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Incorporating economic indicators and market sentiment effect into US Treasury bond yield prediction with machine learning

Zichao Li^{1,*}, Bingyang Wang², Ying Chen³¹ Canoakbit Alliance Inc., Oakville, ON L6H 0H3, Canada² Emory University, Atlanta, GA 30322, USA³ MyMap.AI Inc., Sunnyvale, CA 94086, USA* **Corresponding author:** Zichao Li, zichaoli@canoakbit.com

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Abstract: Accurate prediction of US Treasury bond yields is crucial for investment strategies and economic policymaking. This paper explores the application of advanced machine learning techniques, specifically Recurrent Neural Networks (RNN) and Long Short-Term Memory (LSTM) models, in forecasting these yields. By integrating key economic indicators and policy changes, our approach seeks to enhance the precision of yield predictions. Our study demonstrates the superiority of LSTM models over traditional RNNs in capturing the temporal dependencies and complexities inherent in financial data. The inclusion of macroeconomic and policy variables significantly improves the models' predictive accuracy. This research underscores a pioneering movement for the legacy banking industry to adopt artificial intelligence (AI) in financial market prediction. In addition to considering the conventional economic indicator that drives the fluctuation of the bond market, this paper also optimizes the LSTM to handle situations when rate hike expectations have already been priced-in by market sentiment.

Keywords: bond; machine learning; recurrent neural networks; long short-term memory; market sentiment

1. Introduction

In the dynamic landscape of financial markets, US Treasury bonds hold a pivotal role as benchmarks for interest rates, influencing a wide range of economic activities and financial instruments. Predicting Treasury bond yields accurately is essential for investors, policymakers, and financial analysts, as these predictions inform critical decisions in both investment strategies and economic policymaking. US Treasury bonds, including Treasury bills, notes, and bonds serve as key instruments for the federal government to finance its operations. Their yields are sensitive to multiple factors such as GDP growth rates, inflation, employment data, and monetary policy actions by the Federal Reserve (Ang and Bekaert, 2006; Engle and Granger, 1987; Fama, 1984). These variables introduce significant volatility, make the prediction of yields a complex and nuanced problem. Traditional prediction methods often fall short in capturing the intricate temporal dependencies inherent in financial data, highlighting the need for more sophisticated approaches.

Recent advancements in machine learning offer promising solutions to these challenges. Specifically, Recurrent Neural Networks (RNN) and Long Short-Term Memory (LSTM) models have shown exceptional capabilities in handling sequential data and time series prediction (Bagastio et al., 2023; Graves et al., 2009; Hochreiter and Schmidhuber, 1997; Xu et al., 2024). These models are particularly adept at

capturing temporal dependencies, which are crucial in understanding the fluctuations in Treasury bond yields. RNNs process data in sequences, maintaining a hidden state that evolves over time, while LSTM networks enhance this by including memory cells that can preserve information over extended periods, addressing the limitations of standard RNNs. However, all previous efforts failed to improve the solution from those intrinsic business factors that influence the market. None of them explicitly addressed the correlation between the performance and the choice of economic indicators.

This paper explores the application of these advanced machine learning techniques in forecasting US Treasury bond yields. By integrating economic indicators and policy variables into the RNN and LSTM models, we aim to improve the accuracy and reliability of yield predictions. Our approach represents a significant step forward in the application of artificial intelligence (AI) in financial market analysis. This research contributes to the AI transformative trend by demonstrating the efficacy of RNN and LSTM models in predicting bond yields, thereby highlighting the benefits of integrating sophisticated AI models into financial forecasting. In the year 2022/2023, US Fed introduced a few larger than expected rate hike that drag the yield from 2% to more than 5%, this has not been seen since the 1970s. We are going to introduce our approaches to address this unexpected yield hike that puzzles traditional yield predictors.

In summary, this paper addresses the complex challenge of predicting US Treasury bond yields by leveraging the power of RNN and LSTM models. We incorporate key economic and policy variables into our analysis, demonstrating how these factors can significantly enhance predictive accuracy. Our findings underscore the transformative potential of AI in the financial sector, advocating for its broader adoption in legacy banking to drive efficiency and innovation.

2. Review of literature

US Treasury bonds, including Treasury bills, notes, and bonds, play a crucial role in financial markets as benchmarks for interest rates and indicators of economic sentiment. These instruments are pivotal in determining borrowing costs across various sectors of the economy, influencing investment decisions and monetary policy formulations (Campbell and Shiller, 1991; Fama, 1984). There are three types of US treasuries: bills, notes and bonds. Treasury bills, which also known as T-bill have the shortest maturity terms from four weeks to a year. Treasury notes (T-notes) mature between two and ten years. Bonds typically mature in 20–30 years from issuance.

The yields of Treasury bonds are influenced by a complex interplay of economic indicators, monetary policy and fiscal policy. Economic indicators are factors such as GDP growth rates, inflation, and unemployment figures affect investor expectations and market sentiment (Stock and Watson, 2001, 2003). Monetary policy is decisions by central banks, particularly the Federal Reserve in the US, regarding interest rates and quantitative easing policies, have direct impacts on bond yields (Bernanke and Kuttner, 2005; Hamilton, 1983). Fiscal policy are government spending and tax policies also influence bond yields by affecting the supply and demand dynamics in the bond market (Elmendorf and Mankiw, 1999; Ramey, 2011).

