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ORIGINAL ARTICLE

Assessing the effect of public capital on growth: An extension of the World Bank Long-Term Growth Model

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ABSTRACT

To analyze the effect of an increase in the quantity or quality of public investment on growth, this paper extends the World Bank's Long-Term Growth Model (LTGM), by separating the total capital stock into public and private portions, with the former adjusted for its quality. The paper presents the LTGM public capital extension and accompanying freely downloadable Excel-based tool. It also constructs a new infrastructure efficiency index, by combining quality indicators for power, roads, and water as a cardinal measure of the quality of public capital in each country. In the model, public investment generates a larger boost to growth if existing stocks of public capital are low, or if public capital is particularly important in the production function. Through the lens of the model and utilizing newly-collated cross-country data, the paper presents three stylized facts and some related policy implications. First, the measured public capital stock is roughly constant as a share of gross domestic product (GDP) across income groups, which implies that the returns to new public investment, and its effect on growth, are roughly constant across development levels. Second, developing countries are relatively short of private capital, which means that private investment provides the largest boost to growth in low-income countries. Third, low-income countries have the lowest quality of public capital and the lowest efficient public capital stock as a share of GDP. Although this does not affect the returns to public investment, it means that improving the efficiency of public investment has a sizable effect on growth in low-income countries. Quantitatively, a permanent 1 ppt GDP increase in public investment boosts growth by around 0.1–0.2 ppts over the following few years (depending on the parameters), with the effect declining over time.

Keywords: *long-term growth; infrastructure; public capital; public investment efficiency*
JEL Classification: *O40, O57, H41*

1. Introduction

Inadequate infrastructure, especially public infrastructure, is often viewed as a key impediment to economic growth and development in low- and middle-income countries. While increasing infrastructure investment has been a part of national development strategies for

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decades, its perceived importance has gained prominence with the rapid development of China and its infrastructure-led growth strategy; as well increased infrastructure-specific finance through new bilateral lending, the Asian Infrastructure Bank, and the Belt and Road Initiative.

Despite the importance of public infrastructure investment, there is wide disagreement about the size and significance of its effect on growth in developing countries (Calderon and Serven, 2014). On the one hand, the needs are clearly great – close to 700 million people do not have access to safe drinking water and 1.2 billion are without electricity – and so one should expect a sizable impact.[1] Several papers have estimated large returns to infrastructure investment – most frequently cited, Aschauer (1989). However, as infrastructure investment is endogenous – for example, growth for other reasons might generate public revenues which allow the construction of infrastructure – many of those empirical studies lack causal validity and estimated impacts are implausibly large. Many other papers have found insignificant or negative impacts (Bom and Ligthart, 2014), possibly because public investment in developing countries often fails to generate productive capital due to corruption and the presence of "white elephants" (Pritchett, 2000).

Perhaps less appreciated is that there is a great deal of confusion in the empirical and policy discussion about the dynamics and mechanisms through which public infrastructure investment would affect growth. For example, empirical studies (and policy reports) are often vague about whether it is the level of infrastructure that affects growth, or whether infrastructure investment (and hence changes in infrastructure levels) affects growth. Likewise, empirical studies often have difficulty estimating when the boost to growth might occur (whether the size of the effect will increase or decrease over time) and what country-level factors determine the impact on growth (as different studies are for particular countries or reflect a cross-country average). All these aspects are crucial for evaluating the effectiveness of a country's public investment-led growth plans.

This paper makes contributions in two areas to try to address these gaps. First, we develop a model of the effect of public investment on long-term growth – called the Long-Term Growth Model Public Capital Extension (LTGM-PC) – that is simple enough to be solved in an Excel spreadsheet without macros (which is provided as a companion to this paper on the website www.worldbank.org/LTGM).[2] Unlike coefficients estimated in most empirical studies, the LTGM-PC allows for the effect of extra public investment to vary across countries and over time within the same country. In the model, the effect of an increase in public investment (or the quality of that investment) and the full dynamic growth path depend on country-specific factors such as the scarcity of public capital (relative to gross domestic product [GDP]) and some crowding in of private investment. The model also allows for the fact that the public capital stock might be of low-quality construction, which is a practical concern in many developing countries.

More technically, our model builds on the celebrated Solow-Swan growth model and another World Bank Excel-based tool known as the Long-Term Growth Model (LTGM) (Loayza and Pennings, 2018; and Hevia and Loayza, 2012), which we refer to as the Standard LTGM. However, in the Solow-Swan model (and Standard LTGM), capital is simply an aggregate, and so those models cannot simulate the specific effect of an increase in public investment. In contrast, in the LTGM-PC, total capital is split into public and private portions. The LTGM-PC retains many other realistic growth drivers and features of the Standard LTGM, including other growth fundamentals (human capital, TFP, demographics, and labor market participation by gender), and also implications for

1 http://www.worldbank.org/en/news/feature/2016/04/16/spending-more-and-better-essential-to-tackling-the-infrastructure-gap.

2 The relevant data and parameters for all countries are already pre-loaded into the LTGM-PC spreadsheet.

poverty rates. Section 2 presents the model and Section 3 describes how it is implemented in the Excel-based tool.

Despite being theoretical, the paper draws extensively on the empirical literature to guide the choice of parameters. The most important parameter is the elasticity of output to public capital, ϕ, which we call the usefulness of public capital. In Section 5, we review the evidence from two meta-analyses and other literature, which suggests an elasticity of $\phi = 0.17$ for essential infrastructure and $\phi = 0.10$ for generic public capital like buildings (though users can also specify its value). We also calculate the country-specific scarcity of public capital using a new public capital database from the IMF's Fiscal Affairs Department.

However, we could not find a suitable measure of the fraction of public capital that is of high quality (we use "efficiency" and "quality" interchangeably).[3] Hence, in Section 4, we develop a new cardinal measure of efficiency, the Infrastructure Efficiency Index (IEI), to quantify the extent to which public capital is of high quality in different countries. The IEI is based on estimates of the fraction of roads that are in poor condition, water that never reaches its final customers, and electricity that is lost through transmission and distribution.

Our second contribution is to document how the quantity and quality of public capital vary across countries with different levels of development, and how this affects the impact of new public and private investment on growth (Section 6). This analysis is conducted through the lens of the LTGM-PC and utilizes the cross-country data on the IEI and public capital stocks collected for the Excel-based tool.

Surprisingly, we find that the effect of an extra 1 ppt of GDP of public investment on growth is roughly constant across different levels of development.[4] This puts us at odds with optimistic commentators claiming that sizable "infrastructure gaps" mean a larger growth dividend from public investment in low-income countries. However, it also puts us at odds with pessimistic commentators who claim that the low efficiency of public investment in developing countries – due to corruption and mismanagement, for example – means that such projects have little effect on growth.[5] Overall, a 1 ppt increase in public investment as a share of GDP increases growth by 0.1–0.2 ppts in our model, depending on the calibration. As public investment is typically around 5% of GDP and usually <10% of GDP, higher public investment alone cannot turn a slow-growing country into a tiger economy.

Instead, developing countries are short of private capital, both relative to GDP and in absolute terms. Private capital as a share of GDP in low-income countries is only two thirds of that in middle-income countries, and almost half that in high-income countries. By our calculations, this means the return to private capital is highest in low-income countries, relative to both advanced countries and also relative to the return on public capital. This stems from the relatively low levels

3 Other indices like the World Economic Forum's infrastructure quality index or the IMF Public Investment Efficiency Index (PIE-X) include survey-based scores or distance to the frontier analysis, which means that a quality or efficiency score does not reflect the cardinal or absolute fraction of public capital operating as it should (see Section 4). The literature uses the terms quality and efficiency interchangeably as well.

4 This result follows from measured public capital as a share of GDP being roughly constant across countries with different levels of development (which is possibly overstated in low-income countries with weak governance; Keefer and Knack, 2007).

5 As in Berg *et al*. (2015), the level of efficiency in the LTGM-PC has no effect on the return to new public investment because the low quality of new public investment is exactly offset by greater need for public capital due to the poor quality of past public investment. See Sections 2 and 6.5.

of private investment in low-income countries (whereas public investment in low-income countries is actually larger as a share of GDP).

However, low-income countries also have the most inefficient public investment – with an IEI one-fifth lower than middle-income countries and one-third lower than high-income countries. Even though low-income countries might not be short of measured public capital – as public investment is likely overstated in many low-income countries with poor institutions (Keefer and Knack, 2007) – low-income countries are likely short of efficient public capital that is actually useful in production. This means that in low-income countries (i) the marginal product of efficient public capital – if it could be installed – is extremely high and (ii) there is substantial room for low-income countries to boost growth through increases in efficiency. As high efficiency only affects output through new investment, countries with high existing rates of public investment (and low existing efficiency) have the most to gain. However, efficiency is extremely difficult to increase quickly, and so in practical terms, the return to public investment will still be similar across different levels of development (as claimed above).

1.1. Definitions and related literature

In this paper, we generally equate public capital with infrastructure for simplicity, though we recognize that not all public capital is infrastructure, and not all infrastructure is public. Public capital in the literature is defined as core infrastructure made up of transport (roads, railways, and airports) and utilities (water supply and sanitation, energy, and ICT); and also includes hospitals, education buildings, other public buildings, and public physical assets (Agénor, 2013; Bom and Ligthart, 2014).[6] Although the public sector dominates the provision of infrastructure in low- and middle-income countries, in high-income countries the private sector plays an increasingly important role, including in hybrid categories such as Public-Private Partnerships, or PPPs (International Monetary Fund, 2015). In the literature, infrastructure is generally thought to increase the productivity of private factors much like TFP (see for instance Romp and de Haan, 2007; Serven, 2010; and Straub, 2008) – an approach we take here.

The closest modeling project to ours is the IMF's Debt, Investment and Growth (DIG) Model (Buffie *et al.*, 2012). While the DIG model seeks (in part) to estimate the effect of infrastructure on growth, it also aims to provide more analysis on the fiscal side on how public infrastructure might be financed. The DIG model also accounts for traded and non-traded goods and optimizing consumers (among other things). While the fiscal analysis and other features of the DIG model are missing from the LTGM-PC, the cost of those extra features is in complexity and transparency: For example, the DIG model cannot be solved in a standard Excel spreadsheet. Our default calibration of the usefulness of public capital $\phi = 0.17$, is the same as that used in the the DIG model. The LTGM-PC and the DIG model, in turn, build on an earlier generation of models involving public capital, such as Baxter and King (1993) and Barro and Sala-i-Martin (1992) and more recent models like Leeper *et al*. (2010).

2. A model of long-term growth with public capital

In this section, we provide an overview of the model structure (Sections 2.1–2.3) and some intuition on growth drivers (Section 2.4). Section 3 describes how these model equations enter the LTGM-PC Excel-based tool which enables users to run policy simulations.

6 IMF (2015): public capital is the accumulated value of public investment over time, which is the principal input into the production of public infrastructure, comprising *economic* infrastructure (transport and utilities) and *social* infrastructure (for example, public schools, hospitals and prisons).

2.1. The production function

To analyze the effects of public capital on growth, we adapt the Standard LTGM by splitting aggregate capital stock into public and private portions. We assume a Cobb-Douglas specification, where the two capital stocks have unitary elasticity of substitution, in contrast to being perfect substitutes in the Standard LTGM.[7] Based on the models in Eicher and Turnovsky (2000) and Agénor (2013), we first consider the following production function at time, *t*:

$$Y_t = A_t S_t (K_t^P)^{1-\beta} (h_t L_t)^{\beta} \tag{1}$$

Each firm takes technology (TFP), A_t and public services S_t as given, that is, these are externalities to the firm. K_t^P is the private capital stock, $h_t L_t$ is effective labor, which can be further decomposed into h_t, human capital per worker and L_t, the number of workers. $1–\beta$ and β are private capital and labor income shares. Next, we consider the following specification for public services S_t:

$$S_t = \left[\frac{G_t}{K_t^{P\zeta}} \right]^{\phi} \tag{2a}$$

G_t is the efficient physical public capital stock – the public capital that is actually used in production. ζ captures whether public capital is subject to congestion (or not) – discussed further below. ϕ is the usefulness of public capital (more technically the elasticity of output to efficient public capital).

$$G_t = \theta_t K_t^{Gm} \tag{2b}$$

Due to corruption, mismanagement or pork-barreling, only a fraction $\theta_t \leq 1$ of measured public capital is useful for production. The measured capital stock K_t^{Gm} is what is recorded in international statistical databases, constructed using the perpetual inventory method. θ_t is the average efficiency/quality of the public capital stock. Equations (1), (2a), and (2b) can be written in a more conventional production function as:

$$Y_t = A_t \left(\theta_t K_t^{Gm} \right)^{\phi} (K_t^P)^{1-\beta-\zeta\phi} (h_t L_t)^{\beta} \tag{3}$$

2.1.1. Congestion ($\zeta \in [0,1]$)

In principle, the congestion parameter in Equations (2a) and (3) can take values between ζ=1 (full congestion) and ζ=0 (no congestion). As long as ζ>1, it is the ratio of public capital to private capital that provides public services, rather than the absolute amount of public capital (Barro and Sala-i-Martin, 1992).[8] When there is a large amount of private capital relative to public capital, the public capital becomes "congested" and its benefits diminish. The intuition for this is a road network: When there are too many cars on the road, it becomes jammed, reducing its capacity to add to output.

