\{O = 2\}^{M}\{N_{O}C_{O}^{(1)}\}(L)$ (25)

$$\hat{S} = (N^{(Y)}N) - 1N^{(Y)}U$$
 (26)

$$\hat{S} = [S, N2, N3, ..., N_(M)]^{(Y)} = (N^{(Y)}N)^{-1}N^{(Y)}U$$
(27)

GM(0,N) model:

$$C_{1}^{(1)}(L) = \sum_{O=2}^{M} \{N_{O}C_{O}^{(1)}(L) + S\}$$
(28)

$$\hat{S} = (N^{(Y)}N) - 1N$$
 (29)

These grey prediction models can effectively describe the continuous development of systems and transform unclear grey factors into clearer ones, making them valuable tools for economic and management applications ((Yousuf et al., 2021; Magno et al., 2024):

3. Experiment and low-carbon economy enterprise management mode

3.1. Case data

The case data for this study is a research paper titled "Construction of Lowcarbon Economic Enterprise Management Mode Based on Grey Digital Model" by Han et al. (2023). The paper explores the application of the grey digital model in the construction of a low-carbon economy enterprise management mode.

 Table 1 is obtained after statistical sorting.

Year	Oil consumption	Natural gas consumption	Coal consumption	Waste water	Solid waste	Waste gas
2017	5560.49	408.9	56213.2	593453.21	35291.78	62341.9
2018	6523.71	476.2	60128.3	678823.13	45239.2	73456.2
2019	7645.29	503.1	78231.4	712348.91	58723.8	83458.9

Table 1. Energy consumption and industrial three wastes table.

Detailed information can be obtained from **Table 1**. From 2017 to 2019, energy consumption in the region has been on the rise. It shows that the scale and production of enterprises have increased during this period, and the consumption of energy has also increased. We can find that coal energy is the most consumed energy. In 2017, the consumption in the region reached 56,213.2. In 2018, the coal consumption was 60,128.3. In 2019, the coal consumption was 78,231.4. Coal consumption is forecast to grow in the next few years, but may improve as soon as a low-carbon economy is implemented. With reference to the omission of pollutants, we can see that the discharge of wastewater is the largest, so that more pollutants are discharged. In 2017, the discharge of wastewater from enterprises in the region was 593,453.21, in 2018, there was 678,823.13, and in 2019, there was 712,348.91. It is also in a state of continuous growth, which shows that a large amount of wastewater will be generated when the enterprise is in production.

By initialize the data in **Table 1** to make the data clearer, and then we can get **Table 2**.

Year	Oil consumption	Natural gas consumption	Coal consumption	Waste water	Solid waste	Waste gas
2017	0.7	0.78	0.86	0.97	0.97	0.72
2018	0.89	0.84	0.91	1.08	0.98	0.83
2019	1.0	1.0	1.0	1.2	1.1	1.1

 Table 2. Initialization processing table.

The data on energy consumption and environmental pollution can be found very clearly from **Table 2**. According to the grey model above, we can calculate the correlation coefficient between the two, thus obtaining **Tables 3–5**.

	2017	2018	2019	
Waste water	0.34	0.48	1.0	
Solid waste	0.6	0.57	1.0	
Waste gas	0.48	0.75	1.0	

Table 3. Coal grey connection coefficient.

	Table 4. Petroleum grey correlation coefficient.			
	2017	2018	2019	
Waste water	0.34	0.48	1.0	
Solid waste	1.0	0.9	1.0	
Waste gas	0.9	0.95	1.0	

Table 5. Natural gas grey correlation coefficient.

	2017	2018	2019	
Waste water	0.34	0.5	1.0	
Solid waste	0.74	0.64	1.0	
Waste gas	0.61	0.83	1.0	

From **Tables 3–5**, we can get the information that the correlation between coal, oil and gas and industrial wastewater is not very different. Because a plan to purify wastewater has been proposed since a very early time, the effect of water control by various governments is also good. In 2017, the correlation between oil and wastewater was 0.34. Its correlation with solid waste is 1.0, and its correlation with exhaust gas is 0.9. The correlation between coal and wastewater is 0.34, the correlation with solid waste is 0.6, and the correlation with waste gas is 0.48. The correlation between natural gas and wastewater is 0.34, the correlation with solid waste is 0.74, and the correlation with exhaust gas is 0.61. This shows that different energy sources cause different pollution to the environment. Exhaust gas will cause global warming and threaten the safety of all human beings. However, the discharge of waste water and waste is very polluting to the environment and has seriously affected the natural ecology. This shows that enterprises are still mainly focused on energy consumption, which is not a favourable strategy of sustainable development. Businesses should transform as soon as possible to a low-carbon economic management model.