Recent advancements in machine learning techniques have revolutionized financial market predictions, offered new insights and enhanced predictive accuracy. Cao et al. (2024), Yu et al. (2024) and Zheng et al. (2024) discussed various techniques in applying AI techniques in detecting time-series correlations. Recurrent Neural Networks (RNNs) have been successfully applied to time series forecasting, including stock prices and exchange rates, by capturing sequential dependencies (Graves et al., 2009; Hochreiter and Schmidhuber, 1997). It also has many applications outside of the financial arena (Jin et al., 2024; Liu et al., 2024; Mo et al., 2024; Zhu et al., 2024).

Long Short-Term Memory (LSTM) Network introduced by Hochreiter and Schmidhuber (1997), is an extension of RNNs, improve upon traditional models by better capturing long-term dependencies and mitigating issues such as vanishing gradients (Cho et al., 2014; Gers et al., 2000). It is designed to overcome the limitations of traditional RNNs. LSTMs are particularly adept at learning long-term dependencies, making them suitable for tasks where context and sequence are important. Unlike standard RNNs, which struggle with the vanishing gradient problem, LSTMs can retain information over extended periods, thanks to their unique cell state and gating mechanisms.

In capital markets, RNN and LSTM has been showing effective in predicting stock market (Bagastio et al., 2023; Ding and Qin, 2019; Lopez de Prado, 2018; Tsai et al., 2017; Wang et al., 2024). While RNN and LSTM have shown promising results in stock market, there remains a gap in the literature regarding the comprehensive integration of economic and policy variables into predictive models for Treasury bond yields. None of those studies have explored the combined effects of macroeconomic indicators and policy decisions on yield predictions using advanced machine learning techniques.

All the previous attempt on stock and bond yield prediction focuses on improving the performance by tweaking the learning rate, batch size, number of layers and units, dropout rate, sequence length etc., none of them explored the problem from the business perspective. None of the researchers attempted to address the performance from its business nature and improve training from the economic factors that influence the bond market. We notice that the two LSTM-based bond yield prediction applications (Shu et al., 2019; Ying et al., 2019) conducted in years where interest rate does not have big volatility, and with zero expectation of interest rate hike/decrease soon. Without factoring in potential rate hike/drop expectation, the prediction will meet issues in years like 2022/2023 when unexpected strong CPI number will bring much larger than expected rate hikes. We will try to address this unexpected market trend in 2022 and 2023 with our approaches.

3. Methodology

3.1. Factors and assumptions

The dataset used in this study includes historical data on US Treasury bond yields and key economic indicators sourced from reliable financial databases such as the Federal Reserve Economic Data (FRED) and other authoritative sources. Unlike US short term bills, the focus of our study 5-year treasury notes is usually quoted on its yield (Chen, 2023), i.e., bid yield and ask yield. We use the average of bid yield and

ask yield to get mid yield as the prediction output. Daily or monthly observations of bond yields across various maturities, with a specific focus on the 5-year Treasury bond yield, are collected. Economic indicators considered include: (1) Consumer Price Index (CPI), (2) Producer Price Index (PPI), (3) Employment Situation Report, (4) Gross Domestic Product (GDP), (5) Retail Sales, and (6) Federal Reserve FOMC Meetings.

We are occasionally faced with missing data. Forward filling or interpolation are used to address missing values, as they are two most common ways to fill missing financial data (Brockwell, 2016; Jones, 1987). Forward filing assumes that the last observed value is a reasonable approximation of the missing value, which can be particularly useful in financial time series where values tend to be relatively stable over short periods. Interpolation provides smoother and more continuous estimates for missing values, reducing the likelihood of abrupt changes in the data. As a prerequisite step: all variables are normalized to a standardized scale to facilitate model training. We also use feature engineering in which lagged versions of economic indicators are created to capture historical trends and potential lead-lag relationships with bond yields, crucial for modeling temporal dependencies.

The assumption that we take during our study is that bond yields can only be influenced by economic indicators, monetary policy and fiscal policy. There are more factors like geopolitical tension, foreign investment policy, and global epidemic (Liu et al., 2022). For the consistency of our study, we would not take those into account. Model's performance will be evaluated using Mean Square Error (MSE), Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE). MSE measures the average squared difference between observed and predicted values, sensitive to large errors. RMSE is the square root of MSE. MAE measures average absolute errors, less sensitive to outliers compared to MSE, RMSE. MAPE provides a percentage error, making it useful for comparing across different datasets.

3.2. Recurrent Neural Network (RNN)

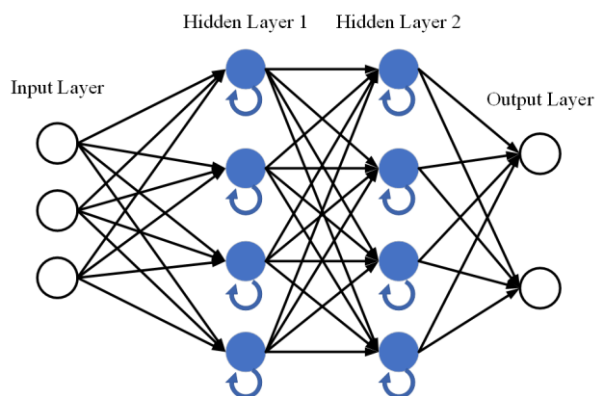


Figure 1. RNN network.