7 In Appendix 1, we compare the effect of investment on growth between the LTGM-PC and Standard LTGM. Appendix A1.5 provides a general idea of how the Standard LTGM differs from the LTGM-PC. All appendices referred to in the main text are available at www.worldbank.org/LTGM.

8 Congestion can also be measured in terms of both private capital and labor supply (see for instance Glomm and Ravikumar (1997)) or aggregate output, but these do not result in substantial changes to the analysis (Eicher and Turnovsky, 2000). Aside from absolute congestion, there can also be relative congestion, in which case, congestion increases only if aggregate usage increases relative to individual usage (see Eicher and Turnovsky (2000) for further details on how relative and absolute congestion affect growth analysis).

In the Excel-based tool, we only allow for two cases, $\zeta \in \{0,1\}$ for simplicity. In our main "congestion" specification, ζ=1. This means that K_t^{Gm} must grow faster than K_t^P to have a positive effect on output. In this scenario, there is decreasing returns to scale to private inputs (private capital and effective labor), and constant returns to scale to all inputs. In the appendices and in some parts of the paper we take the alternative assumption that ζ=0: Public capital is a pure public good. When ζ=0, there are constant returns to scale to private inputs but increasing returns to scale to all inputs, though as we assume $\phi + (1-\beta) < 1$, endogenous growth through capital accumulation is ruled out. ζ=0 is a polar case – in reality, almost all public goods are characterized by some degree of congestion.

2.1.2. The efficiency/quality of public capital ($\theta \in [0,1]$)

$\theta \in [0,1]$, reflecting that "a dollar's worth of public investment spending often does not create a dollar's worth of public capital" (Pritchett, 1996) – K_t^{Gm} units of capital act like $G_t = \theta_t K_t^{Gm}$ units, and it is only the latter that is useful for increasing output. That is, productive capital is sometimes not created at all; or supposedly productive capital is created but subject to implementation weaknesses and/or operational inefficiencies such that the cost is higher than the minimum required to build the capital.

More concretely, a low θ most closely resembles poor construction quality which impedes efficient operation of the public capital project. A good example of low quality/efficiency is a corrupt road construction project where the construction firm reduces the thickness of pavement to save money (and pays kickbacks to politicians/bureaucrats). The road surface then deteriorates much more quickly than it should if it was properly constructed, resulting in reduced travel speeds and capacity. This example closely relates to how we measure θ in practice based on the fraction of unpaved roads (or electricity/water transmission losses).

If θ mostly reflects construction quality, readers might wonder about other aspects of the public investment management process, such as poor project selection, excessive public investment in politically sensitive regions, or large vanity projects with little economic value. Unfortunately, it is close to impossible to assess the scale of these problems quantitatively across countries and so they are excluded from our IEI (and from θ), which is discussed in Section 4. To the extent that vanity projects are a different class of public investment (even less essential than other public buildings), it could be argued that they are less useful for producing output and hence have a lower ϕ. However, we would generally prefer adjusting down θ – below that implied by the IEI – which allows for potential improvement in the efficiency/quality of public investment in the future (and is closer to Pritchett's original formulation).

2.1.3. The usefulness of public capital for production ($\phi \in [0,1]$)

The elasticity of output with respect to the public capital stock measures the usefulness of public capital for production, assuming that the project is of maximum quality/efficiency (θ=1). Essential infrastructure such as roads, ports, power and water, tend to have a higher usefulness than less essential forms of public capital, like public buildings, even if these different types are constructed properly. The calibration of values for this parameter is discussed in Section 5.2.

2.1.4. Population and labor force growth

Equation (3) can be translated into per worker terms by dividing both sides by L_t:

$$y_t \equiv \frac{Y_t}{L_t} = A\left[\theta_t \left(L_t\right)^{1-\zeta} k_t^{Gm}\right]^{\phi} (k_t^P)^{1-\beta-\zeta\phi} h_t^{\beta} \tag{4}$$

Where y_t is output per worker and k_t^P is private capital per worker and k_t^{Gm} is measured public capital per worker (note the lower case). $L_t = \varrho_t \omega_t N_t$, where N_t is total population, ω_t is the working age-population ratio, and ϱ_t is the labor participation rate (labor force-to-working age population ratio). The above equation can then be used to calculate growth rates of output per worker from t to t+1:

$$\frac{y_{t+1}}{y_t} = \left[\frac{\omega_{t+1}}{\omega_t}\frac{\varrho_{t+1}}{\varrho_t}\frac{N_{t+1}}{N_t}\right]^{(1-\zeta)\phi}\left[\frac{A_{t+1}}{A_t}\right]\left[\frac{\theta_{t+1}}{\theta_t}\right]^{\phi}\left[\frac{k_{t+1}^{Gm}}{k_t^{Gm}}\right]^{\phi}\left[\frac{k_{t+1}^{P}}{k_t^{P}}\right]^{1-\beta-\zeta\phi}\left[\frac{h_{t+1}}{h_t}\right]^{\beta} \tag{5}$$

Equation (5) can be rewritten in terms of growth rates from t to t+1:

$$1+g_{y,t+1} = [(1+\Gamma_{t+1})^{(1-\zeta)\phi}](1+g_{A,t+1})(1+g_{\theta,t+1})^{\phi}(1+g_{k^{Gm},t+1})^{\phi}(1+g_{k^{P},t+1})^{1-\beta-\zeta\phi}(1+g_{h,t+1})^{\beta} \tag{6}$$

Where the growth rate of a variable x from t to t+1 is denoted by $g_{x,t+1}$ and Γ is the growth rate of the number of workers:

$$1+\Gamma_{t+1}=(1+g_{\varrho,t+1})(1+g_{\omega,t+1})(1+g_{N,t+1}) \tag{7}$$

$1+\Gamma_{t+1}$ drops out from Equation (6) in the congestion default (ζ=1).

To obtain output per capita, y_t^{PC} from Equation (4), $y_t^{PC} \equiv \frac{Y_t}{N_t} = \frac{Y_t}{L_t}\varrho_t\omega_t$. Rewriting this equation in terms of growth rates:

$$1+g_{y,t+1}^{PC} = (1+g_{y,t+1})(1+g_{\varrho,t+1})(1+g_{\omega,t+1}) \tag{8}$$

To obtain output growth, we multiply (8) with population growth:

$$1+g_{Y,t+1} = (1+g_{y,t+1}^{PC})(1+g_{N,t+1}) \tag{9}$$

2.2. Public and private capital accumulation and changes in the efficiency/quality of public capital

The measured quantity of public capital (as in international statistical databases) accumulates according to a standard capital accumulation identity, with the next period's stock coming from the previous period's undepreciated stock, $(1-\delta^G)K_t^{Gm}$ (where δ^G is the public capital depreciation rate) and new public investment, I_t^G.

$$K_{t+1}^{Gm} = (1-\delta^G)K_t^{Gm} + I_t^G \tag{10}$$

The gross growth rate of measured public capital (not per worker) is:

$$K_{t+1}^{Gm} / K_t^{Gm} = (1-\delta^G) + \frac{I_t^G / Y_t}{K_t^{Gm} / Y_t} \tag{11}$$

The growth rate of measured public capital per worker, which enters Equation (6), is:

$$1+g_{k^{Gm},t+1} \equiv \frac{K_{t+1}^{Gm}}{K_t^{Gm}} / \frac{L_{t+1}}{L_t} = \frac{(1-\delta^G)+\frac{I_t^G / Y_t}{K_t^{Gm} / Y_t}}{(1+g_{\varrho,t+1})(1+g_{\omega,t+1})(1+g_{N,t+1})} \tag{12}$$

The stock of efficiency-adjusted public capital (which is actually used in production) evolves based on the previous period's efficiency-adjusted undepreciated stock and efficiency-adjusted new investment $\theta_t^N I_t^G$.

$$G_{t+1} = (1-\delta^G)G_t + \theta_t^N I_t^G \tag{13a}$$

Readers will note that Equation (13a) is the same as Equation (1) in Berg *et al*. (2015), with the efficiency of new investment being θ_t^N rather than ϵ. Consequently, all of Berg *et al*.'s results on the effects of efficiency also go through here (discussed further below). Equation (13a) is also equivalent to Equation (2) in Pritchett (2000), who refers to γ as the efficiency of public investment.[9] Here, one can interpret $1/\theta_t^N$ as the dollar cost of providing an extra dollar of usable public capital. Hence, corruption or other rent-seeking which reduce the quality of public investment effectively increase the cost of a given increase in the productive capital stock as found empirically by Olken (2007) and Collier *et al*. (2015).

θ_t is the average efficiency of existing public capital (rather than the efficiency of new investment). Substituting $G_t = \theta_t K_t^{Gm}$ into Equation (13a) and rearranging as Equation (13b), one can see the θ_{t+1} evolves as a weighted average of the quality of existing public capital θ_t, and the quality of new investment θ_t^N.

$$\theta_{t+1} = \theta_t \frac{\left(1-\delta^G\right)K_t^{Gm}}{\left(1-\delta^G\right)K_t^{Gm} + I_t^G} + \theta_t^N \frac{I_t^G}{\left(1-\delta^G\right)K_t^{Gm} + I_t^G} \tag{13b}$$

As such, the quality/efficiency of the stock of public capital only changes when the quality of new investment projects is different from that of the existing public capital stock: $\theta_t^N \neq \theta_t$.[10] Using Equation (13b), the growth in quality which enters Equation (6) can be written as follows:

$$1 + g_{\theta,t+1} \equiv \frac{\theta_{t+1}}{\theta_t} = \left[\left(1-\delta^G\right) + \frac{\theta_t^N}{\theta_t}\frac{I_t^G / Y_t}{K_t^{Gm} / Y_t}\right] / \left(K_{(t+1)}^{Gm} / K_t^{Gm}\right) \tag{14}$$

9 The measured public capital stock K_t^{Gm} here is G_t^m in Berg *et al*. (2015). Pritchett (2000) refers to K_t^G in his Equation (2) as the efficient capital stock.

10 The treatment of "new" investment versus maintenance expenditure requires some clarification. For instance, Buffie *et al*. (2012), in their macroeconomic model of public investment effects, consider infrastructure investment as encompassing net investment, as well as operations and maintenance; and treat the depreciation rate as exogenous. Kalaitzidakis and Kalyvitis (2004), in their infrastructure-led growth model, specify the accumulation of public capital as a function of new investment, and the depreciation rate depends on maintenance expenditure. Our model is more in line with Buffie *et al*. (2012), in that depreciation is exogenous. Conceptually, I_t^G in our model could include spending on major repairs, which along with new investment helps offset the capital decumulation effects of depreciation. But practically, we note that maintenance spending is typically subsumed under public consumption data and hence is hard to gauge. (From a national accounts perspective, the SNA (1993) notes "ordinary maintenance and repairs to keep fixed assets in good working order are intermediate consumption. However, major improvements, additions or extensions to fixed assets which improve their performance, increase their capacity or prolong their expected working lives count as gross fixed capital formation. In practice, it is not easy to draw the line…Some analysts…would favor a more "gross" method…all such activities are treated as gross fixed capital formation.") User concern about insufficient maintenance spending could thus be reflected as higher depreciation rates. Developing countries tend to spend less on operations and maintenance, which could imply higher depreciation rates than developed countries (Devarajan, Swaroop and Zou (1996), in regression analysis for a sample of developing countries, find public capital expenditure and economic growth to be negatively correlated but that current expenditure has positive effects, illustrating how capital expenditure may have been excessive, while current expenditure insufficient). However, developed countries are more likely to hold a higher share of more sophisticated assets that are subject to faster depreciation, making the net implication for depreciation rates not readily obvious (Arslanalp *et al*., 2010).

The quantity of private capital follows the same accumulation process as public capital. But with δ^P as the private capital depreciation rate, and I_t^P as private investment. The growth rate of private capital per worker is as follows:

$$1+g_{k^P,t+1}=\frac{\left(1-\delta^P\right)+\dfrac{I_t^P/Y_t}{K_t^P/Y_t}}{\left(1+g_{\varrho,t+1}\right)\left(1+g_{\omega,t+1}\right)\left(1+g_{N,t+1}\right)} \tag{15}$$

2.3. Analysis of the drivers of growth

To better understand and simplify the analysis of the drivers of growth, we take a log-linear approximation of Equation (6). Specifically, Equations (12), (14), and (15) are substituted into Equation (6). Then, taking logs and using the approximation ln(1+g)≈g (for small g) we arrive at the following:

$$\begin{aligned} g_{y,t+1}^{PC} &\approx g_{A,t+1}+\beta\left(g_{\varrho,t+1}+g_{\omega,t+1}+g_{h,t+1}\right)-(1-\beta)(g_{N,t+1})+\phi\left[\theta_t^N\frac{I_t^G/Y_t}{\theta_t K_t^{Gm}/Y_t}-\delta^G\right] \\ &+(1-\beta-\zeta\phi)\left(\frac{I_t^P/Y_t}{K_t^P/Y_t}-\delta^P\right) \end{aligned} \tag{16}$$

From Equation (16), one can see that a 1 ppt increase in the public investment share of GDP increases growth the following year by:

$$\frac{\partial g_{y,t+1}^{PC}}{\partial I_t^G/Y_t}=\frac{\phi}{\theta_t K_t^{Gm}/Y_t}\theta_t^N \tag{16a}$$

$\dfrac{\phi}{\theta_t K_t^{Gm}/Y_t}$ is the marginal product of efficient public capital G_t, calculated by taking the derivative with respect to $G_t=\theta_t K_t^{Gm}$. This is multiplied by θ_t^N, such that an increase in public investment has a larger effect on growth when new public investment is more efficient.

