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Article

An econometric model based on moments of high orders of a time series for detecting the crisis in stock markets of USA, Germany and Hong Kong

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Abstract: Many financial crises have occurred in recent decades, such as the International Debt Crisis of 1982, the East Asian Economic Crisis of 1997–2001, the Russian economic crisis of 1992–1997, the Latin American debt Crisis of 1994–2002, the Global Economic Recession of 2007–2009, which had a strong impact on international relations. The aim of this article is to create an econometric model of the indicator for identifying crisis situations arising in stock markets. The approach under consideration includes data for preprocessing and assessing the stability of the trend of time series using higher-order moments. The results obtained are compared with specific practical situations. To test the proposed indicator, real data of the stock indices of the USA, Germany and Hong Kong in the period World Financial Crisis are used. The scientific novelty of the results of the article consists in the analysis of the initial and given initial moments of high order, as well as the central and reduced central moments of high order. The econometric model of the indicator for identifying crisis situations arising considered in the work, based on high-order moments plays a pivotal role in crisis detection in stock markets, influencing financial innovations in managing the national economy. The findings contribute to the resilience and adaptability of the financial system, ultimately shaping the trajectory of the national economy. By facilitating timely crisis detection, the model supports efforts to maintain economic stability, thereby fostering sustainable growth and resilience in the face of financial disruptions. The model's insights can shape the national innovation ecosystem by guiding the development and adoption of monetary and financial innovations that are aligned with the economy's specific needs and challenges.

Keywords: time series; statistical moment; point valuation; assets

1. Introduction

Existing probability distributions have some limitations when modeling lifetime data. First, these distributions cannot model non-monotonic hazard functions. In particular, the exponential distribution can only model a constant hazard rate function, while the gamma distribution can only model a monotonically increasing failure rate function. Second, existing probability functions, even when modeling the data, do not fit real data well. In practice, many datasets follow a shape other than a constant or monotonically increasing failure function. For example, the accident rate lifetime of electronic equipment follows a pattern that is a non-monotonic hazard rate function.

The paper introduces a novel spatial model based on generalized extreme value distributions (GEV) and tree ensemble models to estimate the maximum concentration of particulate matter smaller. Extreme value theory has the modern modeling algorithms for estimating the risk of energy crisis, and at its core are two different sampling techniques, namely block maximum and peak exceeding threshold. However, the uncertainties in estimates obtained by these sampling techniques are too large to be widely used. One of the main reasons for this problem is the wrong selection of extreme values and the lack of a suitable and efficient sampling mechanism. This study proposes a hybrid modeling framework of machine learning and extreme value theory to risk of a crisis in the market of stock indexes by identifying extreme values through efficient sampling techniques. More specifically, machine learning methods replace traditional sampling techniques with anomaly detection techniques, since anomalies are data points that do not match the rest of the data, much like defining extreme values. In this study, six representative machine learning-based unsupervised anomaly detection algorithms were tested. These include minimum covariance determinant, one-class support vector machines, k-nearest neighbors, local outlier factors, and connectivity-based outlier factors. The extreme values identified by these algorithms were then fitted to the distribution of extreme values for univariate and bivariate frameworks. The algorithms were tested on extensive data collected. The results show that the proposed hybrid model consistently outperforms conventional extreme models using block maxima and threshold peaks as underlying sampling techniques. The proposed hybrid modeling framework represents a methodological advance in market crisis identification and opens new avenues for exploring the possibility of applying machine learning techniques to existing a crisis in the market of stock indexes.

A literature review is presented in Section 2. Mathematical background, definitions and algorithm are described in Section 3. Section 4 contains the computational results for solving real problems based on the indicator of detection of a crisis situation in the stock indexes. Some concluding remarks are made in Section 5. In what follows, we use the terminology on series and finance from the book (Werner and Sotskov, 2006).

2. Related literature

In particular, many countries have been hit by multiple economic crises. Many financial crises have occurred over the recent decades, such as The International Debt Crisis 1982, The East Asian Economic Crisis 1997–2001, the Russian Economic Crisis 1992–1997, The Latin American Debt Crisis 1994–2002, Global Economic Recession 2007–2009 to name a few. Financial crisis can be recognized as the outcome of the spread of financial disturbances through market linkages (Bouri, 2023; Cui and Maghyereh, 2023; Mandal and Thakur, 2023; Zhang et al., 2023).