RNNs are designed to capture sequential dependencies in time-series data. The basic architecture of an RNN is shown in **Figure 1**. The network consists of a sequence of hidden layers, each of which has an output and a feedback connection to the

previous layer. The feedback connection allows the network to remember information from previous time steps, which is essential for processing sequential data. The output of the final hidden layers is used to make a prediction about the current step. The network is trained by adjusting the weights of the connections between the layers so that the system makes favorite predictions.

In the context of bond yield prediction, RNNs sequentially process each input while maintaining an internal state or memory. The model equation can be expressed as in Equation (1):

$$y_t = \beta_0 + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \dots + \epsilon_t \quad (1)$$

where symbols are defined as:

- y_t : predicted bond yield at time t
- β_0 is the intercept term, which represents the baseline yield when all predictor values are zero.
- $\beta_1, \beta_2, \beta_3, \dots$: Coefficients for the lagged economic indicators at time $x_{t-1}, x_{t-2}, \dots, x_{t-n}$. These coefficients measure the impact of past values of economic indicators on the current bond yield.
- x_{t-i} : Lagged values of economic indicators at the time $t - i$. For example, x_{t-1}, x_{t-2} can represent the lagged values of the economic indicators such as CPI and GDP.
- ϵ_t : Error term at time t , capturing the unexplained variation in bond yields.

Let's take an illustrative example of 5-year treasury bond yields using lagged values of CPI and GDP coefficient. Suppose the RNN model parameters are:

$\beta_0 = 4.5$ as the current expected yield as baseline yield

$\beta_1 = 0.4$: coefficient for CPI from the previous period ($x_{t-1} = 0.02$).

$\beta_2 = 0.3$: coefficient for GDP from the previous period ($x_{t-1} = 0.03$).

Using these values, the predicted bond yield y_t is:

$$y_t = 4.5 + (0.4 \times 0.02) + (0.3 \times 0.03) = 4.5 + 0.008 + 0.009 = 4.517$$

If the actual observed bond yield at time t is 4.53, then the error term ϵ_t is:

$$\epsilon_t = y_{actual} - y_{predicted} = 4.53 - 4.517 = 0.013$$

This error term $\epsilon_t = 0.013$ represents the portion of the bond yield not explained by our model. This could be due to market volatility, or factors not captured in the model.

Without making the problem too complicated, we fix the number of training examples per batch, the length of the input sequence and the number of features in each input vector.

3.3. LSTM

It is worth emphasizing that neural networks frequently encounter the challenges of vanishing gradients and exploding gradients during the training process. The Long Short-Term Memory Network (LSTM) is a recurrent neural network trained using backpropagation through time that addressed the vanishing gradient issue. Compared to RNN, LSTM networks have memory blocks connected through layers.

3.3.1. LSTM architecture

As shown in **Figure 2**, an LSTM unit receives three vectors (three lists of numbers) as input. Two vectors come from the LSTM itself and are generated by the LSTM at the previous instant (instant $t - 1$). These are the cell state (C) and the hidden state (H). The third vector comes from outside. This is the vector X (called input) submitted to the LSTM at instant t .

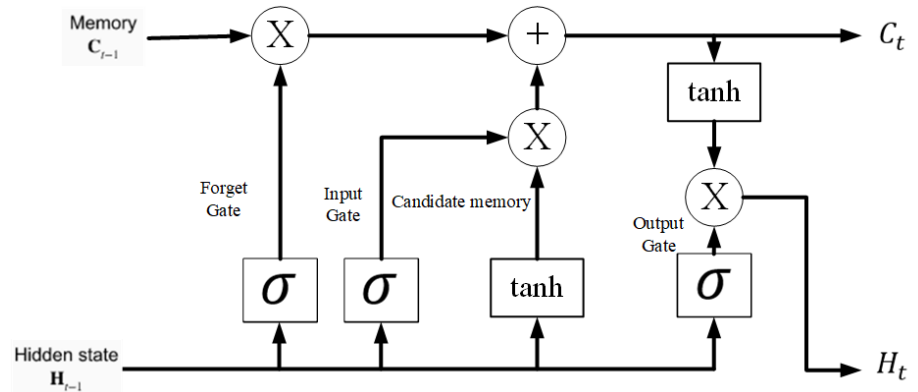


Figure 2. LSTM architecture.

Given the three input vectors (C , H , X), the LSTM regulates, through the gates, the internal flow of information and transforms the values of the cell state and hidden state vectors. Vectors that will be part of the LSTM input set in the next instant (instant $t + 1$). Information flow control is done so that the cell state acts as a long-term memory, while the hidden state acts as a short-term memory.

In practice, the LSTM unit uses recent past information (the short-term memory, H) and new information coming from the outside (the input vector, X) to update the long-term memory (cell state, C). Finally, it uses the long-term memory (the cell state, C) to update the short-term memory (the hidden state, H). The hidden state determined in instant t is also the output of the LSTM unit in instant t . It is what the LSTM provides to the outside for the performance of a specific task. In other words, it is the behavior on which the performance of the LSTM is assessed.

There are three types of gates within a unit. Each gate within a block unit uses the sigmoid activation units to control whether it is being triggered or not, making the change of state and addition of information flowing through the block conditional (Brownlee, 2022). The forget gate conditionally decides what information to throw away from the block. The input gate decides which values from the input to update the memory state. The output gate decides what to output based on input and the memory of the block.

3.3.2. Hyperparameter tuning

The LSTM approach we used here was similar to that of Van Houdt et al. (2020). But we added some Bayesian optimization in LSTM hyperparameter tuning as we found the straightforward approach defined in the study of Van Houdt et al. (2020) needs some calibration for parameters. We made the following improvements as shown in **Table 1**.