3.2. Experiments based on grey mathematical model

The researchers first provide an overview of various enterprise management models, including the affectionate, friendship, warmth, randomization, and institutionalization models, as well as the evolution of these models from management to governance. They then delve into the connotation of a low-carbon economy, highlighting its characteristics of advance, innovation, and procedural nature (Klymchuk, 2021; Liu, 2020).

The researchers introduce the grey mathematical model, including the concepts of whitening of grey numbers, whitening weight functions, and the three main grey prediction models: GM(1,1), GM(1,M), and GM(0,N) (Yousuf et al., 2021; Magno et al., 2024):

3.3. Experiments based on grey mathematical model

The researchers conducted experiments to examine the application of the grey mathematical model in constructing a low-carbon economy enterprise management mode. They utilized the three main grey prediction models—GM(1,1), GM(1,M), and GM(0,N)—to analyze the performance and accuracy of the proposed model.

In the GM(1,1) model, the researchers established the following equations (Yousuf, M. U., et al., 2021; Magno, F., et al., 2024):

$$F^{((O))}(L_{(O)}) = C(L_{(O)}) - C(L_{(O)}-1_{(O)}) \neq 0$$
(30)

$$F^{(0)}(L_{(0)}) = LIM(C(L_{(0)})-C(L_{(0)}-1_{(0)}))$$
(31)

where $F^{((O))}(L_{(O)})$ represents the first-order accumulated generating operation (AGO) of the original sequence $C(L_{(O)})$, and $C(L_{(O)})$ and $C(L_{(O)})$ are the data points in the original sequence.

For the GM(1,M) model, the researchers used the equations (Zhu & Qian, 2015; Yang, 2019):

$$C_{1}^{(0)}(L) + \{SX\}_{1}^{(1)}(L) = \sup\{O = 2\}^{M}\{N_{O}C_{O}^{(1)}\}(L)$$
(32)

$$\hat{S} = (N^{(Y)}N) - 1N^{(Y)}U$$
 (33)

$$\hat{S} = [S, N2, N3, ..., N_(M)]^{(Y)} = (N^{(Y)}N) - 1N^{(Y)}U$$
(34)

where $C_{1}^{(0)}(L)$ is the original sequence, $\{SX\}_{1}^{(1)}(L)$ is the first-order accumulated generating operation, $C_{0}^{(1)}(L)$ are the data points in the first-order accumulation, and \hat{S} is the estimated parameter vector.

Finally, for the GM(0,N) model, the researchers employed the following equations (Liu, 2020; Zhou, 2017):

$$C_{1}^{(1)}(L) = \sum_{0 = 2}^{M} \{N_{0}C_{0}^{(1)}(L) + S\}$$
 (35)

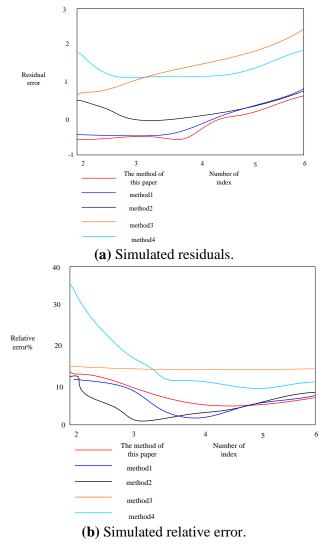
$$\hat{\mathbf{S}} = (\mathbf{N}^{\wedge}(\mathbf{Y})\mathbf{N}) - 1\mathbf{N} \tag{36}$$

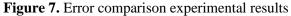
where $C_{1}^{(1)}(L)$ is the first-order accumulated sequence, $C_{0}^{(1)}(L)$ are the data points in the first-order accumulation, and \hat{S} is the estimated parameter vector.

The researchers then conducted comparative experiments to assess the level of performance and accuracy of the suggested model based on grey mathematical approach. They analyzed various statistical indicators, such as Mean Absolute Percentage Error (MAPE), Mean Squared Error (MSE), Mean Absolute Error (MAE), and the coefficient of determination (R^2), to assess the effectiveness of the grey digital model in constructing the low-carbon economy enterprise management mode.