However, in most cases, it is prudent to assume that the efficiency of new investment is the same as past investment, $\theta_t^N=\theta_t$. In this case, the effect of a 1 ppt GDP increase in public investment is the marginal product of measured public capital, $\phi/(K_t^{Gm}/Y_t)$. To calculate how many extra percentage points of GDP of public investment an economy needs to increase growth by a percentage point, simply invert this ratio $(K_t^{Gm}/Y_t)/\phi$. We call this the public marginal incremental capital-to-output ratio (ICOR), because it is a close analog of the traditional concept of the ICOR.

An analogous expression is available for private capital:

$$\frac{\partial g_{y,t+1}^{PC}}{\partial I_t^P/Y_t}=\frac{1-\beta-\zeta\phi}{K_t^P/Y_t} \tag{16b}$$

The public capital portion of Equation (16) in brackets is equal to the net growth rate of efficient public capital G_t. This can be further decomposed into an increase in quality $\left[\dfrac{\theta_t^N-\theta_t}{\theta_t}\right]\dfrac{I_t^G/Y_t}{K_t^{Gm}/Y_t}$ and an increase in quantity $\frac{I_t^G / Y_t}{K_t^{Gm} / Y_t} - \delta^G$. The increase in quantity is simply the net growth rate of the measured public capital stock $g_{K^{Gm},t+1} = K_{t+1}^{Gm} / K_t^{Gm} - 1$ (which is not affected by the level of θ).

$$g_{G,t+1} = \left[\frac{\theta_t^N}{\theta_t} \frac{I_t^G / Y_t}{K_t^{Gm} / Y_t} - \delta^G \right] = \underbrace{\left[\frac{\theta_t^N - \theta_t}{\theta_t} \right] \frac{I_t^G / Y_t}{K_t^{Gm} / Y_t}}_{Quality\,increase} + \underbrace{\frac{I_t^G / Y_t}{K_t^{Gm} / Y_t} - \delta^G}_{Quantity\ increase\,(=g_{K^{Gm},t+1})} \tag{17}$$

One will note that if $\theta_t^N = \theta_t$ – that is the efficiency of public capital is constant – then the level of public capital efficiency θ_t does not appear at all in Equations (16a), (16b), or (17) and so does not affect growth. This surprising result, which also appears in Berg *et al*. (2015), is due to two exactly offsetting forces in the production function. First, lower quality/efficiency naturally means that there is a smaller increase in efficient public capital for each extra 1 ppt of public investment. Second, in economies with lower efficiency, the stock of efficient public capital is scarcer, and hence has higher marginal product.[11]

From Equation (16), TFP growth $g_{A,t+1}$ has the largest direct effect on growth. The effect of most other factors depends on the labor share, β<1. The larger is β, the lower is the effect of private capital accumulation on growth. For both public and private capital accumulation, holding all else constant, the same level of investment-to-output becomes less efficient as the capital-to-output ratio rises.

3. Implementing public capital in the LTGM-PC

	Sub-model 1	Sub-model 2	Sub-model 3
Public investment/GDP	Input	Output	Input
Private investment/GDP	Input	Input	Output
Growth target	Output	Input	Output
National savings/GDP	Output	Output	Input

In practical terms, using the LTGM-PC involves choosing the path for several inputs in the future (exogenous variables), and then the LTGM-PC calculates the future implied path of the outputs (endogenous variables). The LTGM-PC has three sub-models (1–3) where the endogenous and exogenous variables in the model are switched. Other growth drivers – growth in TFP (A), human capital (h), labor participation rate ϱ, working age-population ratio (ω), and population (N), respectively, are always exogenous, as in the Standard LTGM. The LTGM-PC also allows for output growth to affect poverty rates, as in the Standard LTGM. More technically, the LTGM-PC has three "state" variables, which are predetermined at any point and change slowly over time: The public and private capital stocks (usually expressed as a ratio to GDP) and the average efficiency of installed public capital.

11 Countries with different efficiency levels will have the same marginal product of public investment given the same parameters and initial conditions. The marginal product of public investment depends on the marginal product of efficient public capital and the translation of public investment to efficient capital and can be expressed as follows (using Equation (3) and (13a)):

$$\partial Y_{t+1} / \partial I_t^G = \left(\partial Y_{t+1} / \partial \theta_{t+1} K_{t+1}^{Gm}\right)\left(\partial \theta_{t+1} K_{t+1}^{Gm} / \partial I_t^G\right) = (\phi Y_{t+1} / \theta_{t+1} K_{t+1}^{Gm}) \theta_t^N$$

Note that θ_{t+1} and θ_t^N cancel out if efficiency is unchanged.

In this paper, we mostly use Sub-model 1, where future paths of public and private investment (as a share of GDP) are exogenous and the path of GDP (or GDP per capita) is endogenous. Alternatively, this can be reversed, and Sub-model 2 can calculate the required public investment ratio to achieve a specified growth target (given an exogenous private investment share). In both Sub-models 1 and 2, savings rates are calculated as a residual for an assumed path of the current account balance to GDP ratio. In Sub-model 3, the user instead specifies national savings rates and public investment rates as exogenous, with the model calculating implied private investment and growth rates.

3.1. Sub-model 1: Growth given public and private investment

In Sub-model 1, per capita output growth ($g_{y,t+1}^{pc}$) is generated by Equation (8), based on GDP per worker growth ($g_{y,t+1}$) from Equation (6). The components of Equation (6) are:

- The future growth rates of the labor participation rate ($g_{\varrho,t+1}$), the working age-population ratio ($g_{\omega,t+1}$), population ($g_{N,t+1}$), human capital ($g_{h,t+1}$), and pure TFP ($g_{A,t+1}$), which are exogenous and can be determined by the user.
- The growth rate of measured public capital per worker $\left(g_{k^{Gm},t+1}\right)$ which is given by Equation (12), using the growth rate of the public capital stock (Equation (11)) as an intermediate step.
- Private capital per worker growth $\left(g_{k^{P},t+1}\right)$ as given by Equation (15).
- The growth rate of the efficiency of public capital ($g_{\theta,t+1}$) as given by Equation (14) using the growth rate of the public capital stock (Equation (11)) as an intermediate step.

Finally, the model is closed by updating public capital-to-output using Equation (18) and the private capital-to-output ratio using Equation (19) (with the growth rates in per-worker terms):

$$\frac{K_{t+1}^{Gm}}{Y_{t+1}} = \frac{K_t^G}{Y_t}\frac{(1+g_{k^{Gm},t+1})}{1+g_{y,t+1}} \tag{18}$$

$$\frac{K_{t+1}^{P}}{Y_{t+1}} = \frac{K_t^P}{Y_t}\frac{(1+g_{k^{P},t+1})}{1+g_{y,t+1}} \tag{19}$$

3.2. Sub-model 2: Public investment required to generate a target growth rate (given a constant private investment rate)

Sub-model 2 is particularly useful for assessing the feasibility of a public investment-led growth strategy. Specifically, one can ask what rates of public investment would be required to generate a given target growth rate, assuming a path for private investment. Across countries, public investment is typically around 6% of GDP and more than 90% of countries have public investment rates <12% of GDP.[12] As such, if a growth strategy required an increase in public investment rates of more than a few percent of GDP, it should be regarded as ambitious, and in some cases unrealistic. In practice, the required rates of public investment using Sub-model 2 are often extremely high or low if the target growth rate is not close to that achieved by the economy under business-as-usual public investment rates.

12 Figures for 2016, with the investment share of GDP from the World Bank MFMOD database on the public investment share from the IMF FAD databases. Cross-country mean is 6.47% and cross-country median is 5.4% of GDP.

To find the required public investment share to achieve the target per capita growth rate:

- First rearrange Equation (8) to calculate required GDP growth per worker:

$$1+g_{y,t+1}=\frac{1+g^{PC}_{y,t+1}}{\left(1+g_{\varrho,t+1}\right)\left(1+g_{\omega,t+1}\right)} \tag{20}$$

- Then rearrange Equation (6) to calculate the combined growth rate of efficiency-adjusted public capital per worker ($\theta_t K_t^{Gm}/L_t$) required to generate the target growth in GDP per worker in Equation (20). Note here that $g_{A,t+1}$, $g_{h,t+1}$, and Γ_{t+1}) are all always exogenous, and the growth rate of private capital per worker $g_{k^P,t+1}$ is calculated using Equation (15) (as the private investment share is exogenous in Sub-model 2).

$$\left(1+g_{\theta,t+1}\right)\left(1+g_{k^{Gm},t+1}\right)=\frac{\theta_{t+1}K^{Gm}_{t+1}/L_{t+1}}{\theta_t K^{Gm}_t/L_t}=\left[\frac{1+g_{y,t+1}}{\left(1+g_{A,t+1}\right)[(1+\Gamma_{t+1})^{(1-\zeta)\phi}]\left(1+g_{k^P,t+1}\right)^{1-\beta-\zeta\phi}\left(1+g_{h,t+1}\right)^{\beta}}\right]^{1/\phi} \tag{21}$$

- Finally, one can rearrange Equation (13a) in per worker terms to solve for the investment share of GDP (recall that Γ is the growth rate of the number of workers, as in Equation (7)):

$$I^G_t/Y_t=[\left(1+g_{\theta,t+1}\right)\left(1+g_{k^{Gm},t+1}\right)\left(1+\Gamma_{t+1}\right)-(1-\delta^G)]\frac{K^{Gm}_t/Y_t}{\theta^N_t/\theta_t} \tag{22}$$

- As before, one also needs to update the state variables ($\theta_t, K^{Gm}_t, K^P_t$). Equation (14) updates the efficiency of public capital for $t+1$ and Equation (19) updates the private capital-to-output ratio. Equations (11) and (12) calculate the growth rate in the public capital stock (per worker) implied by the rate of public investment and Equation (18) updates the new public capital-to-output ratio.

3.3. Sub-model 3: Growth given savings and public investment rates

Any growth strategy involving an increase in public investment rates needs to take account of the fact that greater public investment needs to be funded by either domestic or foreign savings. In the absence of policies to increase national savings (or increase access to foreign savings), an increase in public investment will crowd out private investment, resulting in a smaller increase in growth than would otherwise be the case if there was no savings constraint. This mechanism is captured in Sub-model 3, where the user specifies the national savings rate as well as a path for public investment.

- Private investment is then calculated by:
  - Equation (23), if the user chooses to specify the current account balance,

$$\frac{I^P_t}{Y_t}=\frac{S_t}{Y_t}-\frac{CAB_t}{Y_t}-\frac{I^G_t}{Y_t} \tag{23}$$

  - Or Equation (24) if instead, they specify a path for external debt as a share of GDP (see the description of the Standard LTGM for a derivation of these equations).[13]

$$\frac{I^P_t}{Y_t}=\frac{S_t}{Y_t}+\frac{FDI_t}{Y_t}+\frac{D_t}{Y_t}-\frac{D_{t-1}/Y_{t-1}}{\left(1+g^{PC}_{y,t}\right)\left(1+g_{N,t}\right)}-\frac{I^G_t}{Y_t} \tag{24}$$

13 Strictly speaking, in the LTGM, external debt, *D*, can capture all foreign portfolio assets and liabilities, which means *D* could be decreasing if the country is accumulating foreign assets, for example through a sovereign wealth fund.

Where CAB_t is the current account balance, FDI_t is inbound foreign direct investment and D_t is end-of-year external debt.

- Once private investment is determined, the rest of the equations are the same as in Sub-model 1.

4. Evidence on the efficiency/quality of the public capital stock, θ

This section develops the Infrastructure Efficiency Index (IEI) to measure the proportion of public capital spending that delivers useful public capital/infrastructure services. For example, power lines and power plants might not deliver electricity to households and businesses, dams and pipelines might not be able to deliver water due to leaks, and roads may be in poor condition (such as being unpaved) (World Bank, 1994). The IEI combines these measures into a single index for all countries.

4.1. Constructing the IEI

An index of infrastructure efficiency for measuring θ_t in the model above should have several key features: (i) informative about many countries; (ii) simple and transparent in its construction; and (iii) cardinal rather than conveying a relative rank/score. A cardinal index is needed because a doubling of θ_t in our model doubles the efficient public infrastructure stock, whereas a doubling in score or rank could mean the increase in the efficient public infrastructure stock is smaller or greater than double. While there are other investment efficiency indices in the literature (surveyed below in Section 4.2), none of them have all three features. Our new IEI does, following a similar methodology as Rioja (2003)[14] but for as many countries as possible subject to data availability.