Most literature on using LSTM to explore financial instrument pricing (Weng and Wu, 2024a, 2024b) has the issue of finding suitable parameter values. BO is efficient in exploring the hyperparameter space, often requiring fewer evaluations than random or grid search. By modeling the objective function's uncertainty, BO can make informed decisions about where to sample next. The use of acquisition functions helps in focusing the search on areas that are likely to yield better performance.

Table 1. Bayesian optimization in LSTM hyperparameter tuning.

1. Preprocessing Data
<ul style="list-style-type: none">• Normalize or scale the input features.• Split the data into training and testing sets.
2. Define Hyperparameter Space. Specify the range and types of hyperparameters to optimize (e.g., number of units in LSTM layers, dropout rate, learning rate).
3. Initialize Bayesian Optimization. Create a surrogate model (e.g., Gaussian Process) to approximate the objective function (validation loss of the model), followed by choose an acquisition function to decide which hyperparameters to try next.
4. Iterate Over Hyperparameter Selection. Repeat the following steps for a specified number of iterations or until convergence:
<ul style="list-style-type: none">• Select Hyperparameters: Use the acquisition function to choose the next set of hyperparameters to evaluate.• Train the LSTM Model: Train the LSTM model using the selected hyperparameters. Evaluate the model on validation data to obtain the validation loss.• Update the Surrogate Model: Update the surrogate model with the new validation loss obtained from the current hyperparameters.
5. Select best Hyperparameters. Identify the set of hyperparameters that yielded the lowest validation loss during the optimization process.
6. Retrain final model. Train the final LSTM model using the best hyperparameters on the full training dataset. Evaluate the final model on the test set to estimate its performance.

3.3.3. Key LSTM input features: Economic indicator events

Several key economic indicators and announcements can significantly influence US Treasury bond yields. These indicators are closely watched by investors and analysts for insights into the health of the economy and potential shifts in monetary policy. We use the following major ones as key input features:

- Consumer Price Index (CPI): This data is typically released around the 10th to the 15th of each month for the previous month's data. CPI measures the average change over time in the prices paid by urban consumers for a basket of consumer goods and services. Both core CPI (excluding food and energy) and headline CPI (including all items) are important for understanding inflation trends.
- Producer Price Index (PPI): This data is usually released around the 10th to the 15th of each month for the previous month's data. PPI measures the average change over time in the selling prices received by domestic producers for their

output. It provides insights into inflationary pressures at the wholesale level. It is a leading economic indicator.

- **Employment Situation Report (including Nonfarm Payrolls):** Non-farm Payroll released on the first Friday of each month for the previous month's data. This report includes the unemployment rate, the number of new jobs created (nonfarm payrolls), and other labor market indicators.
- **Gross Domestic Product (GDP):** GDP data is released quarterly, with an advance estimate released about a month after the end of the quarter, followed by revised estimates in the subsequent months. GDP measures the total value of goods and services produced in the economy. It provides a broad overview of economic performance and can impact bond yields by influencing expectations for growth and inflation.
- **Federal Reserve FOMC Meetings (Policy Decisions and Statements):** FOMC meetings occur eight times a year, with scheduled announcements followed by press conferences by the Fed Chair. The Federal Open Market Committee (FOMC) sets monetary policy, including decisions on interest rates. Statements and forecasts made during these meetings can significantly affect bond yields as they signal the Fed's outlook on the economy and inflation. This event may not always bring a significant impact, as members of FOMC may imply meeting's sentiment ahead of the meeting or during interviews between the meeting. Meeting minutes will be released one month after the meeting.
- **Retail Sales:** This is usually released around the 12th to the 15th of each month for the previous month's data. Retail sales measure the total receipts at stores that sell merchandise and related services to final consumers. Strong retail sales indicates economic growth and potential inflationary pressures. As this data is usually released around the same time as CPI, it may not bring large impact unless revealing opposite trend as CPI.

Among these six economic drivers, FOMC meetings are the wild card that don't occur on a monthly basis and its impact varies with a proceeding/trailing a stronger than expected CPI report. We will show how LSTM handles better than RNN in this kind of scenario in the next session. In addition, we have deliberately trained LSTM to reject yield skyrocketing scenario where a better-than-expected CPI followed by a higher PPI in $t - 1$, as yield is likely to have priced in the higher CPI after a high PPI in previous period.

4. Discussion

4.1. Data and running statistics for the model

Before running our model, we first prepare training data by downloading daily open, high, low, close for US Treasury 5-Year yield from Yahoo Finance (Yahoo, 2024). All the four categories' data are the mid yield (average of bid and ask yield). We use open mid yield (8:00 am Eastern Standard Time daily) to compare with our prediction. The open yield reflects analyst's predicted sentiment of the market as most economic data release is announced at 8:30am. We also predicted the close yield data which was compared with the actual close mid yield at 3 pm (US Eastern Standard

Time). Data for US public holidays and weekends are not available. Outliers can significantly skew results, leading to inaccurate predictions and poor model performance. We use z-score to inspect the data from (Yahoo, 2024) and we do not find data with Z-score greater than 3 or less than -3 that needs to flag as outliers. On each day, two dataframes are supplied to the learning process, in which each dataframe consists of date stamp, open/close flag, yield values. Scheduled indicator events discussed in section 3.3.3 are also created as event dataframe and supplied to the model.