The results of the experiments demonstrated that the values of MAPE, MSE, and MAE for the model proposed in the study were the lowest compared to other models, indicating its high prediction accuracy. Additionally, the R^2 value of the

proposed model was the highest, suggesting its superior performance in capturing the underlying patterns and dynamics of the low-carbon economy enterprise management mode (Klymchuk, 2021; Liu, 2020).





In order to prove that the method in this paper has a good prediction effect in constructing a low-carbon economic enterprise management model, we have done a comparative experiment between the grey digital model and several other models. This paper uses the Grey Mathematical Model and several specific forms of it, including the GM (1, 1) Model, the GM (1, M) Model, and the GM (0, N) Model. The following is a brief description of the differences between these models and others not mentioned. Grey Mathematical Model. This is a general term for a model based on gray system theory that is used to deal with situations of incomplete information or small samples of data. Grey models are used to generate data series (e.g., cumulative, cumulative subtraction, etc.) to explore the underlying patterns in the data, and then build differential equation models for prediction or analysis. GM (1, 1) Model. This is the most widely utilized type of grey model, where "1, 1" denotes a first-order differential equation and a variable. It is suitable for data series

with strong exponential patterns, especially those processes that show monotonic changes. The model generates (1-AGO) by accumulating the original data series once, weakening the randomness of the data to show more obvious patterns, and then builds a first-order differential equation to make predictions. GM (1, M) Model. It is similar to GM (1, 1), but "M" indicates that more than one variable is considered in the model. This particular model can be used for analyzing multiple correlated factors. This model is suitable for analyzing the effects of multiple related factors on a variable. By building a first-order differential equation with multiple independent variables, the trend of the dependent variable can be predicted. GM (0, N) Model. "0, N" indicates that the model is a zero-order differential equation and contains N variables. Unlike general multiple linear regression models, the GM (0, N) is modeled on the basis of a 1-AGO series of the original data. The model is suitable for analyzing the static relationship of several independent factors that influence the dependent variable. The result is shown in **Figure 7**.

(a) Residual Error Comparison

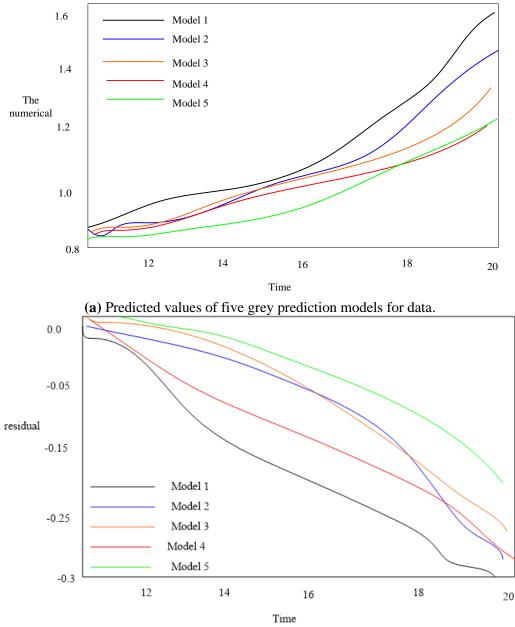
This plot compares the residual errors of the model proposed against four other models (labeled "method1", "method2", "method3", and "method4"). The model in this paper shows the smallest residual errors across the index values, indicating higher prediction accuracy.

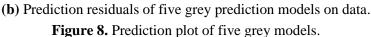
(b) Relative Error Comparison

This plot compares the relative errors of the same five models. The model demonstrates the lowest relative errors, further confirming its superior predictive performance compared to the other methods.

The information can be obtained from Figure 7. The qualitative comparison of the residual curve and the relative error curve shown in the figure shows that it is easy to know that method 3 and method 4 have larger simulation errors. The method model in this paper has the smallest simulation error, and the method 1 and method 2 have similar simulation errors. In other words, the simulation where our method's accuracy surpasses that of other methods. Because it can dynamically change its structure according to the characteristics of unknown system parameter changes to better adapt to the evolution trend of data. The simulation performance and adaptive performance of the grey model put forward in this work are superior to the conventional model. The grey model has many excellent properties, and it is a more generalized representation of previous models. Moreover, the predictive properties of the model show that it has a strong predictive ability for perturbed sequences. It can predict approximate exponential sequences with steady-state, constant-speed perturbations, and acceleration perturbations without error. In fact, it can be found that in order to improve the prediction accuracy, there are many types of models. It includes many of our common prediction models. However, after experimental comparison, the error of the grey digital model is still the smallest. This shows that the model is very suitable for the construction of low-carbon economic enterprise management mode.