A crisis can have some positive effects and many negative effects on entrepreneurship and enterprises. Negative impacts are led to a decline in business volume and profitability, affect the characteristics of entrepreneurs and make it possible for some enterprises to survive (Doern, 2016). On the other hand, a crisis can provide an impetus for the development of opportunities and resources create opportunities for entrepreneurship based on resource holes and promote the development of innovative products and services (Linnenluecke and McKnight, 2017). Since it is difficult to predict a crisis, it is required to find some indicators of a crisis at an early stage. We will develop a crisis indicator based on probability theory and mathematical statistics (An and Mikhaylov, 2020; Mikhaylov, 2021, 2022; Mikhaylov et al., 2023; Moiseev et al., 2023; Mutalimov et al., 2021; Saqib et al., 2021; Yumashev and Mikhaylov, 2020).

The purpose of our study is to analyze time series based on higher-order moments and develop a crisis indicator that allows detecting a crisis. The article is devoted to the construction of a statistical model for predicting crises associated with critical periods of economics. The relevance of the research is associated with the crises accompanying the economic development of modern society. The economic crisis is defined by a noticeable decline in the production and stock indexes and is accompanied by a significant decline in business activity. The rest of this paper is organized as follows (An et al., 2020, 2024).

The main contribution to the crisis models is presented in the study of Krugman (1979) and Siregar et al. (2004). The indicators used for predicting banking crises include credit-related indicators, macroeconomic indicators, asset or property-related indicators, house price, property prices, and market-related indicators (interest rates).

With regards to stock market turbulences, variables from stock markets, bond markets, exchange rates, and additional variables such as VIX index, oil prices, LIBOR rate, gold price (Chatzis et al., 2018; Kratz, 2019; Schulze, 2004).

Common models used to predict financial crises in the past were based on logic models and signal extraction methods. Machine learning models belonging to the Bayesian networks such as hidden Markov model and switching linear dynamic system have outperformed the traditional logic model and signal extraction method (Dabrowski et al., 2016). These models have a drawback in their complex implementation; however, they effectively illustrate early warning systems (Batten et al., 2019; Osiyevskyy and Dewald, 2018).

3. Materials and methods

We next propose an econometric model of the crisis situation on the basis of moments of high orders. A moment is a point estimate of the central trend of some characteristic of a random variable (Stepanova et al., 2024; Mikhaylov, 2023;).

3.1. Moments of higher orders

Consider the two main types of moments of random variable, namely: initial and central. The initial moment of the i-th order ν_i is the expected values of the i-th degree of the random variable *X*:

$$\nu_i = M\left[X^i\right] \tag{1}$$

The central moment of the i-th order is the expected value of the i-th degree of deviation of the random variable X from the initial moment of the first order ν_1 according to the following equality

$$\mu_i = M\left[(X - \nu_1)^i\right] \tag{2}$$

If the moments of the k-th order are determined, then all the moments of the lower orders k' are determined as well, where $1 \leq k' < k$. Note that the moments are calculated only for the finite interval estimates of the random variable.

From the linearity property of mathematical expectation, it follows that the central moments can be expressed through the initial moments according to the formula:

$$\mu_k = \sum_{s=0}^{k} (-1)^s C_k^s \nu_{k-s} \nu_1^s \tag{3}$$

For examples, we obtain:

$$\mu_0 = 1, \mu_1 = 0, \mu_2 = \nu_2 - \nu_1^2, \mu_3 = \nu_3 - 3\nu_2\nu_1 + +2\nu_1^3, \mu_4 = \nu_4 - 4\nu_3\nu_1 + 6\nu_2\nu_1^2 - 3\nu_1^4$$

Next, we will use the absolute initial and central moments:

$$\nu_{abs\,i} = M\left[|X|^i\right], \mu_{abs\,i} = M\left[(|X - \nu_1|)^i\right] \tag{4}$$

along with the following initial moment and central point:

$$\overline{\overline{\nu_i}} = \sqrt[i]{M[X^i]} \tag{5}$$

$$\overline{\overline{\mu_i}} = \sqrt[i]{M[(X - \nu_1)^i]} \tag{6}$$

The absolute adduced moments are determined as follows:

$$\overline{\overline{\nu_{abs\,i}}} = \sqrt[i]{M(|X|^i)}, \overline{\overline{\mu_{abs\,i}}} = \sqrt[i]{M[(|X - \nu_1|)^i]} \tag{7}$$

Note that the above adduced moment corresponds to the exponentiation to the *i*-inverse (negative) power (in other words, the calculation of the root of the degree *i*). The adduced moment is a characteristic of the same dimension as the random variable itself. So, in some cases, it is more convenient to use the adduced moment for a data analysis.