Specifically, we construct the index using more recent data for three indicators – (i) electricity transmission and distribution losses (percentage of output); (ii) water losses (percentage of provision); and (iii) paved roads (percentage of total roads). Ideally, we would like to have used the percentage of paved roads in good condition as a subcomponent of the IEI. However, this statistic is no longer available. As such, we follow Calderon and Serven (2010) who use paved roads (percentage of total roads) as an indicator of the quality of road networks in their analysis of the quantity and quality of infrastructure services in Latin America. We nevertheless recognize that unpaved roads are not always undesirable or inefficient and may depend on country-specific geographic features. Electricity losses reflect inefficiency at the transmission and distribution stages - which is what our index intends to measure. Losses also capture electricity delivered but not paid for which can be attributable to theft and unmetered supply. For some countries the second type of non-technical losses can be large. Nevertheless, this can still to some extent be seen as related to dysfunctional infrastructure in terms of construction and management (or operational inefficiency) (Jimenez *et al*., 2014). We also do not include telecommunications in the composite index since fixed-telephone line faults may no longer be relevant due to the rising importance of mobile telephony, and data on the quality of mobile phone service are not as extensive.

In calculating the IEI, we want to include the latest data, but recognize that infrastructure losses for a single year can be very noisy. As such, we take the average of the index for the latest available

14 Rioja (2003) uses the physical infrastructure losses reported in World Bank (1994), weighted by corresponding infrastructure stock shares (from Ingram and Fay (1994)), to proxy the efficiency parameter for public capital stock in seven Latin American countries and five industrialized countries. The infrastructure loss indicators comprise electricity power transmission and distribution losses (% of output), faults per 100 main telephone lines per year, percentage of paved roads not in good condition, and water losses (% of total provision). Calderon and Serven (2004, 2010) construct an infrastructure quality index based on the first principal component of electricity losses, the share of paved roads, and the waiting time for telephone line installation.

year and the post-2000 average, using a weighted average of water losses, power losses, and paved roads:

$$Individual\ country\ IEI = \frac{1}{2}\left(\sum_{n=1}^{3} w_n I_{n,avg(2000-latest)} + \sum_{n=1}^{3} w_n I_{n,latest\,available\,value(starting\,2000)}\right)$$

Where:

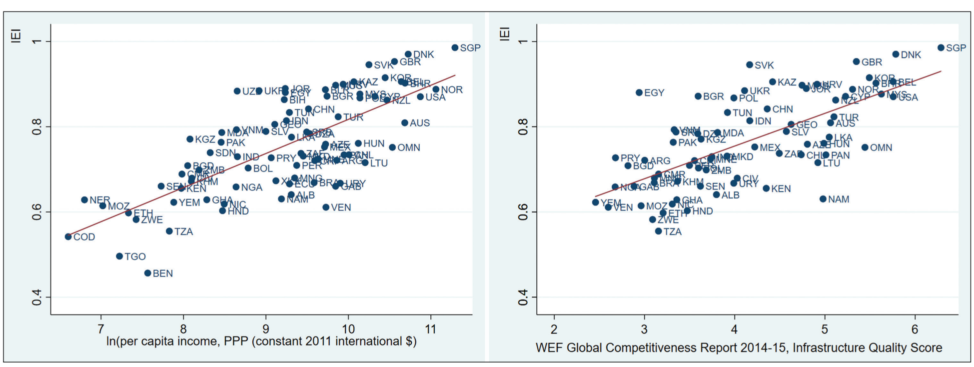
- I_n equals the portion of efficient infrastructure type, n, calculated as 100 - electricity transmission and distribution losses (percentage of output), 100 – water losses (percentage of provision), and paved roads (percentage of roads), respectively.
- *avg (2000-latest)*: The average of available values of the infrastructure indicator, from 2000 until the latest data point.
- *latest available value (starting 2000)*: The latest value available, the cutoff being the year 2000 (a country is excluded if its latest data point is before 2000).
- w_n = Infrastructure stock weight associated with each infrastructure, n. The weights are based on Fay and Yepes (2003) and vary with income groups but not over time (Table 1).

Graph 1 shows that the IEI has the expected properties, rising with GDP per capita (correlation: 0.72); and the World Economic Forum (WEF) Global Competitiveness Report's survey-based infrastructure quality indicator (correlation: 0.68). According to the IEI, efficiency is highest in high-income countries (including OECD members) with an average of 84%, followed by middle-income countries (77% for upper-middle-income countries and 74% for lower-middle-income countries), and low-income countries (58%). Further details on the sources of data, IEI summary statistics as well as discussion on robustness checks for the index are provided in Appendix 2.

Table 1. The composition of infrastructure stocks by income group, Fay and Yepes (2003)

Infrastructure type	Low income (%)	Middle income (%)	High income (%)
Electricity	28.13	55.87	44.70
Water	15.93	11.50	5.24
Roads	55.93	32.64	50.06
Total	100.00	100.00	100.00

Notes: Weights have been normalized, based on initial weights from Fay and Yepes (2003), Table 2 (page 2), which also include rail and telecommunications. The weights calculated by Fay and Yepes are for the year 2000 and are based on estimations of the monetary value of infrastructure stocks, using best practice prices (unit costs).



Graph 1. Infrastructure Efficiency Index (IEI) - correlation with per capita income and infrastructure quality score

4.2. IEI versus other measures of public investment efficiency

In this section, we briefly discuss some of the other measures of public investment efficiency that is available. These serve as useful checks against the IEI, but do also face some limitations for use in our model – primarily that they indicate relative performance rather than being a cardinal measure.

Afonso *et al*. (2005, 2010) and various IMF papers, starting with Albino-War *et al*. (2014) take the approach of distance to frontier, where inefficiency is measured relative to best performing peer countries. In the case of the papers by Afonso *et al*. (2005, 2010) which, respectively, cover 23 industrial countries (2005 paper), and 23 emerging and new EU member states (2010 paper), the output measure is a composite of public sector performance based on a series of quantitative and qualitative socioeconomic indicators, while the input measure is public sector spending (i.e., more than just public investment).[15] For our model analysis purposes, we find that the results from these two papers may not be suitable because; one, the outcome variable encompasses broad, indirect macroeconomic outcomes; and two, efficiency is compared within a small group of countries – this may be particularly worrisome in the case of the emerging market/new EU member sample.[16]

On the other hand, the IMF Public Investment Efficiency Index (PIE-X) covers more than 100 countries. The output variables are directly related to infrastructure – a quantity index (physical infrastructure coverage and provision of social services[17]) and a survey-based quality index,[18] respectively, as well as a hybrid of the two; while the input variable is public capital stock per capita.[19] While individual country efficiency scores have not been published, group averages of the quantity indicator suggest that advanced economies are 70% efficient (infrastructure output could be increased by 30% for the same amount of public capital input), emerging markets are 60% efficient, while low-income developing countries are at about 45% efficiency. Nevertheless, these are still *relative* performance indicators rather than cardinal indicators of quality: A score of 70% does not mean that 30% of infrastructure stock is not productive but rather the economy is operating 30% below the best performer in its peer group.

Aside from the above measures of inefficiency, there is also the IMF Public Investment Management Index (PIMI) (Dabla-Norris *et al*., 2012), a purely qualitative indicator based on scores for individual country performance in terms of the investment process (project appraisal, selection, implementation, and evaluation). While it provides information on relative performance across 71 developing countries and shows a positive correlation with GDP per capita and indicators of governance quality, it is not a cardinal indicator of the proportion of public capital that is productive, and only measures the quality of input process. Despite this, Gupta *et al*. (2014) normalize the index on a 0–1 scale and use it as a measure of efficiency-adjusted public capital effects on growth based on a sample of 52 countries. They find that upper-middle-income countries have on average 57% efficient public capital stock against 46% for lower-middle-income countries, and 38% for low-income countries.[20]

15 Qualitative indicators of corruption, red tape, judiciary efficiency, public infrastructure quality; and quantitative indicators - shadow economy size, secondary school enrollment, education achievement, infant mortality, life expectancy, as well as broad macroeconomic outcomes – income distribution, growth performance and stability, inflation and unemployment.

16 Sinha (2017) uses the emerging market/new EU member state average efficiency as a reference point for public investment efficiency in Bangladesh.

17 Length of road network, electricity production, access to water, number of secondary teachers and number of hospital beds.

18 Based on the World Economic Forum Global Competitiveness Report survey responses on the quality of key infrastructure services.

19 With GDP per capita as an auxiliary input variable.

20 We group Gupta *et al*. (2014)'s individual country calculations according to the World Bank income classification scheme.

Table 2. Indicators of efficient public capital stock – different methodologies

<table>
<tr><th colspan="2">Infrastructure Efficiency Index (IEI) (this paper)</th><th colspan="4">Public Investment Efficiency Index (PIE-X)</th><th colspan="2">Efficiency-adjusted public capital, using Public Investment Management Index (PIMI)</th></tr>
<tr><th>Country group</th><th>Average</th><th>Country group</th><th>Hybrid, average</th><th>Physical, average</th><th>Survey-based, average</th><th>Country group</th><th>Average</th></tr>
<tr><td>High income (21)</td><td>0.844</td><td>All countries (132)</td><td>0.730</td><td>0.570</td><td>0.800</td><td>Upper middle income (14)</td><td>0.572</td></tr>
<tr><td>Upper middle income (25)</td><td>0.767</td><td>Advanced (26)</td><td>0.870</td><td>0.700</td><td>0.900</td><td>Lower middle income (22)</td><td>0.456</td></tr>
<tr><td>Lower middle income (27)</td><td>0.738</td><td>Emerging markets (62)</td><td>0.730</td><td>0.600</td><td>0.800</td><td>Low income (14)</td><td>0.382</td></tr>
<tr><td>Low income (10)</td><td>0.576</td><td>Low income developing countries (44)</td><td>0.600</td><td>0.450</td><td>0.750</td><td colspan="2" rowspan="2">The capital accumulation process is based on efficient public investment. The portion of public investment deemed as efficient is based on a time-invariant PIMI score (between 1 and 4) that has been normalized (4 to 1).
Source: Gupta et al. (2014).</td></tr>
<tr><td colspan="2">Weighted average of output not lost in delivery (for electricity and water) and paved roads as a percentage of total roads.</td><td colspan="4">Based on efficiency frontier analysis with public capital stock per capita as input and infrastructure quantity and/or quality as output. The hybrid indicator is a simple mean of the physical and survey-based indicators.
Source: IMF (2015).</td></tr>
</table>

Table 2 summarizes the PIE-X and PIMI and how they compare against the IEI. While caution should be exercised in comparing outcomes in absolute terms across different methodologies, one crucial takeaway is that all methodologies point to a gap between high-income countries and low-income countries. Thus, there appears to be substantial room for improvement in efficiency in low-income countries which could lead to better growth performance.

5. Model calibration

5.1. Setting up the baseline and scenarios

To make empirical comparisons between the Standard LTGM and the LTGM-PC as well as within the LTGM-PC (public versus private investment, increasing quantity versus quality of public investment), we run simulations using both models for a sample of 147 countries. To solve each model, we need to first input baseline parameters, initial conditions, and paths of variables for individual countries. Data sources and calculations are detailed in Table 3.

5.2. Calibration of select parameters

5.2.1. The usefulness of public capital (elasticity of output with respect to public capital)

The effect of public investment on growth is most sensitive to the elasticity of output to public capital, ϕ, which we calibrate carefully in this subsection based on two meta-analysis studies – Bom and Ligthart (2014) and Nunez-Serrano and Velazquez (2016), and a recent paper by Calderon *et al*. (2015). Meta-analyses are necessary due to the controversy in the literature on this parameter with a range of studies with different samples, definitions, and methodologies. Our default value is

Table 3. Baseline parameters, initial conditions, and paths of variables

Constant parameters	**Model(s)**	**Source/Calculation**
Labor share β	Standard LTGM LTGM-PC	Latest individual country β, based on Penn World Tables (PWT) version 8.1, or the Global Trade Analysis Project (GTAP) database. Labor shares <0.45 or >0.7 were trimmed to those values to remove outliers.
Elasticity of output to public capital ϕ	LTGM-PC	0.170 (default) and 0.100 for all countries. Section 5.2 discusses this calibration.
Congestion ζ	LTGM-PC	1 (congestion) or 0 (pure public good) is assumed for all countries.
Efficient public capital stock θ	LTGM-PC	Infrastructure Efficiency Index (IEI) – Section 4. Unchanged efficiency is assumed for all countries.
Aggregate depreciation rate δ	Standard LTGM	Latest value from PWT 8.1.
Public capital and private capital depreciation rates δ^G, δ^P	LTGM-PC	δ^G=2% for all countries, which is the average depreciation rate for structures in the PWT 8.1. δ^P, individual country-specific, is the residual from the weighted average calculation.
Initial conditions		
Initial capital-to-output ratio $\frac{K_0}{Y_0}$	Standard LTGM	Individual country values are based on the most recent data in PWT 8.1. (Capital stock at constant 2005 national prices (2005 US$ mil.)/Real GDP at constant 2005 national prices (2005 US$ mil.). *K/Y* <1.5 or >3.5 were trimmed to those values to remove outliers.
Initial measured public capital-to-output and private capital-to-output ratios $\frac{K_0^{Gm}}{Y_0},\frac{K_0^P}{Y_0}$	LTGM-PC	Individual country values are derived by applying 2015 shares of public and private capital calculated by the IMF Fiscal Affairs Department to PWT data on K_0/Y_0. IMF data: http://www.imf.org/external/np/fad/publicinvestment/.
Projected paths		
TFP growth g_A	Standard LTGM LTGM-PC	Individual country average value over 2001–2010 based on PWT 8.1 (or applying PWT methodology with GTAP labor shares). Missing TFP in PWT calculated by Barrot (2016) using GTAP labor share. Negative TFP growth adjusted up to zero.
Human capital growth rate g_h	Standard LTGM LTGM-PC	Individual country average value over 2001–2010 from PWT 8.1, based on years of schooling and returns to education.
Growth in labor participation rate g_ϱ	Standard LTGM LTGM-PC	International Labor Organization (ILO) individual country data for 2014, available from the World Development Indicators (WDI) database.
Population growth g_N	Standard LTGM LTGM-PC	World Bank compilation of data sourced from the United Nations Population Division and country censuses. Projections for 2016–2050.
Growth in the working age-to-population rate g_ω	Standard LTGM LTGM-PC	World Bank estimates based on United Nations Population Division data on age and sex distribution of the population. Projections for 2016–2050.