Now, we need to decide the input size which refers to the number of previous time steps used as input features to predict the future values. This is often referred to as the “look-back” period or “window size”. According to Hochreiter and Schmidhuber (1997), the input size should cover at least one full cycle of the seasonality. We are going to try different “look-back” sizes in our LSTM model empirical analysis. The prediction horizon is the number of future times steps the model aims to forecast. It depends on the forecasting goal and the data characteristics. We are going to use prediction size 1 and 30 in our analysis.

Bias in data can arise from various sources, including sampling bias, measurement errors, and temporal biases. We address the temporal bias by ensuring the training datasets include a balanced representation of different economic cycles, where 2022 data shows consensus economic growth, while 2023/2024 data indicates some signs of recessions. We adopt stratified sample by ensuring that each economic indicator’s different strata (e.g., low, medium, high CPI periods) are adequately represented.

Past machine learning practices suggest periodic Bayesian Optimization for hyperparameters. Since bond yield volatility occurs only after major economic events, we run Bayesian optimization after significant economic events (Federal Reserve announcement and when CPI release surprised consensus). For both RNN and LSTM models, the following hyperparameters are considered for optimization: The number of units per layer range from 32 to 256, while the number of layers ranges from 1 to 3. The sequence length has three options 30, 60, 250. Our experiment shows that 20-time steps is a good choice for our case. Learning rate which is the step size for updating the model weights during training ranges from 0.001 to 0.01. A smaller learning rate allows for more fine-grained updates, while a larger one speeds up training. We found that when using 250 days history to train, 0.01 would be a good learning rate to ensure all learning can be completed within 1 h and 21 min. If we adjust the learning rate to 0.001, all processes can be completed within 250 min. Batch size ranges from 32 to 128. We use dropout rate between 0.1 to 0.5, this represents the fraction of the input units to drop for regularization. For activation function for hidden layers, we only used tanh. The loss function employed is Mean Squared Error (MSE), which is standard for regression problems, as it measures the average squared difference between predicted and actual values. This configuration ensures that the models are well-equipped to learn from the data and make accurate yield predictions.

The RNN for our work is implemented in TensorFlow using the Python Kera module. Our LSTM mechanism and subsequent improvement are implemented using Python TensorFlow library. The running hardware is NVIDIA GeForce GTX 1080 and AMD Ryzen 7 CPU with 16GB of memory. For 5 epochs, time per epoch is approximately 10–25 min on a single GPU (including I/O and lead time to retrieve

input events through API), so the total training time varies from 50 to 125 min. Due to business requirement to finishing training run within 2 h, we can only set to max 5 epochs for our test.

4.2. Running RNN model

We first run the RNN model described in section 3.2. with input size 30 days, 60 days and 250 days for comparison and the results are shown in **Table 2**. The prediction horizon we use is 1.

Table 2. Comparison of prediction error with RNN with different number of input days.

DAYS	MSE	RMSE	MAE	MAPE
30	0.1243	0.3526	0.3076	7.47%
60	0.0839	0.2897	0.2478	6.00%
250	0.0518	0.2276	0.1946	4.74%

As noted in many other literatures, RNNs suffer from the problem of vanishing gradients. Even if we added the training days to 250, the performance did not improve from 60 days of training. We turned our hope to LSTM.

4.3. Result from LSTM model

The following are the running result from the LSTM methodology we employed in section 3.3. With three variations of input size (30-day, 60-day and 250-day training), we conduct daily open yield and close yield production from January 2023 to June 2024. We got promising results especially with 250-days input. The Mean Absolute Percentage (MAPE) is consistently around 2% for both daily open yield prediction and close yield prediction (see **Tables 3** and **4**). Our LSTM model usually produces better results in open yield prediction than close yield prediction, as economic announcement introduces uncertainty to the market.

Table 3. Comparison of prediction error on 5-year open yield using LTSM with different number of input days.

DAYS	MSE	RMSE	MAE	MAPE
30	0.0572	0.2392	0.2081	5.08
60	0.0146	0.1208	0.1031	2.51
250	0.0092	0.0959	0.082	2.00

Table 4. Comparison of prediction error on 5-year close yield using LTSM with different number of input days.

DAYS	MSE	RMSE	MAE	MAPE
30	0.0502	0.2241	0.1948	4.74
60	0.0191	0.1382	0.1205	2.95
250	0.0127	0.1127	0.0955	2.34

Despite promising results, we find that the errors on Close Yield are partially due to yield fluctuation in the period after economic data release get affected by market sentiment at period t . Market sentiment can be influenced by many external factors, notably speech from FOMC members, economic data in other countries. We will discuss our solutions in the next section.

4.4. Adjustment to LSTM to incorporate market sentiment price-in effect

Past literature like that of Bagastio et al. (2023) and Xu et al. (2024) did not discuss scenarios when market sentiment already priced-in a rate hike, how would LSTM cells handle other economic indicators in the input vector. We cannot ignore this market sentiment effect, as conventional LSTM implementation is likely to give prediction output of yield move in same the direction of CPI announcement pointing to, which is contrary with what market moved in many cases in 2023. For example, when stronger than expected non-farm payroll and PPI data is posted a few days before CPI data release, this will bring the market sentiment for rate hike, hence alleviating the treasury yield. The strong CPI result may not bring much fluctuation anymore. In some trading days, market sentiment plays a stronger role than economic indicators (Mo et al., 2024; Piñeiro-Chousa et al., 2021). We need to improve our learning mechanism as suggested in other works (Jin et al., 2024; Wang et al., 2024; Zhong et al., 2024). Market sentiment also affects bond yield spreads, it introduces complicity, and we don't take the yield spreads as input feature consideration here.