With the help of these moments, it is possible to describe the average tendency of the series, scattering and other features of the variation of a certain feature.

3.2. Mathematical background

The paper uses point estimates for mathematical expectation, variance and moments of a higher order. We next prove the following theorem.

$$M[X] = \sum_{i=1}^{\infty} p_i x_i \tag{8}$$

Theorem. *Let statistic* $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ *be regarded as an estimation of the mathematical expectation μ of the general population X. Then the estimation* $\bar{X}$ *is unbiased, consistent, and effective in the class of all linear estimations.*

Proof of Theorem. We show the intransigence of the estimation $\bar{X}$. The following equations follow from the properties of the mathematical expectation:

$$M(\bar{X}) = M\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n}\sum_{i=1}^{n} M(X_i) = \frac{1}{n} n\mu = \mu \tag{9}$$

Thus, it follows from the definition of an unbiased estimation that the assessment of $\bar{X}$ is unbiased.

We next prove the consistency of the estimation $\bar{X}$. Since the sample $x_1, x_2, \dots, x_n$ is a sequence of independent equally distributed quantities $X_1, X_2, \dots, X_n$ with finite variance (in the case of financial time series, the variance can only be a finite value), according to the strengthened law of large numbers for any $\varepsilon > 0$, it follows that:

$$P(|\bar{X} - \mu| \le \varepsilon) \xrightarrow{P} 1 \ if \ n \to \infty \tag{10}$$

In other words, the considered estimation $\bar{X}$ converges in probability to the estimated parameter. Therefore, the assessment of $\bar{X}$ is valid.

We prove the effectiveness of the evaluation of $\bar{X}$. The variance of the effective evaluation of the parameter is the minimum among the variances of all linear estimates. From the definition of linear valuation follow equality:

$$D\left(\tilde{\theta}\left(\overrightarrow{X_n}\right)\right) = D\left(\sum_{i=1}^{n} \alpha_i X_i\right) = \sum_{i=1}^{n} D(\alpha_i X_i) = \sum_{i=1}^{n} \alpha_i^2 D(X_i) = \sigma^2 \sum_{i=1}^{n} \alpha_i^2) \tag{11}$$

We determine at what values α_i the Equation (11) reaches a minimum. To this end, we calculate the conditional minimum of the function $\sum_{i=1}^{n} \alpha_i^2$ with the constraint $\sum_{i=1}^{n} \alpha_i = 1$.

$$L(\alpha_1, \alpha_2, \ldots, \alpha_n, \lambda) = \sum_{i=1}^{n} \alpha_i^2 + \lambda\left(\sum_{i=1}^{n} \alpha_i - 1\right) \tag{12}$$

From the following necessary conditions of the extremum function:

$$\begin{cases} \dfrac{\partial L}{\partial \alpha_i} = 2\alpha_i + \lambda = 0, \;\; i = \overline{1, n} \\ \dfrac{\partial L}{\partial \lambda} = \sum_{i=1}^{n} \alpha_i - 1 = 0 \end{cases} \tag{13}$$

we obtain $\lambda = -\frac{2}{n}, \alpha_i = \frac{1}{n}, i = \overline{1, n}.$

Therefore, the minimum is reached at $\alpha_i = \frac{1}{n}$

$$D\left(\tilde{\theta}\left(\overrightarrow{X_n}\right)\right) = \sigma^2 \sum_{i=1}^{n} \alpha_i^2 = \sigma^2 \sum_{i=1}^{n} \frac{1}{n^2} = \frac{\sigma^2}{n} = D(\bar{X}) \tag{14}$$

Thus, the estimation of the mathematical expectation $\bar{X}$ is unbiased, consistent and effective. Theorem is proven. □

Statistics can be used as an estimation of the variance

$$\hat{\sigma}^2\left(\vec{X}_n\right) = \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2 \tag{15}$$

The Equation (15) is valid but biased one. Negative bias does not play a significant role in large samples, however, when *n* approaches 1000, it is better to move on to an unbiased estimate of variance as follows:

$$s^2\left(\overrightarrow{X_n}\right) = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2 \tag{16}$$