To better illustrate the model's applicability in this study, we used five grey prediction models to predict the data of the low-carbon economy enterprise management model. The result is shown in **Figure 8**.





Information can be obtained from **Figure 8**, the anticipated values of the data parameters of the five grey models for the construction of a low-carbon economy enterprise management model. It is evident that the method model has the lowest forecast accuracy, while the method 2 model has a higher prediction accuracy than the method 1, but lower than that of the method 3. Both method four and method five have fairly high prediction accuracy. But in general, the accuracy of method five is slightly higher than that of method four, because the latter fits the original data better. We also give a dotted line plot of the residuals. It can be found that the prediction of accuracy on each of the five models for the sequence of enterprise management patterns is consistent. Although the above error comparison experiments are also carried out, the data obtained are not accurate enough.

According to the reference data, the error indicators are MAPE, MSE, and MAE. The smaller their values, the better the prediction accuracy of the model. The larger the value of r^2 , the better the performance of the model. In the following, we will obtain the numerical value of the model according to these indicators. The result is shown in **Figure 9**.

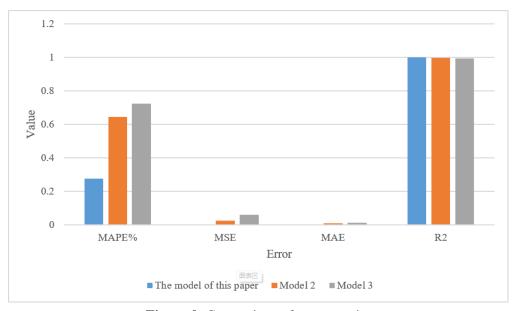


Figure 9. Comparison of error metrics

The consistently lower error metrics and higher R^2 value for the proposed model confirm its effectiveness in constructing low-carbon economy enterprise management modes compared to alternative approaches.

The data used in the experiments to evaluate the performance of the grey digital model in constructing a low-carbon economy enterprise management mode was obtained from through surveys, reports, or other means of data gathering within the scope of their case study. The data used in this case study, serves as an effective example to showcase the capabilities of the grey digital model and its applicability in the realm of sustainable enterprise management.

The information can be obtained from **Figure 9**. The values of MAPE, MSE and MAE of the model in this paper are all the lowest, indicating that the model's prediction accuracy is very high. And the r^2 of the model in this paper is also the highest, indicating that the performance of the model is very superior. The MAPE value of the model in this paper is 0.275, the MSE is 0.001, and the MAE is 0.003, which are much lower than other models. r^2 is 0.9997, which is much higher than other models. The MAPE value of Model 2 was 0.645, the MSE was 0.025, the MAE was 0.008, and the r^2 was 0.9962. The MAPE value of Model 3 was 0.723, the MSE was 0.061, the MAE was 0.011, and the r^2 was 0.9932. The good performance ability of the model in this paper just proves that it is very suitable for the construction of enterprise management mode. It believes that using this model to participate in model construction can achieve very good results.

Below we conduct a comparative analysis of the model grey generation operator. Different from other commonly used grey accumulation techniques, the damped accumulation proposed in this paper is a unique data conversion method. Through a damped trend parameter, information is provided in various weights related to the time factor. In order to compare the differences between the four new grey generation operators, we combined the model in this paper with four different grey generation operators to obtain the prediction results of the data. It replaces the traditional first-order accumulation data generation operation with four grey generation operators, and other modelling calculation steps remain unchanged. When the cumulative parameter is 1, the calculation results of the four cumulative grey models are equivalent to the traditional first-order cumulative grey prediction model. It brings the data into four models, and thus obtains the prediction results of the four grey generation operators. The results are shown in **Figure 10**.

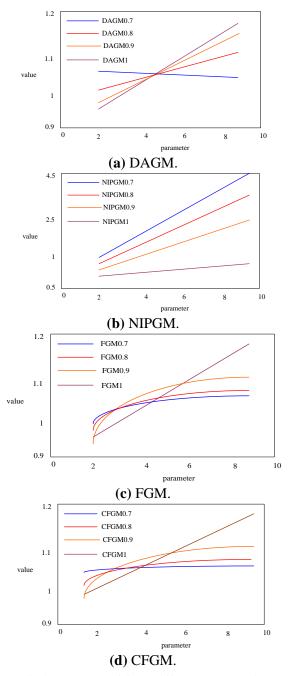


Figure 10. Prediction results of four different cumulative grey models.