We introduce a feature that captures the market's consensus or expectations about interest rate changes. The consensus can be downloaded from CME FedsWatch Tool (CME, 2024) or other financial information sources. Another preparation step is, we encode a CPI sensitivity matrix by adjusting how CPI changes impact the model based on the context provided by the market sentiment feature. Adjust the input data to reflect the fact that a CPI increase won't lead to an additional rate hike if the market has already priced in a 25-basis point hike. We also implemented an attention mechanism that can help the LSTM model learn to focus on relevant factors, such as ignoring CPI changes when the market has already priced in a rate hike.

In the input gate, we adjust to minimize the flow of CPI information into the cell state under these conditions. Adjust the forget gate to remember more of the past cell state when the market sentiment indicates a priced-in hike. In the meantime, we adjust the output gate to modulate the contribution of the current cell state to the output based on market sentiment.

The following result are plot from the prediction after we incorporate market sentiment effect for 250-day input size on the next day open and close yield daily. Compared to result in **Tables 3** and **4**, the result improved a lot for close yield prediction with all other parameters holding the same. For open yield prediction, incorporating market sentiment has reduced the MSE from 0.0092 (**Table 3**) to 0.0071 (**Table 5**); while for the case of close yield prediction improves from 0.0127 (**Table 4**) to 0.0072 (**Table 6**).

Table 5. Prediction error on 5-year open yield with market sentiment.

DAYS	MSE	RMSE	MAE	MAPE
30	0.0482	0.2195	0.1879	4.60
60	0.0162	0.1273	0.1091	2.65
250	0.0071	0.0843	0.0718	1.75

Table 6. Prediction error on 5-year close yield with market sentiment.

DAYS	MSE	RMSE	MAE	MAPE
30	0.0491	0.2216	0.1887	4.56
60	0.0141	0.1187	0.1008	2.47
250	0.0072	0.0849	0.0713	1.73

4.5. Refinement from taking annual CPI and PPI into consideration

During LSTM learning 2022 and 2023 data, we found that as 2022 first half have dramatic CPI increases for consecutive months, a 0.4% month-to-month increase in February 2023 does not really bring much spike on 5-year yield as compared to any other 0.4% month-to-month increase, as February 2023 had a 0.9% month-to-month increase. A 0.4% actually brings down the annual CPI increase, so it may act as a cool-down or stabilizing input feature. Please refer to **Table 7** for more information obtained from Trading Economics (2023).

Table 7. Monthly and annual inflation rate in 2022 and 2023 obtained from trading economics.

Month (2023)	CPI	2022 Monthly Inflation Rate (%)	Annual Inflation Rate (%)	Month (2022)	CPI2	2022 Monthly Inflation Rate (%)2	2022 Annual Inflation Rate (%)
January	299.17	0.5	6.4	January	281.148	0.6	7.5
February	300.84	0.4	6	February	283.716	0.9	7.9
March	301.836	0.3	5.1	March	287.504	1.3	8.5
April	303.363	0.4	4.9	April	289.109	0.6	8.3
May	304.127	0.3	4	May	292.296	1.1	8.6
June	305.109	0.2	3	June	296.311	1.4	9.1
July	305.691	0.2	3.2	July	296.276	0	8.5
August	307.026	0.4	3.7	August	296.171	-0.03	8.3
September	307.789	0.2	3.7	September	296.808	0.2	8.2
October	307.671	0	3.2	October	298.012	0.4	7.7
November	307.051	-0.2	3.1	November	297.711	-0.1	7.1
December	306.746	-0.1	3.4	December	296.797	-0.3	6.5

Source: Trading Economics (2023).

We developed an input feature called CPI Anomaly Indicator which indicates whether the current month's CPI change is above or below the long-term average, adjusted for historical volatility. In the hyperparameter adjustments as described in section 3.3.2., we increased the look-back period to ensure the model captures longer-term trends, and the added some additional weight to address the correlation between

the monthly and annual CPI changes. In addition, we use L2 regularization to prevent overfitting. We have improved the output gate and forget gate by taking annual CPI and PPI into consideration. This has helped to further improve our prediction for certain economic announcement dates (as shown in **Tables 8** and **9**), but the effect on non-economic announcement date is rather limited.

Table 8. Prediction error on 5-year open yield with consideration of annual rate.

DAYS	MSE	RMSE	MAE	MAPE
30	0.0528	0.2298	0.1991	4.8511
60	0.0142	0.1192	0.1016	2.4775
250	0.007	0.0837	0.0706	1.7177

Table 9. Prediction error on 5-year close yield with consideration of annual rate.

DAYS	MSE	RMSE	MAE	MAPE
30	0.0547	0.2399	0.2037	4.9704
60	0.0149	0.1221	0.1042	2.5443
250	0.0069	0.0831	0.0699	1.7116

We have plotted our prediction on both open and close yield versus the actual ones in **Figures 3** and **4**.