As an assessment of the initial moments are selective initial moments:

$$\hat{v}_k = \frac{1}{n} \sum_{i=1}^{n} X_i^k \tag{17}$$

The Equation (17) is unbiased, which is proved similarly as for the evaluation of $\bar{X}$ in Theorem. Shifted assessments of central moments are selective central moments:

$$\hat{\mu}_k = \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^k) \tag{18}$$

With the help of some additions, one can get consistent and unbiased estimates for the central points of the sample:

$$M_2 = \frac{n}{n-1} \hat{\mu}_2 \tag{19}$$

$$M_3 = \frac{n^2}{(n-1)(n-2)}\hat{\mu}_3 \quad (20)$$

$$M_4 = \frac{n(n^2-2n+3)\hat{\mu}_4 - 3n(2n-3){\hat{\mu}_2}^2}{(n-1)(n-2)(n-3)} \quad (21)$$

A standardized estimation (or z-estimation) is a dimensionless statistical indicator that is a measure of the relative spread of an observed quantity. This estimate shows how many standard deviations can fit into the spread of the value relative to its average value. As a consequence, a standardized random variable is a random variable whose expected value is equal to 0 and whose standard deviation (i.e., the second-order central adduced moment) is equal to 1. Being a dimensionless quantity, a standardized estimate becomes convenient for comparing values of different dimensions, for example, for different currencies. If a random variable x has a mathematical expectation of μ and a standard deviation of σ, then its standardized z-estimation will be determined as follows:

$$z = \frac{x-\mu}{\sigma} \quad (22)$$

On the basis of z-estimation, the concept of a standardized moment of the i-th order is introduced as follows:

$$\beta_i = M\left[\left(\frac{X-\nu_1}{\sqrt{\mu_2}}\right)^i\right] \quad (23)$$

A standardization of moments is needed not only to avoid difficulties with scale and dimensionality. This procedure also provides additional information about the distribution of a random variable. After the standardization of a random variable, all its values are conditionally divided into two groups: those lying within one standard deviation of the σ and those that do not lie within these limits.

If $x - \mu \geq \sigma$, then $z = \frac{x-\mu}{\sigma} \geq 1$. Therefore, some of the z values will be greater than 1 (and so not located within the σ), and the other part will be less than 1. Then, when elevated to the i-th degree, these two groups of values will behave differently. Values within one σ will decrease and tend to 0, while the second group of values will increase significantly. Thus, the difference between the values of the two groups will become more and more significant. Later, the practical application of this property will be shown. The initial and central moments of even orders are absolute.

As the order of the moment increases, its sensitivity to emissions and deviations increases. The higher the order of moment, the more accurately it captures deviations from the central trend of the series and the less it reflects the central trend itself. In a symmetric distribution, any central moment of odd order is zero. To proceed to a dimensionless quantity and obtain an absolute characteristic, an asymmetry coefficient γ_1 is introduced, which is a standardized moment of the third order presented:

$$\gamma_1 = \frac{\mu_3}{\sigma^3} \quad (24)$$

The selective asymmetry coefficient would be as follows:

$$\widehat{\gamma_1} = \frac{M_3}{M_2^{\frac{3}{2}}} = \frac{\sqrt{n(n-1)}}{n-2}\left(\frac{\hat{\mu}_3}{{\hat{\mu}_2}^{\frac{3}{2}}}\right) \quad (25)$$

We present the following three properties of the asymmetry coefficient:

- If the distribution is symmetrical, then $\gamma_1 = 0$;
- The inequality $\gamma_1 > 0$ holds, if the cubes of positive deviations outweigh the cubes of negative deviations. In this case, the distribution has a left asymmetry, that is, the long part of the distribution curve is located to the right of the average value;
- The opposite inequality $\gamma_1 < 0$ holds, if the distribution has a right asymmetry;
- The central moment of the fourth order μ_4 (excess) characterizes the sharpness of the peak of the random variable distribution, as well as the frequency of occurrence of values that are distant from the mean value.