Relevant information can be obtained from **Figure 10**. When the accumulation parameter is less than 1, the anticipated value sequence of NIPAGO is much larger than the exponential rate of the conventional first-order accumulation model in this paper. Obviously, it is extremely unreasonable to only consider the information priority and ignore the change of the growth rate of the predicted value. On the other hand, depending on fractional accumulation and differential reduction calculations, FAGO and CFAGO can perform nonlinear processing on the initial sequence. It has a high performance in data fitting, but the prediction effect of the model cannot be guaranteed. The figure shows that the trend of the prediction results of the two fractional order models shows an increase first and then a decrease. This is inconsistent with the monotonous growth trend of the traditional grey model. Different from the above three grey generation operators, the DAGO proposed in this chapter can effectively change the exponential growth rate of the prediction data by adjusting the damping parameters, while the prediction result sequence remains monotonic. With the decrease of the damping parameter, the exponential growth trend of the prediction results decreases continuously, and even the concaveconvexity changes.

The researchers concluded that the construction of a low-carbon economy enterprise management mode based on the grey digital model can provide new insights and strategies for enterprises to accelerate their transition towards a low-carbon economy. The grey mathematical model's ability to effectively utilize limited and imprecise data makes it a valuable tool for enterprises in their efforts to adapt to the changing environmental and regulatory landscape (Li et al., 2023; Mierzwiak R, 2024; Rajagopal et al., 2022).

Businesses can effectively operationalize the grey digital model proposed in the study to construct a low-carbon economy enterprise management mode through a comprehensive implementation framework. This approach can guide enterprises in navigating the complex transition towards sustainability and optimizing their resource utilization and environmental impact.

The first step is to assess the enterprise's current state and challenges. Businesses should evaluate their existing management model and its alignment with low-carbon principles, identify key pain points, data limitations, and obstacles in the transition towards a low-carbon economy. Understanding the enterprise's energy consumption patterns, emissions, and environmental impact is crucial for informing the subsequent steps.

Next, the enterprise should adopt the grey digital model by familiarizing the management team with its underlying concepts and capabilities. Determining the appropriate grey prediction models, such as GM (1, 1), GM (1, M), and GM (0, N), based on the enterprise's data availability and needs is a crucial decision. Establishing robust data collection, processing, and analysis processes is essential to ensure the model's inputs are reliable and accurate.

The third step involves integrating the grey digital model into the enterprise's decision-making framework. Leveraging the model's ability to handle incomplete and uncertain data, businesses can generate accurate forecasts and insights to guide resource allocation, energy efficiency improvements, and emissions reduction strategies. By incorporating the model's outputs into the enterprise's strategic

planning, target setting, and performance monitoring, the low-carbon transformation can be seamlessly woven into the organization's overall operations and objectives.

Continuous monitoring and adjustment of the grey digital model's performance is the fourth step. Enterprises should regularly review the model's alignment with their evolving low-carbon goals, refine its parameters and assumptions based on changing market conditions, technology advancements, and regulatory requirements. Establishing feedback loops and a culture of experimentation and adaptation can help drive continuous improvements in the model's effectiveness and the enterprise's low-carbon transition.

In addition to the implementation framework, businesses should consider the following best practices for effective operationalization of the grey digital model:

- 1) Secure Top Management Commitment: Gain the full support and buy-in from the enterprise's leadership team to ensure the successful adoption of the grey digital model and align the low-carbon transformation with the organization's strategic objectives.
- 2) Build Cross-Functional Collaboration: Engage stakeholders from various departments to foster a collaborative approach to data collection, analysis, and decision-making, and enhance the understanding and utilization of the grey digital model across the organization.
- 3) Invest in Data Management Capabilities: Develop robust data governance frameworks to ensure the quality, reliability, and accessibility of the data required for the grey digital model, and leverage digital technologies and automation to streamline data-related processes.
- 4) Prioritize Continuous Learning and Improvement: Establish mechanisms for regular review, evaluation, and refinement of the grey digital model's performance, and encourage a culture of experimentation, adaptation, and knowledge-sharing to drive continuous improvements in the model's effectiveness and the enterprise's low-carbon transition.

By following this comprehensive implementation framework and best practices, businesses can effectively operationalize the grey digital model to construct a lowcarbon economy enterprise management mode. This approach can help enterprises navigate the complex and dynamic landscape of environmental sustainability, optimize their resources, and accelerate their transition towards a more sustainable and resilient future.