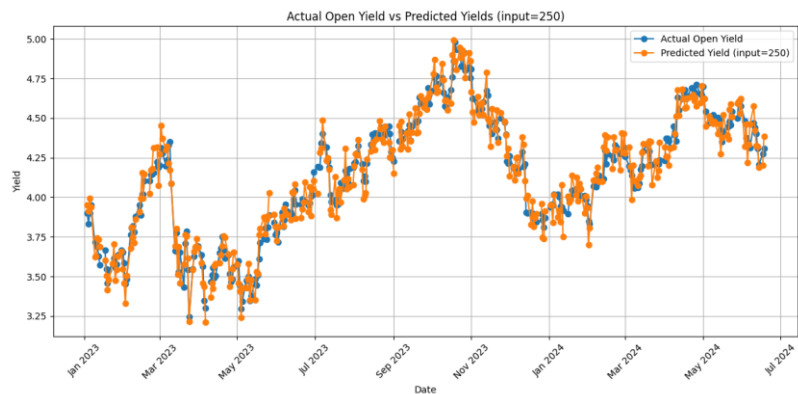


Figure 3. Actual open yield vs predicted yield for 5-year US treasury.

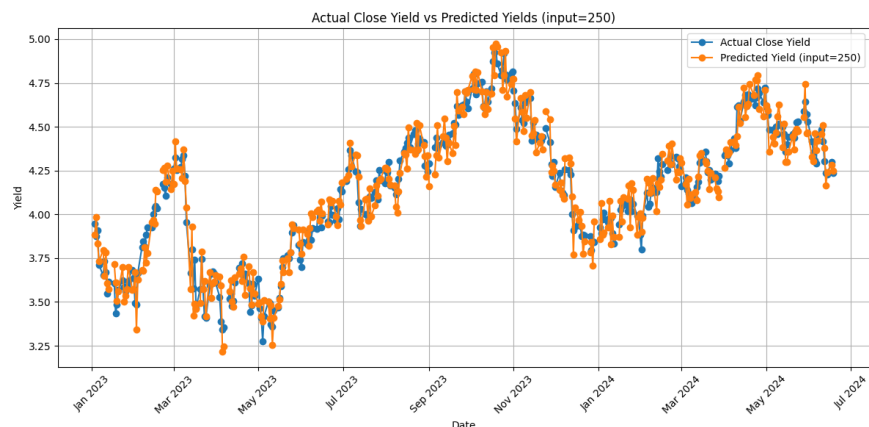


Figure 4. Actual close yield vs predicted yield for 5-year US treasury.

4.6. Comparison with existing method

In order to compare with a close study conducted by Shu et al. (2021) (denoted as SHU) that has available result for 10-Year US Treasury Yield, we are going to run our model which incorporates methodology in section 4.4 and 4.5 (CANOAK), as well as conventional RNN (RNN) and conventional LSTM (LSTM). We are going to test with the year 2019 (when SHU computed their data), when no rate hike/shrink occurred. We are also going to compare the performance of the year 2000 (when multiple rate decrease occurred) and 2022 (when multiple rate increases occurred), when bond yield volatility occurred. We have also used the learning rate 0.005 so that we can have the same available result. Hyperparameter adjustment is not applied there as we try to mimic the same environment as in the study of Shu et al. (2021). Please take note that 10-year treasury yield reflects longer term economic outlook, its value is less volatile to 5-year, and prediction accuracy is much higher. MSE is used for comparison as that’s the only error metrics available in the study of Shu et al. (2021). Since data was repeatedly trained 15 times by Shu et al. (2021), we also set out epoch number as 15.

The loss (MSE) for our approach is much better in the year 2020 and 2022, when there is an unexpected rate drop and increase respectively (**Table 10**). The training on market sentiment and a weighted average input factor processing for annual and monthly figure are the main source of these improvements. In 2019, there is no significant market sentiment on rates policy, and annual/monthly CPI expectation does not deviate much from each other, therefore our model performs just in par with SHU.

Table 10. Comparison between SHU vs RNN vs CANOAK.

	MSE (Learning Rate 0.01)			MSE (Learning Rate 0.001)		
	2019	2020	2022	2019	2020	2022
SHU	0.0104	0.0298	0.0445	0.0126	0.0298	0.0445
RNN	0.0245	0.0369	0.0537	0.311	0.0516	0.0393
CANOAK	0.0109	0.0118	0.0123	0.0124	0.0193	0.0083

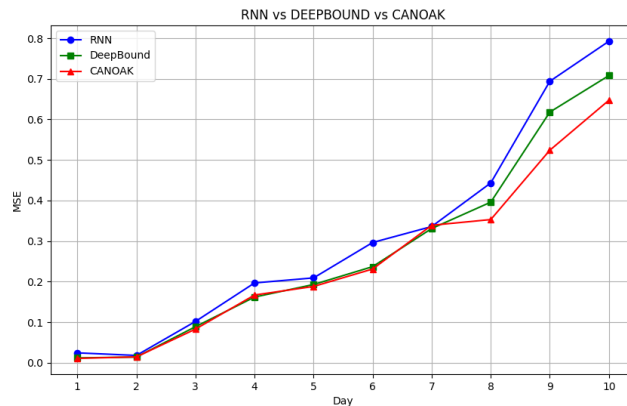


Figure 5. Comparison between RNN vs DEEPBOUND vs CANOAK.

As displayed in **Figure 5**, we compare with the existing DeepBond algorithm introduced by Ying et al. (2019) for predicting yield for the coming 10 trading days in 2019–2023. We take the average MSE and MAE as the original literature also used

these two measures. We use input size equal 60 to match the literature. Our performance excels DeepBond from 4th day to 10th day in terms of MSE for the yield prediction of year 2019–2023.