By analogy with asymmetry, the transition to a dimensionless value is made by switching to a standardized moment of the 4th order (9) and introducing an excess coefficient γ_2 determined as follows:

$$\gamma_2 = \frac{\mu_4}{\sigma^4} - 3 \quad (26)$$

Sampling rate (coefficient) of excess based on (8) and (9) is determined as follows:

$$\widehat{\gamma_2} = \frac{M_4}{M_2^2} - 3 = \frac{n^2 - 1}{(n-2)(n-3)}\left(\frac{\hat{\mu}_4}{\hat{\mu}_2{}^2} - 3 + \frac{6}{n+1}\right) \quad (27)$$

We present the following three properties of the excess coefficient:

- If the distribution is normal, then $\gamma_2 = 0$;
- The inequality $\gamma_2 > 0$ holds, if the peak of the distribution in the vicinity of the average is sharper and sharper, and the distribution curve has heavy tails;
- The opposite inequality $\gamma_2 < 0$ holds, if the peak of the distribution is smoother, the distribution curve has less heavy tails. The total moment (this is the moment of the entire time series) has a numerical value and is calculated for the whole series. The estrange (this is the moment corresponding to a certain time interval) has a numerical value and is calculated for the series X^*, which is a subset of the original series X.

Urgent moments serve to observe the dynamics in the time series. In the case of a large observation period, the general moment does not provide information about the behavior of the series, since deviations and outliers are averaged or completely mutually destroyed, and the total moment is simply a number. Therefore, they move on to a sequence of urgent moments, which can be represented graphically. From changes in the magnitude of the estrange moment in this sequence, for example, for a month, it is possible to draw conclusions about the behavior of a series and changes in the characteristics of its distribution.

4. Computational results

We use the following data for analysis: stock market indices such as S&P500, DAX and Hang Seng as well as daily closing prices of ICE-Brent and USD/RUB currency at an interval of 3 years. S&P500 tracks the performance of 500 large companies listed on stock exchanges in the United States, DAX is a stock market index consisting of the 40 major German blue chip companies trading on the Frankfurt Stock Exchange and Hang Seng is the main indicator of the overall market performance in Hong Kong. Oil price is used as it is important indicator of an upcoming crisis.

USD/RUB currency is used as an example of indicator of the economic situation within a given country.

We calculate return as a difference between daily closing prices

$$R_{i,t} = P_{i,t} - P_{i,t-1}$$

At this stage of the study, the time series is analyzed by the classical raw moments of higher order. We calculate 10 fixed raw moments and 10 reduced fixed raw moments. The observation period for a fixed moment is 30 days. Some results are shown in **Figures 1** and **2**.

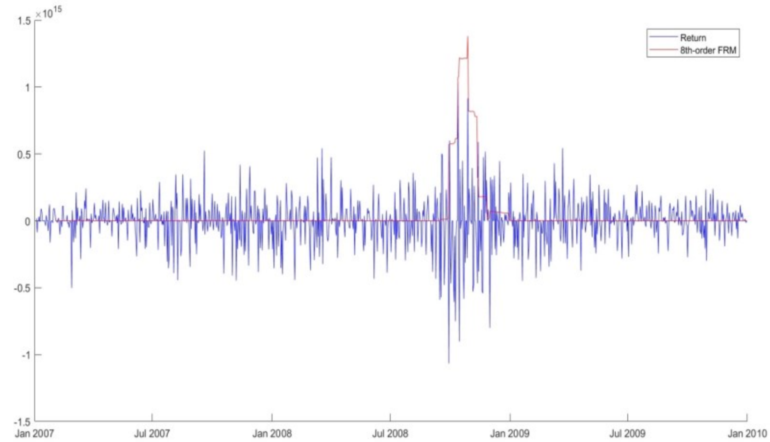

Figure 1. Return (blue) and the fixed raw moment of the 8th order (red) for S&P500 index during crisis of 2008.

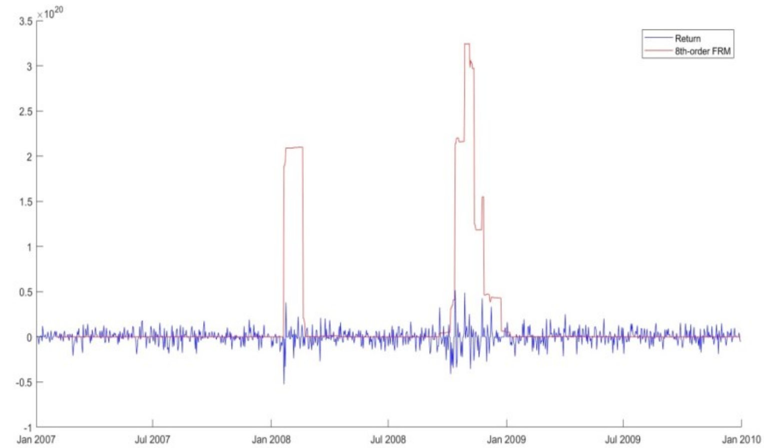

Figure 2. Return (blue) and the fixed raw moment of the 8th order (red) for DAX index during crisis of 2008.