4. Conclusion

The study demonstrates the potential of the grey digital model in constructing a low-carbon economy enterprise management mode. The researchers provide a comprehensive overview of various enterprise management models and the connotation of a low-carbon economy, laying the foundation for their application of the grey mathematical model.

The researchers' experiments based on the grey prediction models, including GM (1, 1), GM (1, M), and GM (0, N), showcased the effectiveness of the grey digital model in addressing the challenges associated with the construction of a low-carbon economy enterprise management mode. The statistical analyses, such as

MAPE, MSE, MAE, and R², revealed indicated the suggested model performed better overall and in terms of prediction accuracy than alternative methods. (Chicco et al., 2021; Klymchuk, 2021; Liu, 2020).

The key findings of the study indicate that the grey digital model can provide enterprises with a robust and reliable framework for transitioning towards a lowcarbon economy. The model's ability to handle incomplete, inexact, and limited data sets makes it particularly valuable in the context of environmental and sustainabilityrelated management challenges (Gölgeci et al., 2023; Javanmardi et al., 2023; Ozkan et al., 2023).

Furthermore, the researchers emphasize that the construction of a low-carbon economy enterprise management mode based on the grey digital model can offer new strategies and insights for enterprises to accelerate their transformation towards a more sustainable and eco-friendly business model. This approach aligns with the broader worldwide initiatives to reduce climate change and advance sustainable development (Yang, 2019; Zhang, 2020).

The study's results have significant consequences for both academic research and practical business applications. From a philosophical standpoint, the researchers contribute to the existing literature on enterprise management models and the integration of low-carbon principles into organizational structures and decisionmaking processes (Klymchuk, 2021; Liu, 2020). Practitioners, on the other hand, can leverage the insights provided in this study to guide their enterprises' transition towards a low-carbon economy, leveraging the grey digital model as a strategic tool for data-driven decision-making and sustainable business practices (Adaga et al., 2023; Gökalp et al., 2021).

The proposed low-carbon economic enterprise management model, based on the grey digital model, offers significant advantages over traditional methods. Compared to conventional management strategies, which often rely on linear models and historical data, the grey digital model excels in handling incomplete and uncertain datasets, thereby providing more accurate forecasts and insights. This model's performance was demonstrated through comparative experiments, where it achieved superior values in key statistical indicators such as Mean Absolute Percentage Error (MAPE), Mean Squared Error (MSE), and the coefficient of determination (R^2), outperforming other models.

However, while the grey digital model presents a robust framework for managing low-carbon enterprises, it is not without limitations. For instance, its reliance on the quality and availability of data can pose challenges, particularly in regions where data collection mechanisms are underdeveloped (Ogbu et al., 2024). Additionally, the complexity of the model may require specialized knowledge and training for effective implementation, which could hinder its adoption among smaller enterprises lacking the necessary resources (Khanzode, 2021). Thus, while the proposed model offers a promising approach to fostering low-carbon transitions, its practical application will require addressing these limitations to ensure broader usability and effectiveness in diverse economic contexts.

In conclusion, the research offers a valuable contribution to the field of lowcarbon economy enterprise management. The application of the grey digital model demonstrates its potential in helping enterprises navigate the complex and evolving landscape of environmental sustainability, ultimately paving the path to a future in which the economy is more robust and sustainable.

Author contributions: Conceptualization, YH and XH; methodology, XH; software, XH; validation, YH, VWEK and XH; formal analysis, YH; investigation, YH; resources, YH; data curation, YH; writing—original draft preparation, YH; writing—review and editing, YH and WW; visualization, YH; supervision, XX; project administration, YH; funding acquisition, YH. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the NSFC Project "Research on Long-term Mechanism for Solving Relative Poverty in Ethnic Minority Areas" (No. 20BMZ14).

Data availability: Upon request, the corresponding author will provide the data used to support the study's conclusions.

Conflict of interest: The authors declare no conflict of interest.