(Contd...)

Table 3. ***(Continued)***

Constant parameters	Model(s)	Source/Calculation
Investment-to-output ratio $\frac{I}{Y}$	Standard LTGM	Individual country data are based on the most recent data in WDI (Gross fixed capital formation (constant 2010 US$)/GDP [constant 2010 US$]).
Public investment-to-output and private investment-to-output ratios $\frac{I^G}{Y}, \frac{I^P}{Y}$	LTGM-PC	Individual country values are derived by applying the 2015 shares of public and private investment calculated by the IMF FAD to WDI data on *I/Y*. IMF data: http://www.imf.org/external/np/fad/publicinvestment/

Notes: The sample of countries is guided by the availability of human capital data. For missing data in other variables, these are interpolated for the affected countries based on the median value of available data for the corresponding country groups by income. An initial sample of 151 countries was reduced to 147 to exclude countries that were outliers in terms of the incremental growth effect of investment versus its analytical difference when the LTGM-PC and Standard LTGM were compared.

$\phi = 0.17$, which is the upper bound across the two meta-analyses and should be applied to "essential infrastructure" or "productive capital" only and is the same as in the IMF's DIG model (Buffie *et al*., 2012). As such, our default calibration should be viewed as being relatively optimistic about the growth effects of public capital. An alternative value (which could be viewed as a lower bound) is $\phi = 0.10$ for all public capital. This latter value is slightly larger than the estimates in Calderon *et al*. (2015) of $\phi = 0.07-0.10$.

Bom and Ligthart (2014) look at 68 empirical studies[21] which cover the period 1983–2008. These studies are based on the production function approach and measure public capital in monetary and stock terms. Bom and Ligthart's meta-analysis indicates an average elasticity of output to public capital of 0.106, which is higher in the long run and for core public capital, but lower for aggregate public capital/national government level (compared to the regional/local government level).

Nunez-Serrano and Velazquez (2016) consider 145 papers[22] which cover the period 1983–2011. The empirical studies scrutinized are predominantly those that take a production function approach (85% of the sample) and include studies that use non-monetary (17%) and flow (7%) measures of public capital. They find an average elasticity of output to public capital of 0.132, which is higher in the long run, 0.161. Although with less statistical significance, there is also an indication that the elasticity value is higher with productive public capital. The distinction between monetary and non-monetary measures of public capital does not have a discernible effect. A summary of select results from the respective meta-analyses of Bom and Ligthart (2014) and Nunez-Serrano and Velazquez (2016) is presented in Table 4.

Neither of the two meta-analyses contain a discussion of potential differences in between developed and developing countries. However, Calderon *et al*. (2015),[23] using a relatively extensive cross-country sample, find that the long-run elasticity of infrastructure does not seem to vary with countries' per capita income levels, infrastructure endowment or population size.

21 Of the 68 studies, five cover country groups (of which one is exclusively developing countries) and 63 are country-specific (all advanced economies).

22 Of the 145 studies, 26 cover country groups (of which five are exclusively developing countries) and 119 are country-specific (of which five are developing countries).

23 In the study, infrastructure is measured as a composite of several physical infrastructure indicators (as opposed to the monetary value of capital stock). A panel data set is used, comprising 88 countries that cut across different income levels and infrastructure endowments.

Table 4. Elasticity of output to public capital, ϕ ("usefulness")

Study	ϕ value
Bom and Ligthart (2014):*	
Core infrastructure[a]	0.170
Total public capital	0.122
Nunez-Serrano and Velazquez (2016):*	
Productive capital[b]	0.175
Total public capital	0.161
Calderon *et al*. (2015) (infrastructure):*	0.070–0.100

Notes: [a]*Refers to roads, railways, airports, and utilities.* [b]*Refers to capital aimed at health, education, housing and community services, energy installations, communication, and transport infrastructure. Values are computed based on results reported in Table 3, Section B.a. of Nunez-Serrano and Velazquez (2016). *Long run estimates. Note that most studies control for the total or private capital stock.*

5.2.2. Initial capital-to-output ratios and depreciation rates

Our default data source for aggregate data on capital stocks and depreciation rates is the PWT 8.1, which is also used in the Standard LTGM. Unfortunately, PWT 8.1 does not include the split into public and private capital, so we rely on data from the IMF for the relative shares of public and private capital stocks (IMF 2015 and http://www.imf.org/external/np/fad/publicinvestment/).[24] Given different computation methodologies, aggregate *K/Y* ratios differ between the PWT 8.1 and the IMF and are on average lower in the latter case.[25] We assume that public capital is mostly structures, and so apply the 2% structures depreciation rate from PWT 8.1. The private capital depreciation rate is a residual determined by the country-specific PWT 8.1 depreciation rate for aggregate $\mathrm{K}: \delta = \delta^{G} \dfrac{K^{Gm}}{K} + \delta^{P} \dfrac{K^{P}}{K}$.

5.2.3. Congestion (ζ=1) versus pure public good (ζ=0) and actual versus measured TFP growth

Users of the LTGM-PC must choose either a calibration with congestion (ζ=1) or without it (pure public good ζ=0). Unless users strongly feel that public capital in their country is not congested, we recommend using the congestion calibration. We use this ourselves as the default for the results in this paper and the Excel-based tool. The reason is that without congestion, actual TFP in the model (A_t in Equation 1) tends to depart from measured TFP using standard growth accounting exercises. In any practical application of the LTGM-PC, the actual TFP growth rate is one of the most important assumptions – and also the most difficult to calibrate. When using the congestion specification, the actual TFP growth rate is similar to what one would get applying a standard growth accounting exercise. In contrast, without congestion, the measured TFP growth

24 We use the following data from the IMF to calculate the shares: general government capital stock and private capital stock in billions of constant 2011 international dollars.

25 Both sources of data use variations of the perpetual inventory method to estimate aggregate capital stocks and do not take account of the destruction/damage of capital from wars/conflicts (which is naturally difficult to measure). Wars/conflicts also reduce output, perhaps faster than they destroy capital, and so the aggregate *K/Y* ratio may rise as a country enters a war/conflict. In the reconstruction period, it would not be surprising if measured public capital it extremely high because it fails to take account of the public capital destroyed in the conflict.

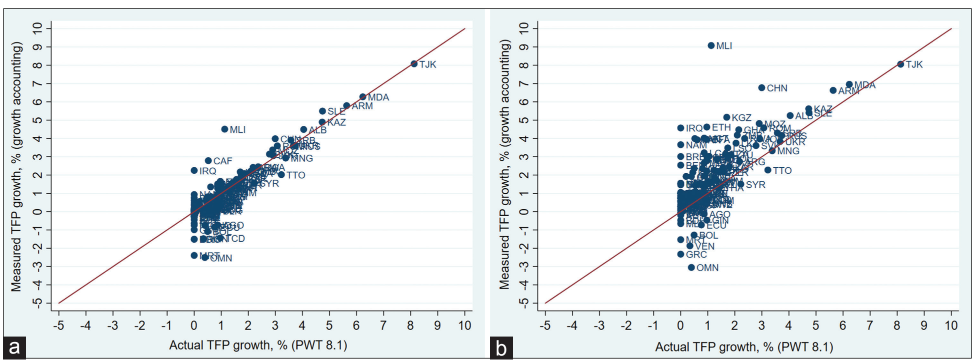
rate is above the actual TFP growth rate, and so cannot be used to guide the calibration of the actual TFP growth rates without some kind of adjustment.

To see this, for each country we used the LTGM-PC to generate a growth path by assuming that actual TFP growth was the same as that recorded in PWT 8.1 (over a 10-year average).[26] Then, we performed a standard growth accounting exercise on the generated growth rates (given growth rates of other growth fundamentals such as human capital and the total capital stock), to generate measured TFP growth. We then compared the measured TFP growth to the inputted actual TFP growth.

Graph 2 plots actual (X-axis) versus measured (Y-axis) TFP growth for the congestion (ζ=1, Graph 2a) and pure public good (ζ=0, Graph 2b) calibrations. As one can see, generally actual and measured TFP growth are very similar (close to the 45-degree line) for the congestion specification, but less so for the pure public good specification. Quantitatively, the mean absolute deviation (MAD) between actual and measured TFP growth for congestion is around 0.6 ppts, whereas for the pure public good specification it is twice as large (1.2 ppts), and also measured TFP growth is biased upward.[27]

This result can be shown analytically with some mild assumptions. Let measured TFP growth be $g_A^{meas} = g_Y - \beta\left(g_h + g_L\right) - \left(1-\beta\right)g_K$ (where g_K is the growth rate of the total capital stock), and actual TFP growth in the LTGM-PC (from Equation (3), rearranged in growth rates) be $g_A^{actual} = g_Y - \beta\left(g_h + g_L\right) - \phi\left(g_\theta + g_{K^{Gm}}\right) - \left(1-\beta-\zeta\phi\right)g_{K^P}$. For standard growth accounting to inform our TFP growth assumptions, we need $g_A^{meas} = g_A^{actual}$ which implies $\phi\left(g_\theta + g_{K^{Gm}}\right) + \left(1-\beta-\zeta\phi\right)g_{K^P} = \left(1-\beta\right)g_K$. If we assume that (i) there is no trend growth in efficiency (g_θ=0, which must be true in the long run as θ is bounded above by 1), and (ii) all capital stocks grow at approximately the same nonzero rate $(g_K \approx g_{K^{Gm}} \approx g_{K^P} \neq 0)$, then $g_A^{meas} = g_A^{actual} \Rightarrow \zeta = 1$.

A corollary is that if growth rates of the different types of capital are similar (and other fundamentals are the same), then the growth rate of GDP generated by the LTGM-PC is consistent with a growth rate generated by a canonical neoclassical model with aggregate capital like the Standard LTGM. We show this numerically in Appendix A1.3.



Graph 2. Measured TFP growth from LTGM-PC (ϕ=0.17) based on growth accounting versus actual TFP growth. (a) Congestion (ζ=1). (b) Pure public good (ζ=0).

Notes: Excludes outliers, Liberia, and Macao, the former showing large negative measured TFP growth under congestion, and the latter large positive measured TFP growth in the pure public good setting. Table 3 provides details on actual TFP growth. Measured TFP growth is obtained using traditional growth accounting (with total capital stock) based on the initial (2017) baseline output growth rates of the congestion and pure public good calibrations.

26 To be clear, the exercise is not dependent on the source of assumed actual TFP growth, but just that its distribution across countries is reasonable.

27 The MAD between actual and measured TFP growth when public capital was assumed to be less useful (ϕ=0.10) was also smaller for the congestion specification (0.6 ppts) than the pure public good specification (1.0 ppt).

6. Results: Stylized facts and the effect of an increase in investment on growth

6.1. Stylized facts on the infrastructure gap and the return to public/private investment

It is often argued that there is a large public infrastructure gap in developing countries, with current public infrastructure falling far short of what is needed. From a human development perspective, this is definitely true, based on figures like 700 million people without safe drinking water and 1.2 billion people without electricity (quoted in the Introduction). However, does this public infrastructure gap mean that the return to new public investment in developing countries is much higher than that in developed countries? Put another way, are developing countries particularly short of public capital relative to their level of development? What about if we adjust for the lower quality of public capital in developing countries? And are they short of public infrastructure relative to, say, private capital?

In this section, we answer these questions through the lens of our calibrated model for representative countries at various stages of development based on the World Bank classification:[28]

- Low income (LI) - GNI PC<\$1,000
- Lower middle income (LMI): \$1,000< GNI PC<\$4,000
- Upper middle income (UMI): \$4,000< GNI PC<\$12,000
- High income (HI): GNI PC>\$12,000.

Parameters for each "representative country" are the within-group medians (Table 5)[29], taken from an overall sample of 108 countries with complete (non-interpolated) data. A caveat here is that the sample of LI countries with complete data is quite small (only 12 countries), and so there is a chance that results for that group might change with better data. We report results using the default congestion setting (ζ=1) for essential infrastructure (ϕ=0.17) – with robustness to other parameters reported in Appendix 3. In addition to answering the questions above, this also provides a guide to how the LTGM-PC might be used in specific countries.