The **Figures 1** and **2** clearly show the outliers in October 2008. However, the use of FRM is inconvenient for non-homoscedastic series, since they have a large dimension. For this reason, we will use reduced raw moments of higher order.

Now it is clear that reduced raw moments represent the outliers in time series and the higher is order of the moment the more accurately it captures deviations. In **Figures 3–7** the results for other assets are represented.

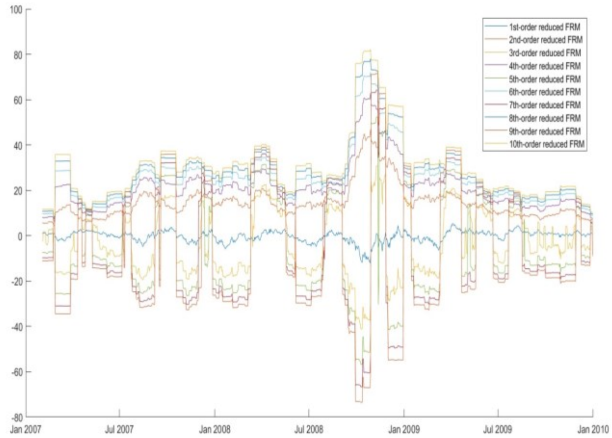

Figure 3. Reduced raw moments of 1–10 orders for S&P500 index.

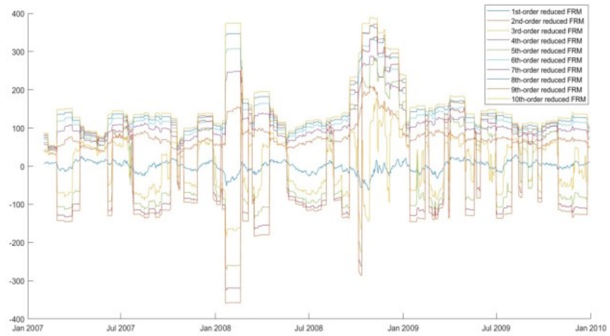

Figure 4. Reduced raw moments of 1–10 orders for DAX index.

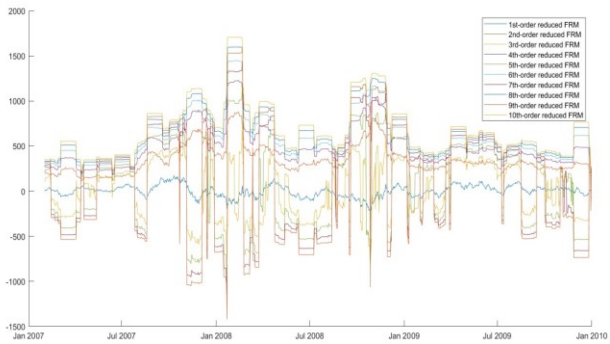

Figure 5. Reduced raw moments of 1–10 orders for Hang Seng index.

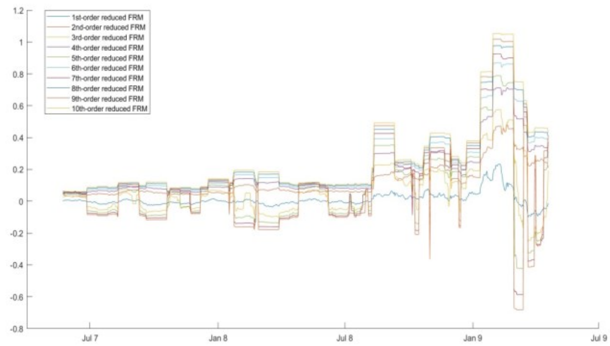

Figure 6. Reduced raw moments of 1–10 orders for USD/RUB currency.