References

- Adaga, E. M., Okorie, G. N., Egieya, Z. E., et al. (2023). The role of big data in business strategy: a critical review. Computer Science & IT Research Journal, 4(3), 327–350.
- Aftab J, Abid N, Cucari N, et al. (2022). Green human resource management and environmental performance: The role of green innovation and environmental strategy in a developing country. Business Strategy and the Environment, 32(4), 1782–1798. https://doi.org/10.1002/bse.3219
- Challoumis, C. (2024). Building A Sustainable Economy-How AI Can Optimize Resource Allocation. In: Proceedings of the XVI International Scientific Conference. pp. 190 224.
- Chicco D, Warrens MJ, Jurman G. (2021). The coefficient of determination R-squared is more informative than SMAPE, MAE, MAPE, MSE and RMSE in regression analysis evaluation. PeerJ Computer Science, 7, e623. https://doi.org/10.7717/peerj-cs.623
- Chu W, Vicidomini M, Calise F, et al. (2022). Recent Advances in Low-Carbon and Sustainable, Efficient Technology: Strategies and Applications. Energies, 15(8), 2954. https://doi.org/10.3390/en15082954
- Drahos, P. (2021). Survival governance: Energy and climate in the Chinese century. Oxford University Press, USA.
- Gökalp MO, Gökalp E, Kayabay K, et al. Data-driven manufacturing: An assessment model for data science maturity. Journal of Manufacturing Systems. 2021; 60: 527 546. https://doi.org/10.1016/j.jmsy.2021.07.011
- Gölgeci I, Gligor DM, Bayraktar E, et al. (2023). Reimagining global value chains in the face of extreme events and contexts: Recent insights and future research opportunities. Journal of Business Research, 160, 113721. https://doi.org/10.1016/j.jbusres.2023.113721
- Han, Y., Han, X., & Wen, W. (2023). Construction of low-carbon economic enterprise management mode based on grey digital model. Springer Nature. Unpublished work.
- Hariram NP, Mekha KB, Suganthan V, et al. (2023). Sustainalism: An Integrated Socio-Economic-Environmental Model to Address Sustainable Development and Sustainability. Sustainability, 15(13), 10682. https://doi.org/10.3390/su151310682
- Ibn-Mohammed T, Mustapha KB, Godsell J, et al. (2021). A critical analysis of the impacts of COVID-19 on the global economy and ecosystems and opportunities for circular economy strategies. Resources, Conservation and Recycling, 164, 105169. https://doi.org/10.1016/j.resconrec.2020.105169
- Islam T, Islam R, Pitafi AH, et al. (2021). The impact of corporate social responsibility on customer loyalty: The mediating role of corporate reputation, customer satisfaction, and trust. Sustainable Production and Consumption, 25, 123 135. https://doi.org/10.1016/j.spc.2020.07.019
- Javanmardi E, Liu S, Xie N. Exploring the Challenges to Sustainable Development from the Perspective of Grey Systems Theory. Systems. 2023; 11(2): 70. doi: 10.3390/systems11020070
- Jayatilake, S. M. D. A. C., & Ganegoda, G. U. (2021). Involvement of Machine Learning Tools in Healthcare Decision Making.

Journal of Healthcare Engineering, 2021, 1–20. https://doi.org/10.1155/2021/6679512

- Khanzode, A. G., Sarma, P. R. S., Mangla, S. K., et al. (2021). Modeling the Industry 4.0 adoption for sustainable production in Micro, Small & Medium Enterprises. Journal of Cleaner Production, 279, 123489. https://doi.org/10.1016/j.jclepro.2020.123489
- Klymchuk, O. (2021). Modern enterprise management models. International Journal of Management and Economics, 67(1), 45– 57.
- Li, F., Xu, X., Li, Z., et al. (2021). Can low-carbon technological innovation truly improve enterprise performance? The case of Chinese manufacturing companies. Journal of Cleaner Production, 293, 125949. https://doi.org/10.1016/j.jclepro.2021.125949
- Li, J., Wang, Y., Zi, Y., et al. (2022). Whitening-Net: A Generalized Network to Diagnose the Faults Among Different Machines and Conditions. IEEE Transactions on Neural Networks and Learning Systems, 33(10), 5845–5858. https://doi.org/10.1109/tnnls.2021.3071564
- Li, X., Wu, X., & Zhao, Y. (2023). Research and application of multi-variable grey optimization model with interactive effects in marine emerging industries prediction. Technological Forecasting and Social Change, 187, 122203. https://doi.org/10.1016/j.techfore.2022.122203
- Liu, Y. (2020). Exploring the connotation of low-carbon economy. Environmental Economics, 11(3), 67–75.
- Luo, J., Zhuo, W., Liu, S., & Xu, B. (2024). The optimization of carbon emission prediction in low carbon energy economy under big data. IEEE Access.
- Magno, F., Cassia, F., & Ringle, C. M. (2024). A brief review of partial least squares structural equation modeling (PLS-SEM) use in quality management studies. The TQM Journal, 36(5), 1242–1251. https://doi.org/10.1108/tqm-06-2022-0197
- Mierzwiak, R. (2024). Methodological Aspects of Grey Systems Theory in Management Research. In: Series on Grey System. Springer Nature Singapore. pp. 33 - 60.
- Ogbu, A. D., Iwe, K. A., Ozowe, W., & Ikevuje, A. H. (2024). Geostatistical concepts for regional pore pressure mapping and prediction. Global Journal of Engineering and Technology Advances, 20(1), 105 117.
- Okujeni, S., & Egert, U. (2023). Structural Modularity Tunes Mesoscale Criticality in Biological Neuronal Networks. The Journal of Neuroscience, 43(14), 2515–2526. https://doi.org/10.1523/jneurosci.1420-22.2023
- Ozkan, S., Romagnoli, S., & Rossi, P. (2023). A novel approach to rating SMEs' environmental performance: Bridging the ESG gap. Ecological Indicators, 157, 111151. https://doi.org/10.1016/j.ecolind.2023.111151
- Rajagopal, N. K., Saini, M., Huerta-Soto, R., et al. (2022). Human Resource Demand Prediction and Configuration Model Based on Grey Wolf Optimization and Recurrent Neural Network. Computational Intelligence and Neuroscience, 2022, 1–11. https://doi.org/10.1155/2022/5613407
- Salvi, M., Jensen, K., Stoermer, E., et al. (2022). Towards a green & digital future. Publications Office of the European Union.