We find no evidence that *measured* public capital is particularly scarce for LI or LMI countries relative to GDP. In fact, public capital as a share of output is relatively constant across various levels of development at around K^{Gm}/Y =0.92 (±0.05), with LI countries having the highest K^{Gm}/Y and LMI countries being in the middle of the group (Table 5 Panel A). If anything, it is HI countries that are relatively short of public capital, as their ratio of K^{Gm}/Y =0.86 is the lowest.

How do we square this with the narrative of infrastructure gaps above? The first answer is that developing countries are short of productive public capital (G_t/Y_t), rather than measured public capital (K_t^{Gm}/Y_t) – an issue we revisit in Section 6.4. As argued in Keefer and Knack (2007), developing countries with poor institutions tend to have higher rates of public investment, which the authors argue are inflated to provide rents and kick-backs.

The second answer is that developing countries have a shortage of public capital relative to their development aspirations, but not relative to their current development level. That is, people in LI countries have many unmet needs with public infrastructure capital being in just as short supply as everything else.

28 Cutoffs are expressed in GNI per capita (to 2 significant figures), calculated using the World Bank Atlas method.
29 Replication files for the main Tables and Figures are available from the authors upon request.

As the "usefulness" of public capital does not vary with income (Calderon *et al*., 2015), the stability of the public capital-to-output ratio across income levels also means that the marginal product of measured public capital (MPK^{Gm}) is not relatively larger in developing countries. As we will see in the next section, this is the effect of an expansion of public investment on growth in the short-run with unchanged efficiency. Specifically, we find that with our relatively generous calibration of usefulness (ϕ=0.17), that the MPK^{Gm} is around 18.5% (±1%), which varies inversely with the public capital-to-output ratio (Table 5, Panel B). After subtracting the depreciation rate of 2% (constant across countries), this yields a return to new public investment of around 16.5% (±1%) which also does not vary systematically across levels of development. The relatively high absolute returns stem from the high assumed usefulness of public capital in our default calibration. If instead, we assumed ϕ =0.10 (for public buildings), then the return to public investment falls to around 9% (±1%), though the ranking across income groups is unchanged.

While some might interpret the lack of a higher return to public investment in LI countries as negative, we are more sanguine. It also means that development banks need not refrain lending for infrastructure projects in countries with poor implementation capacity – as they are often encouraged to do – because that low capacity also means the projects are even more in need.[30]

In contrast, it seems that LI countries have a shortage of private capital for their level of development, and the scarcity of private capital falls with per capita income. Specifically, K^P/Y is around 1.25 for LI countries, around 1.80 for middle-income countries and 2.25 for HI countries (80% higher than that of LI countries). Measured public capital is also the largest share of total capital in LI countries (44%) and the lowest share in HI countries (28%). This reflects the fact that private financial markets are typically underdeveloped in LI countries and so the private sector finds it difficult to raise funds for investment. In some countries, insecure property rights also reduce their incentive to invest in the first place.

A consequence is that the marginal product of private capital is the highest in LI countries (25%), which is double that in HI countries (12%), with middle-income countries in between. Note that the MPK^P does not vary exactly inversely with the K^P/Y across income groups, due to cross-country variation in the capital share 1-β.[31] After subtracting depreciation (around 5%), the return to private investment is the highest in LI countries (around 20%), lower in middle-income countries (13–17%), and lowest in HI countries (7%). Interestingly, the return to private capital for LI countries is actually higher than for public investment (20% versus 16%), suggesting that if savings are scarce, governments need to be careful that public investment does not crowd out private investment.

As today's capital stocks reflect past investment, one might expect that public investment would make up a larger share of total investment in LI countries – which is exactly what we find. Public investment is 37% of total investment in LI countries, double the share in HI countries (18%), and with middle-income countries in between (23%).[32] Translated to a share of GDP, public investment spending increases steadily with lower income per capita: 4% of GDP in HI countries, 5% in UMI countries, 6% in LMI countries, and 7% of GDP in LI countries. Keefer and Knack (2007) argue that this is likely due to poor quality governance in developing countries, rather than the level

30 For example, Keefer and Knack (2007) argue that their "results therefore signal the need for donor agencies to exercise particular caution in supporting public investment in countries with a weak institutional environment."

31 The return to private capital is even higher with ϕ=0.10 (though its ranking across groups is unchanged), as the penalty for reducing the congestion of public capital is lower.

32 These figures are average investment rates over 2001-15. In steady state $K^j / Y = I / Y / (g_Y + \delta^j)$ for $j=G,P$. If the countries in Table 5 were in steady state, headline GDP growth rates would need to be ≈3% for HI countries, ≈3.8% for UMI countries, ≈4.8% for LMI and ≈5.6% for LI countries, which are fairly close to what we observe.

Table 5. Median values of baseline parameters and paths of variables by income group

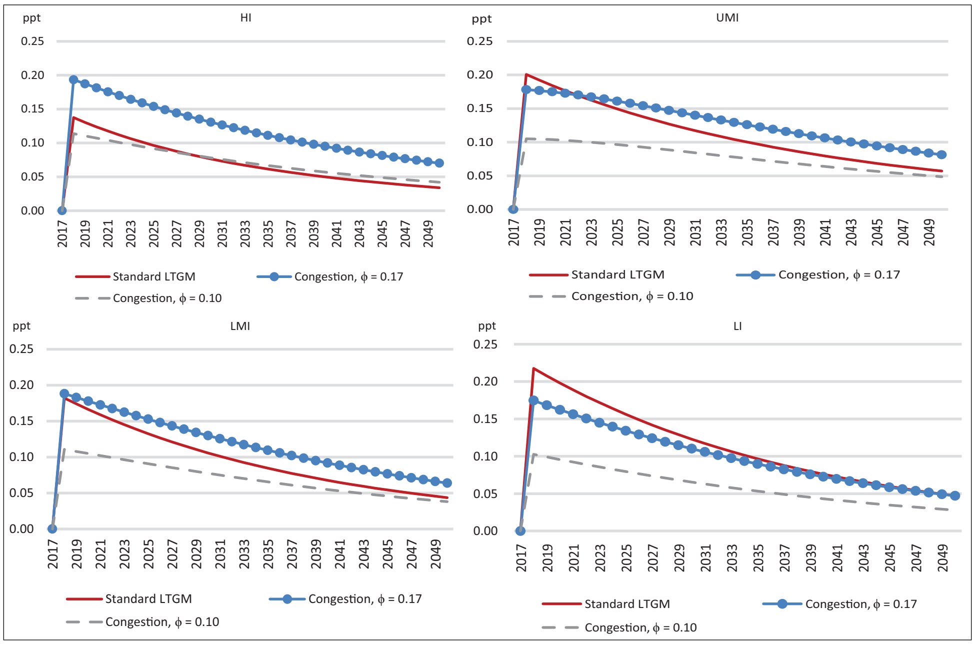
Parameter/variable	Note	HI	UMI	LMI	LI
A. Capital and investment-group medians*					
Labor share β	(1)	0.561	0.450	0.503	0.520
Public capital share of total capital K^{Gm}/K		0.277	0.356	0.330	0.440
Capital-to-output ratio (measured)					
Total K_0/Y_0	(2)	3.110	2.706	2.722	2.208
Public K_0^{Gm}/Y_0	(3)	0.863	0.965	0.900	0.972
Private K_0^{P}/Y_0	(3)	2.247	1.741	1.822	1.235
Depreciation rate					
Total δ		0.040	0.038	0.042	0.035
Public δ^G		0.020	0.020	0.020	0.020
Private δ^P	(4)	0.047	0.048	0.053	0.047
Public investment share of total investment I^G/I	(7)	0.177	0.219	0.243	0.369
Investment-to-output ratio					
Total I/Y	(5)	0.223	0.223	0.236	0.201
Public I^G/Y	(6)	0.039	0.049	0.057	0.074
Private I^P/Y	(6)	0.183	0.174	0.179	0.127
No of countries		*48*	*29*	*19*	*12*
B. The return to investment (with ϕ=0.17; ζ=1)*					
Marginal product of measured public capital $MPK^{Gm} = \phi / \left(K_t^{Gm} / Y_t\right)$		0.197	0.176	0.189	0.175
Return to public investment ($MPK^{Gm}-\delta^G$)		0.177	0.156	0.169	0.155
Marginal product of private capital $MPK^P = \left(1-\beta-\phi\right) / \left(K_t^P / Y_t\right)$		0.120	0.218	0.180	0.251
Return to private investment ($MPK^P-\delta^P$)		0.072	0.170	0.127	0.204

Notes: Table 3 provides a general description of the sources of data. The sample size for all variables is guided by the consistent availability for individual countries of PWT 8.1 data for labor share, capital-to-output ratio, human capital growth, and TFP growth. Appendix 3, Table A3.1 provides median values of other key variables used in the simulations.
Countries are classified according to the 2018 World Bank Classification of countries by income for the 2017 calendar year.
**Multiply by 100 to obtain parameter/variable values in percent terms (%).*
(1) For β, individual country values <0.45 were increased to 0.45 and those >0.70 reduced to 0.70.
(2) For K_0/Y_0, individual country values <1.5 were increased to 1.5 and those >3.5 reduced to 3.5.
(3) $(K_0^{Gm})/Y_0$ and $(K_0^{P})/Y_0$ are derived based on the median values of K_0/Y_0 and K^{Gm}/K.
(4) δ^P is derived based on the median values of δ, δ^G and K^{Gm}/K.
(5) Median of 15-year averages across countries over 2001–2015. I/Y is gross fixed capital formation (% of GDP) from WDI, except for Qatar, for which the same variable is tabulated from the IMF FAD Investment and Capital Stock Database.
(6) I^G/Y and I^P/Y are derived based on the median values of I/Y and I^G/I.

of income *per se*. Consistent with earlier results, private investment is particularly lacking in LI countries – around 13% of GDP versus 17–18% of GDP in the other three income groups.

6.2. The effect of an increase in public investment on growth

Graph 3 shows the effect of a permanent 1 ppt GDP increase in public investment on growth. In the year following the shock, growth increases by the marginal product of measured public capital (Equation (16a) with $\theta_t^N = \theta_t$), which as argued above is around 0.17–0.19 ppts and is surprisingly similar across countries with different levels of development (blue dotted line with ϕ=0.17, congestion specification (ζ=1)).



Graph 3. Incremental output growth from a 1 ppt increase in public investment in the LTGM-PC (congestion, ζ=1)

The second thing to note from Graph 3 is that the increment to growth falls over time. This is because as public capital accumulates, K^{Gm}/Y increases, which reduces the marginal product of measured public capital. This is intuitive: One would expect an increase in public investment to become less effective in boosting growth over time as "infrastructure needs" are met. There is some heterogeneity across income groups: The boost to growth is slightly less persistent in LI countries. This could be because these countries have a lower capital share ($1-\beta$), and so the marginal product of private capital which is "crowded in" declines more quickly (in addition to marginally higher K^{Gm}/Y which means the marginal product of public capital dwindles faster over time).

Finally, the effect of an increase in public investment in the LTGM-PC (with ϕ = 0.17 and ζ=1) is on average similar to the effect of a same-sized increase in the aggregate investment in the Standard LTGM (red solid line) for all but HI countries. Specifically, the effect in the LTGM-PC is very similar to the Standard LTGM for middle-income countries, slightly lower for LI countries, and higher for HI countries. The latter is because HI countries tend to have the lowest share of total capital owned by the public sector. The effect of public investment on growth is naturally not only much smaller in the LTGM-PC when ϕ = 0.10 (dashed gray line) but also much smaller than the comparable effect in the Standard LTGM (except in HI countries). Greater consistency with the Standard LTGM is one reason we prefer the ϕ = 0.17 calibration over the ϕ = 0.10 calibration.[33]

6.3. The effect of an increase in private investment on growth

Graph 4 reports the effect of a 1 ppt of GDP expansion in private investment on growth, which is largest for LI countries. In the first period following the shock, growth increases by the MPK^P

33 With the pure public good setting (Appendix 3, Graph A3.1), the immediate effect of higher public investment is broadly similar to that of the congestion setting, but is more persistent and higher than the congestion setting (and hence the Standard LTGM).