4.7. ANOVA test analysis

We have conducted an ANOVA test in **Table 11**. The F -statistic for all years is very low. This suggests that the variance between the group means (predicted yields for different input values) is much smaller compared to the variance within the groups.

Table 11. ANOVA test for input size 30, 60 and 250.

Year	F -statistic	P -value
2019	0.1042	0.901
2020	0.3845	0.681
2021	0.0997	0.9052
2022	0.1836	0.8323
2023	0.0006	0.9994

The F -statistic close to 0 indicates that the differences in means between groups are negligible. The p -values for all years are much higher than 0.05. This indicates that there is no statistically significant difference between the group means for any of these years. High p -values (much greater than 0.05) suggest that the observed differences in predicted yields across different input values could easily have occurred by chance.

For each year from 2019 to 2023, the ANOVA results suggest that the different input values (30, 60, 250) for predicting yields do not produce significantly different results. This means that the choice of input value does not significantly affect the predicted yields for these years. The lack of significant differences implies that any of the input values could be used for predictions without worrying about large deviations in results.

The ANOVA test results indicate that the predicted yields for the input values of 30, 60, and 250 days are not significantly different for the years 2019 to 2023. Therefore, you can use any of these input values for yield predictions in these years without expecting substantial variations in the results.

4.8. Direction for future improvement

We noticed that the result of the prediction may result in a larger gap with the actual yield when there is a new issuance of 5-year treasury note. As the schedule of 5-year treasury note new-issue and re-issue always coincide with the issuance of 2-year bill and 7-year notes, fluctuation in yield for 2-year and 7-year placed a greater role than 3-year, 10-year, 30-year. Moreover, as US treasuries are issued in an auction manner (Sigaux, 2024), issuing quantity and US debt ceiling may also affect the new-issue yield, we did not take those factors into account when constructing the hyper-parameter tuning, therefore, more variation arise.

We have also tried to apply our LTSM techniques to corporate bond yield prediction. However, due to liquidity of the corporate bond market, the spread between bid yield and ask yield is not as consistent like treasury bond. Moreover, corporate

bond depends on more factors compared to those 6 major economic events that we listed in section 3.3.3. Stock price and corporate debt level have added more complexities to input of an LSTM model. We would leave this as a possible future research direction.

There are various kinds of market sentiments used by investors and arbitragers, The CME FedWatch we used in section 4.4 is only one of many interest-rates market consensus. Theoretically, most technical indicators can be used to measure market sentiment for liquid financial instruments like stocks and US treasuries. These market sentiments indicators need to be incorporated in LSTM based learning in a similar fashion as we discussed in section 4. 4, even a contrary indicator (Simon and Wiggins, 2001) can be added with adequate weight in the hyperparameter function. The tuning of weight for each of these market sentiment indicators can be discussed and compared in future research. For less liquid market instruments like corporate bonds, swaps, another school of indicators that measure market liquidity would be more suitable as discussed by Baker and Stein (2004). Unlike regular investors that benefit from market movements, liquidity plays an important role for market makers. The mechanism for hyperparameter tuning and market sentiment incorporations needs to be further investigated in two separate scenarios, e.g., when the liquidity increases vs decreases.

Treasury yield may also be affected by other disruptive events like major fraud in the market like the Archegos Capital's failure (Bouveret and Haferkorn, 2023) or sudden sale of US treasuries due to US Sovereign credit rating changes. For example, on 3 August 2023, Fitch Ratings downgraded US Sovereign credit rating, followed by another downgrade from Moody. Future research can try to incorporate this factor into the RNN or LTSM model. You can find various credit rating changes for US treasuries in the past ten years on the website of the three major credit rating agencies. Graph Neural Network (GNN) would be a good future approach (Peng et al., 2024; Wang et al., 2024).

Bayesian Optimization is a powerful method for hyperparameter tuning, providing a balance between exploration and exploitation and efficiently handling the expensive evaluations often required in deep learning model training. Future authors can explore to use traditional operations research techniques to improve this part like Elhedhli et al. (2017).

5. Conclusion

In this paper, we explored the application of advanced machine learning techniques, specifically Recurrent Neural Networks (RNNs) and Long Short-Term Memory (LSTM) networks, in predicting US Treasury bond yields. This investigation highlights the use of economic indicators, market sentiment and flexible use between annual and monthly economic data can improve US treasury yield prediction.

Our analysis incorporated key economic indicators such as the Consumer Price Index (CPI), Producer Price Index (PPI), Employment Situation Reports, Gross Domestic Product (GDP), Retail Sales, and Federal Reserve FOMC Meeting decisions. We observed that the nuanced relationships between these variables and bond yields can be effectively captured using LSTM networks due to their ability to maintain and utilize historical information over extended periods, thereby overcoming the

limitations of traditional RNNs which often struggle with long-term dependencies. Mean Average Error for 30 days learning period is below 5%, 60 days and 250 days learning period are consistently below 3%.

The LSTM based model (CANOAK) proved superior in accounting for the complex, non-linear relationships inherent in the economic data, particularly in scenarios where short-term fluctuations in indicators like CPI do not directly translate into proportional changes in bond yields. For instance, in the early part of 2022, the US economy experienced significant CPI increases, leading to higher expectations for bond yields. Yet, as demonstrated by the LSTM model, smaller increases in CPI in early 2023 did not lead to similar spikes in bond yields due to the stabilizing effect of previous higher increases. This insight underscores the importance of considering the broader temporal context in economic forecasting. We have addressed this observation by further controlling the Mean Average Error within 2% for 250 days of learning period.

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