Sarker, I. H. (2021). Data Science and Analytics: An Overview from Data-Driven Smart Computing, Decision-Making and Applications Perspective. SN Computer Science, 2(5). https://doi.org/10.1007/s42979-021-00765-8

- Sun, X. (2023). Sustainable Development Integration in Coal Trading Company: Strategies, Challenges, and Societal Implications [Master's thesis]. ISCTE-Instituto Universitario de Lisboa.
- Tiep Le, T., Ngo, H. Q., & Aureliano-Silva, L. (2023). Contribution of corporate social responsibility on SMEs' performance in an emerging market – the mediating roles of brand trust and brand loyalty. International Journal of Emerging Markets, 18(8), 1868 - 1891. https://doi.org/10.1108/ijoem-12-2020-1516
- Woon, K. S., Phuang, Z. X., Taler, J., et al. (2023). Recent advances in urban green energy development towards carbon emissions neutrality. Energy, 267, 126502. https://doi.org/10.1016/j.energy.2022.126502
- Xu, P., Chen, L., & Dai, H. (2022). Pathways to Sustainable Development: Corporate Digital Transformation and Environmental Performance in China. Sustainability, 15(1), 256. https://doi.org/10.3390/su15010256
- Yang, X., Guo, Y., Liu, Q., et al. (2022). Dynamic Co-evolution analysis of low-carbon technology innovation compound system of new energy enterprise based on the perspective of sustainable development. Journal of Cleaner Production, 349, 131330. https://doi.org/10.1016/j.jclepro.2022.131330
- Yang, Z. (2019). Friendship-based enterprise management model: Challenges and opportunities. Journal of Management Science, 15(2), 87–99.
- Yousuf, M. U., Al-Bahadly, I., & Avci, E. (2021). A modified GM(1,1) model to accurately predict wind speed. Sustainable Energy Technologies and Assessments, 43, 100905. https://doi.org/10.1016/j.seta.2020.100905

- Zhang, H. (2020). Warmth-based enterprise management: Strengths and limitations. International Business Review, 25(1), 123 134.
- Zhang, T., Shi, Z.-Z., Shi, Y.-R., et al. (2021). Enterprise digital transformation and production efficiency: mechanism analysis and empirical research. Economic Research-Ekonomska Istraživanja, 35(1), 2781–2792. https://doi.org/10.1080/1331677x.2021.1980731
- Zhang, Z., Hu, G., Mu, X., et al. (2022). From low carbon to carbon neutrality: A bibliometric analysis of the status, evolution and development trend. Journal of Environmental Management, 322, 116087. https://doi.org/10.1016/j.jenvman.2022.116087
- Zou, C., Xue, H., Xiong, B., et al. (2021). Connotation, innovation and vision of "carbon neutrality." Natural Gas Industry B, 8(5), 523–537. https://doi.org/10.1016/j.ngib.2021.08.009