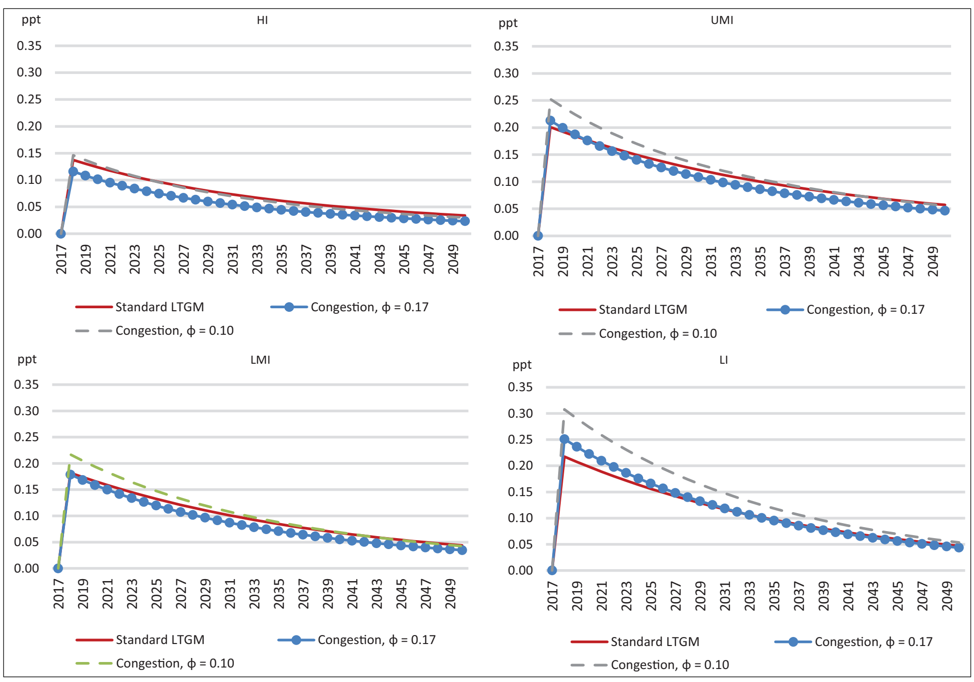
(Equation (16b)) which as argued above is twice as large for LI countries as HI countries (25 ppts versus 12 ppts). As before, the increase in growth falls over time as private capital accumulates, raising K^P/Y and lowering the marginal product of additional private capital. Nonetheless, the effect of higher private investment on growth is quite persistent, verifying our claim above that developing countries are particularly short of private capital.

Comparing different parameterizations: With the congestion specification, a lower ϕ increases the effective output elasticity of private capital $(1-\beta-\phi)$ as it reduces the strength of congestion – increasing the effect of private investment on growth. On average the increment to the growth of private investment is similar with the Standard LTGM (and slightly closer with $\phi = 0.17$). With the pure public good setting (Appendix 3, Graph A3.2), the effect of an increase in private investment is much larger than with the congestion specification, and much larger than the Standard LTGM.

6.4. In which countries is public or private investment more effective for boosting growth?

The previous two sub-sections allowed us to compare the effects of investment (public and private) in the LTGM-PC for income group medians. However, countries within each income group are highly heterogenous. What determines whether the private or public investment has a larger effect on growth in the LTGM-PC for individual countries?

Short Run: Based on Equations (16a) and (16b), the difference in the short-run boost to growth from similar increments to private and public investment-to-output ratios, respectively, is given by Equation (25). One can see that short-run return to private investment is larger if $(1-\beta-\zeta\phi)/\phi > K_t^P / K_t^{Gm}$. This condition is violated (with public investment generating a larger increase in growth) when public investment is relatively useful (high ϕ relative to $1-\beta$) and

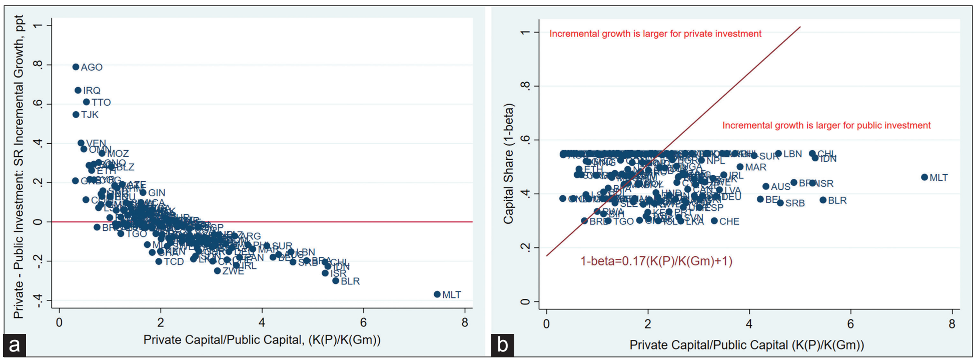


Graph 4. Incremental output growth from a 1 ppt increase in private investment in the LTGM-PC (congestion, ζ=1)

public capital is relatively scarce (high K_t^P/K_t^{Gm}). One can see this as the lower right region of Graph 5b.[34]

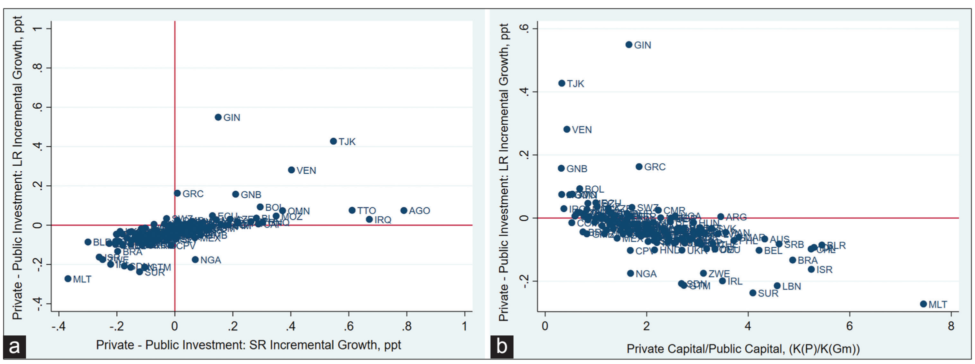
$$\underbrace{\frac{\partial g_{y,t+1}^{PC}}{\partial I_t^P / Y_t}}_{Private} - \underbrace{\frac{\partial g_{y,t+1}^{PC}}{\partial I_t^G / Y_t}}_{Public} = \frac{Y_t}{K_t^P}\left[(1-\beta-\zeta\phi)-\phi\left(\frac{K_t^P}{K_t^{Gm}}\right)\right] \tag{25}$$

The increment to growth from private investment in the short-run is higher than that for public investment whenever private capital stock is less than about double that of the public capital (Graph 5a, with ϕ=0.17, and ζ=1). This occurs for 40% of countries, and the median K^P/K^{Gm} of countries with a higher increment is 1.26. As the marginal product of private capital is higher in the pure public good setting (since there is no congestion), the return to private investment is naturally higher. Specifically, under the pure public good setting (Appendix 3, Graph A3.3(i)), the increment to growth from private investment is higher for two-thirds of countries, and the median K^P/K^{Gm} of countries with a higher increment is 1.60.[35]



Graph 5. Private investment versus public investment in the LTGM-PC – differences in short-run incremental output growth (ϕ=0.17, congestion ζ=1).

Notes: Analytical Difference: $\frac{Y_t}{K_t^P}\left[(1-\beta-\zeta\phi)-\phi\left(\frac{K_t^P}{K_t^{Gm}}\right)\right]$ *Equation (25).*



Graph 6. Private investment versus public investment in the LTGM-PC – differences in long-run incremental output growth (ϕ=0.17, congestion ζ=1)

34 Readers will note that this is a rearrangement of $(1-\beta-\zeta\phi)/(\frac{K_t^P}{Y}) > \phi / \left(\frac{K_t^{Gm}}{Y}\right)$, which is an equivalent condition that the marginal product of private capital is higher than the marginal product of public capital.

35 When ϕ=0.10, more countries record a higher increment to growth from private investment: 84% of countries, under the congestion setting (see Appendix 3, A3.3(ii)).

In the long run, the increment to growth from private investment is higher for only a quarter of countries, and for those countries, the private and public capital stocks are roughly the same size (Graph 6, ϕ=0.17, ζ=1). Under the pure public good setting (Appendix 3, Graph A3.4(i)), the increment to growth from private investment is higher for around 40% of countries, and the median K^P/K^{Gm} of countries with a higher increment is 1.24.[36]

Overall, for our sample of countries and calibration, public investment has stronger effects on growth than private investment for most countries, assuming that the elasticity of output to public capital is relatively high (ϕ=0.17) The approximate cutoff where public investment tends to have stronger effects is when $K^P/K^{Gm}>2$ (or equivalently, $K^{Gm}/K^P<0.5$).[37] In the short run, the boost to growth from public investment is only larger than private investment for most countries when there is congestion.

6.5. The efficiency of public investment

6.5.1. Stylized facts

One of the reasons that public capital might have less effect on growth in developing countries is that it is inefficient. However, as already noted in Section 2, the level of efficiency θ does not affect the marginal product of measured public capital and the effect of additional public investment on growth. For low efficiency countries, only a fraction of new investment might become productive capital, but this is offset by the greater need for public capital given that past investment was also inefficient.[38] However, this does not mean that increasing public investment efficiency cannot have a large effect on growth.

The potential for higher growth through greater efficiency of public investment depends on cuwrrent efficiency being low. In the first row of Table 6, we calculate the median of the IEI for each income group (Section 4 has details on the construction of the IEI). As expected, based on the full set of computed IEI, HI countries have the highest median efficiency, with around 87% of roads being paved or water/electricity reaching their final destination. For middle-income countries, efficiency is about 74%, or one-seventh lower. Efficiency is the lowest for LI countries, where only 59% of roads are paved, or water/electricity reach their final destination; which is about one-third lower than efficiency in HI countries.

One can also use the IEI to calculate the efficient public capital-to-output ratio $\theta K_0^{Gm} / Y_0$. Since the public capital-to-output ratio (K_0^{Gm} / Y_0) is roughly constant across levels of development, but efficiency θ increases with development, the combined efficient public capital-to-output ratio also increases with development. Specifically, the efficient public capital-to-output ratio is around 0.75 in HI countries, 0.73 in UMI countries, 0.66 in LMI countries, and 0.57 in LI countries. This suggests that LI countries do not have a shortage of measured public capital (as argued above), but rather a shortage of efficient public capital.

36 When ϕ=0.10, more countries would record a higher increment to growth from private investment (see Appendix 3, Graph A3.4(ii)).

37 Angola and Iraq are outliers in Graphs 5a and 6a, which may reflect that both have been involved in long-drawn conflicts. See footnote 25, for a further discussion of how K/Y ratios might be affected by war/conflict.

38 Berg *et al*. (2015) show that for the Cobb-Douglas production function (as used here), these two are exactly offsetting. However, even with the CES production function the two factors are mostly offsetting unless public and private capital are extremely complementary (their Figure 3).

Table 6. Efficiency – income group medians (with ϕ=0.17; ζ=1)

Parameter/variable	**HI**	**UMI**	**LMI**	**LI**
Efficient public capital stock θ	0.870	0.753	0.730	0.590
*No of countries***	*21*	*25*	*27*	*10*
Efficient public capital-to-output ratio $G_0 / Y_0 = \theta K_0^{Gm} / Y_0$	0.751	0.726	0.656	0.574
Marginal product of efficient public capital $MPK_e^G = \left[\frac{\phi}{\theta_t K_t^{Gm} / Y_t}\right]$	0.226	0.234	0.259	0.296
Marginal product of efficiency $MPe = \phi\left[\frac{I_t^G / Y_t}{\theta_t K_t^{Gm} / Y_t}\right]$	0.009	0.011	0.015	0.022
$\mu_{SR}=I^G/Y/\theta$ ppts; increase in I^G/Y equivalent to 1 ppt increase in θ_N after 1 year (also equivalent by 2040)	0.045	0.065	0.079	0.125

Notes: Table 3 has a general description of the sources of data.
Countries are classified according to the 2018 World Bank Classification of countries by income for the 2017 calendar year.
**Multiply by 100 (except values in ppts in the last row) to obtain parameter/variable values in percent terms (%).*
***Number of countries for efficiency is based on IEI data.*
Calculations for $\theta K_0^{Gm} / Y_0$, MPK_e^G, MPe, and μ_{SR}, are formed by combining the data in Table 5 with the efficiency θ for each income group (not on a constant group of countries).

How should we interpret the efficient public capital-to-output ratio $\theta K_0^{Gm} / Y_0$ for policy? Equation (26) is the marginal product of efficient public capital (MPK_e^G) - the effect of an extra percentage point of GDP of efficient investment $\theta_t^N [I_t^G / Y_t]$ on growth (the derivative of Equation (16) with respect to $\theta_t^N I_t^G / Y_t$). One can see that the MPK_e^G is inversely proportional to the efficient public capital-to-output ratio $\theta K_0^{Gm} / Y_0$. As such, a low efficient public capital-to-output ratio means that the return to an extra efficient unit of public investment is high.

$$MPK_e^G \equiv \frac{\partial g_{y,t+1}^{PC}}{\partial[\theta_t^N I_t^G / Y_t]} = \frac{\phi}{\theta_t K_t^{Gm} / Y_t} \tag{26}$$

Since LI countries have the lowest $G_t / Y_t = \theta K_0^{Gm} / Y_0$ they also have the highest MPK_e^G = 30%, which is almost double the regular marginal product of public capital from Section 6.1. In contrast, middle-income countries have a MPK_e^G of about 25%, while HI countries, a lower MPK_e^G of 23%.[39] The high MPK_e^G means that if a typical LI country (with low efficiency) was somehow able invest efficiently, the returns for growth would be very high. However, this is a hypothetical scenario. As Berg *et al*. (2015) and others point out, quickly increasing public investment implementation capacity is difficult, largely because public investment capacity has deep determinants – such as poor institutional quality and a lack of relevant bureaucratic human capital.

6.5.2. Increasing efficiency ("investing in investing")

However, it can still be that the efforts to improve efficiency are highly cost-effective, even if they only lead to a slow increase in the average quality of the public capital stock. Here, we consider the

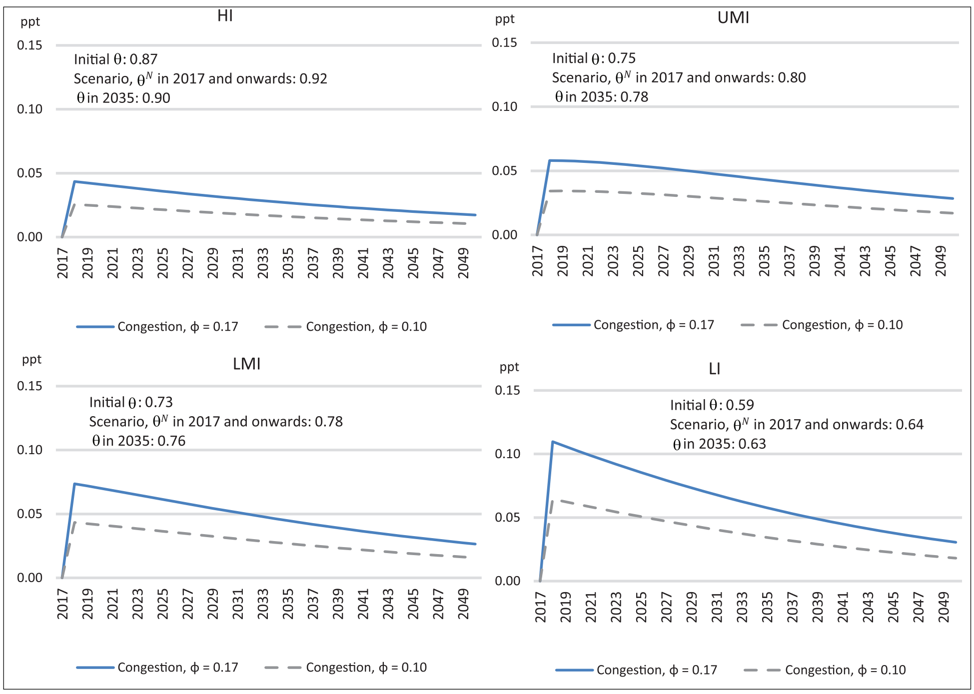
39 The large absolute size of the MPK_e^G stems from the generous calibration of usefulness (ϕ =0.17). Instead, with ϕ =0.10, the MPK_e^G falls to 17% for LI countries, 14-15% in middle-income and 13% HI countries.

effect on growth of a 5 ppt increase in the efficiency of new public investment from 2017 onward (Graph 7), which is roughly a sixth of the gap between the efficiency of low and high-income countries (first row Table 6). We also assume public investment for each income group is at its group median from Table 5 panel A (3.9% for HI countries, through to 7.4% for LI countries). Average efficiency reflects characteristics of the installed public capital stock, and so changes only take place slowly. The effect on growth is strongest for the representative LI country given its low-quality stock, where increased efficiency boosts growth by 0.11 ppts.[40] For middle-income and HI countries, the average efficiency levels are much higher, and so the boost to growth is much smaller – around 0.05 ppts for UMI and HI countries (with ϕ = 0.17), and 0.075 ppts for LMI countries.[41] For LI countries, "catch up" is easier because practices are so far from the frontier.

What determines the increase in growth from an extra unit of efficiency in the short run? Taking the derivative of Equation (16) with respect to θ_t^N, produces the marginal product of efficiency (*MPe*), which is the boost to growth from a 1 ppt increase in the efficiency of new public investment (θ_t^N):

$$MPe \equiv \frac{\partial g_{y,t+1}^{PC}}{\partial \theta_t^N} = \phi \left[\frac{I_t^G / Y_t}{\theta_t K_t^{Gm} / Y_t} \right] \tag{27}$$

In Table 6, one can see that the *MPe* is highest in LI countries (0.022), which is more than double the rate in HI (0.009). UMI and LMI countries are in-between (0.011 and 0.015, respectively). As such a 5 ppt immediate increase in for LI countries will raise growth by 0.022 × 5 ppts=0.11 ppts, which is similar to the boost to growth in the first period in Graph 7. For HI countries, in contrast,



Graph 7. Incremental output growth from a 5 ppt increase in the the efficiency of public investment in the LTGM-PC (congestion, ζ=1)

40 Low base efficiency is important because it is the percentage (not percentage point) increase in efficiency that determines the effect on growth. A fixed 5 ppt increase in efficiency is a larger proportion of a low base.

41 As ϕ is the elasticity of output with respect to efficient public capital, the effect of an increase in the efficiency is much lower with ϕ =0.10 than with ϕ =0.17.

a 5 ppt increase in efficiency would raise growth by a much lower 0.05 ppts (0.009 × 5 ppts). The *MPe* is inversely proportional to the efficient public capital-to-output ratio, is increasing in the usefulness of public capital (ϕ) and also increasing in the public investment to output ratio (I_t^G / Y_t).

The last somewhat surprising result is because an increase in efficiency θ_t^N only affects new investment. Intuitively, in countries with low public investment rates, an increase in the efficiency of new public investment will only have a small short-run effect on the average efficiency of installed capital – and hence on growth – because the new efficient public capital is only a small fraction of the total capital stock. In these countries, a permanent increase in the efficiency of public investment will still boost output, but it will just take more time for these gains to materialize.

6.5.3. Which countries should invest in an increase in the quantity versus quality of public investment?

One can compare the effect on the growth of an extra unit of public investment (Equation (16a) at constant efficiency), and the return to an extra unit of efficiency (Equation (27)). As the two are in different units, it is not appropriate to assess which marginal product is larger (as we did with private and public investment). Instead, we calculate the size of the increase in the public investment rate (μ_{SR} ppts of GDP) equivalent to a 1 ppt increase in efficiency. The larger the value of μ_{SR}, the more effective an increase in investment quality is at boosting short-run growth (relative to boosting the quantity of investment). Setting $MPe=\mu_{SR}\times MPK^{Gm}$ (from Equation (27) and (16a)):

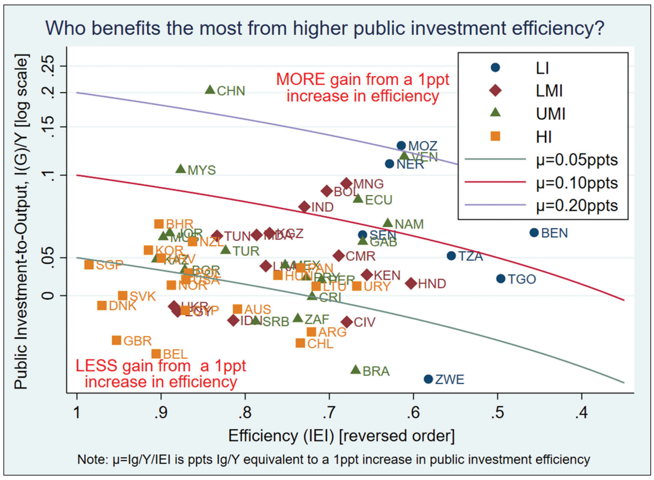
$$\phi\left[\frac{1}{\theta_t}\frac{I_t^G / Y_t}{K_t^{Gm} / Y_t}\right] = \mu_{SR} \times \phi\left[\frac{1}{K_t^{Gm} / Y_t}\right] \tag{28}$$

A nice feature of this comparison is that it does not depend on measured public capital scarcity (K_t^{Gm} / Y_t), or the usefulness of public capital (ϕ), which cancel out in Equation (28). That is, greater scarcity and usefulness increase *MPe* and MPK^{Gm} proportionately and so do not affect the relative effectiveness of quality versus quantity (though they do affect the aggregate size of both marginal products). Rearranging implies:

$$\mu_{SR} = \frac{I_t^G / Y_t}{\theta_t} \tag{29}$$

Equation (29) suggests that increases in investment quality are particularly effective in (i) countries with a high rate of public investment (because only new investment is affected by the improved investment management processes), and (ii) countries with low existing quality of public capital (so the improvement in quality is larger in percentage [not percentage point] terms). For LI countries, $\mu_{SR} = 0.13$ – that is a 1 ppt increase in the quality of public investment has the equivalent effect on short-run growth as a 0.13 ppt increase in the quantity of public investment (I^G/Y). Given that improving investment processes could be almost free (if feasible), saving 0.13 ppts of GDP on public investment expenditure for the same short-run growth outcome is good policy. Of course, improvements in quality are not as powerful elsewhere. For middle-income countries, an extra unit of efficiency is worth $\mu_{SR}\approx 0.07$ ppts of public investment, and for HI countries (with the lowest I_t^G / Y_t and highest quality θ) $\mu_{SR}=0.05$ ppts.

For individual countries with available data on IEI and public investment, Graph 8 plots the size of recent public investment-to-output ratios (15-year average, 2001–2015) on the Y-axis, and



Graph 8. Quantity versus quality of public investment

existing quality (as reflected by the IEI) on the X-axis. The further a country is toward the top right of the figure, the more effective a 1 ppt increase in investment quality is relative to greater investment quantity. Lines represent the locus of points for μ=0.05 ppts, 0.1 ppts, or 0.2 ppts. Specifically, many LI countries have public investment-to-output ratios that are greater than 5% and efficiency levels <0.6 and so fit on the upper right-hand side with the most to gain. Most HI countries are on the lower left-hand side with high efficiency and low public investment rates, suggesting limited gains from higher efficiency. Outliers are China (CHN) and Malaysia (MYS) which appear to have relatively high efficiency levels (≈0.85) but can still make sizable gains given relatively high public investment-to-output ratios of 20% and 11%, respectively. China and Mozambique benefit the most overall, where a 1 ppt higher efficiency is equivalent to ≈0.25 ppts GDP increase in public investment.

Our calculations so far have involved the short-run increase in investment equivalent to a 1 ppt increase in efficiency μ_{SR}. Instead, one might be interested in the permanent increase in investment that generates the same increase in GDP per capita by 2040 as a 1 ppt increase in efficiency – what we call μ_{LR}. Using numerical simulations, we find that the values of μ_{SR} and μ_{LR} are almost identical. This is because the increase in efficiency (and equivalently sized increase in I^G/Y) is small, which means that any non-linearities are second order.

7. Conclusion

In this paper, we develop a new model of public investment and growth – the Long-Term Growth Model Public Capital Extension (LTGM-PC) – which is designed to capture the effect of increases in public infrastructure investment quantity or quality on growth, while at the same time being simple enough to solve in a spreadsheet without macros (the Excel-based tool is provided as a companion to this paper at the website www.worldbank.org/LTGM). Relative to the Standard LTGM, our extension allows public and private capital to enter the production function separately and for public capital to be of poor quality such that only a fraction can be used in production.

The effects of public and private investment on growth in our model vary substantially across countries depending on whether the country is relatively short of public or private capital – but on average are similar to the effect of aggregate investment in the Standard LTGM. We show analytically and numerically that the effect of public investment on growth is higher when the public capital-to-output ratio is lower – that is when public capital is scarce. Conversely, in countries where public capital is abundant relative to other factors – even if it is scarce in absolute terms – public investment will have a smaller effect on growth. The growth impact is also larger when public investment is more useful, such as when it is in the form of essential infrastructure (public investment in other areas will have a lower return).

In contrast with several popular narratives, we find the growth impact of an increase in public investment is very similar across different levels of development. For a typical low- or middle-income country with our default parameters, a permanent 1 ppt of GDP increase in public investment in essential infrastructure tends to boost growth by around 0.18 ppts in the short term, but the boost to growth falls slowly over time as public capital accumulates. Other, less useful types of public investment (like public buildings) have a boost to growth of around 0.1 ppts. In contrast, a permanent 1 ppt of GDP increase in private investment leads to a slightly higher short-term boost to growth of about 0.22 ppts, although the effect tapers off faster over time.

Model simulations also show that there can be substantial growth dividends from improvements in the quality of new public investment. Our new IEI suggests a public capital efficiency loss of about 30 ppts for low-income countries, and 10–15 ppts for middle-income countries (relative to the efficiency of high-income countries). For countries with poor quality public capital and a large public investment share of GDP – such as many low-income countries – an increase in the quality of public investment can be just as effective as a modest increase in the quantity of public investment. For example, for the typical low-income country a 1 ppt increase in efficiency boosts growth by the same amount as a 0.13 ppt of GDP increase in public investment. Despite this, the level of efficiency has no effect on the marginal product of public capital because the low quality of new public investment is exactly offset by the greater need for public capital due to the poor quality of past public investment (as in Berg *et al*. 2015).

In closing, it is worth mentioning a few caveats to our model and stylized facts. To keep the LTGM-PC as simple as possible, we abstract from the effects of the financing of public investment through distortionary taxation. In many cases, this will act as a drag on growth, and so our growth impact should be seen as an upper bound in that context (unless public investment can be financed by reducing unproductive expenditure elsewhere). We also abstract from endogenous private investment and return-seeking international capital flows. These factors might lead to a larger crowd-in of private investment, but they could also amplify any negative impacts of distortionary taxation. Finally, our stylized facts depend on available data and the quality of that data. While we have data on many high- and middle-income countries, the sample size for low-income countries is small, which might increase the volatility of our estimates.

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Appendix

Appendices are available online at www.worldbank.org/LTGM