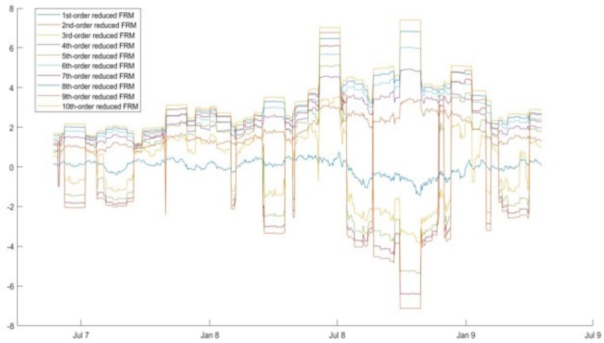

Figure 7. Reduced raw moments of 1–10 orders for ICE-Brent.

The Hang Seng stock market index does not display negative skewness, which is not a common finding for stock market. We also detect positive mean and huge standard deviation for this index which means that the values are spread widely and mostly have continuous growth. Oil returns display higher volatility than stock indices. Positive skewness and mean of USD/RUB currency represents the depreciation of the ruble (Bonato et al., 2020; De Clerk and Savelev, 2021; Finta and Aboura, 2020; Jiang et al., 2018; Jiang et al., 2019; Jun et al., 2017; Kinateder and Papavassiliou, 2019; Majeed and Jamshed, 2023; Mei et al., 2017; Pan et al., 2020; Rapposelli et al., 2023; Singh et al., 2023; Tang and Chen, 2014; Teng and Shang, 2018; Xu et al., 2018; Xu and Shang, 2018).

5. Case of German markets

Figures 8–13 show that the German stock market has high efficiency using the AQT tool: Efficiency falls to 5% significance level only 1 time for the 50 time period; efficiency falls to 5% significance level only 1 time for the 100 time period; efficiency falls to 5% significance level only 1 time for the 150 time period. The GST tool finds that efficiency falls to 5% significance level 6 times for the 50 time period; efficiency falls to 5% significance level only 2 times for the 100 time period; efficiency falls to 5% significance level only 1 time for the 150 time period. These results prove the adaptive nature of stock indices in Germany on the base of AMH, because after widening the window length, the GST test shows that efficiency increases, whilst the AQT results are consistently high too.

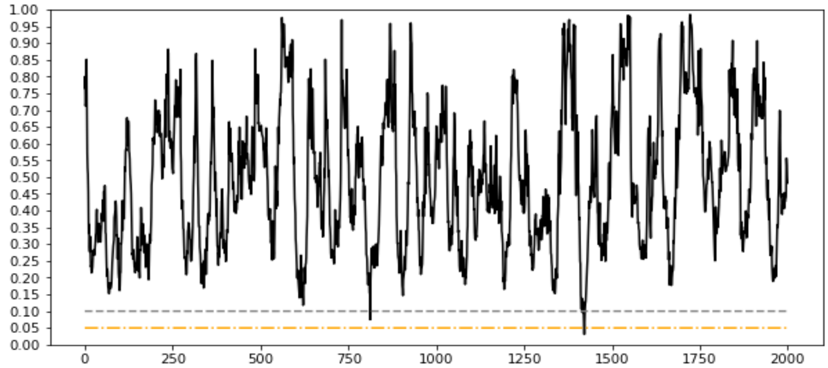

Figure 8. *p* values of the AQT of index DAX in 50 time period.

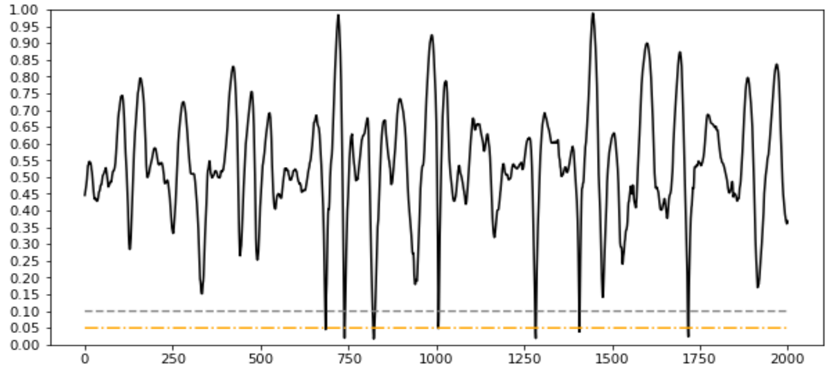

Figure 9. *p* values of the GST of index DAX in 50 time period.

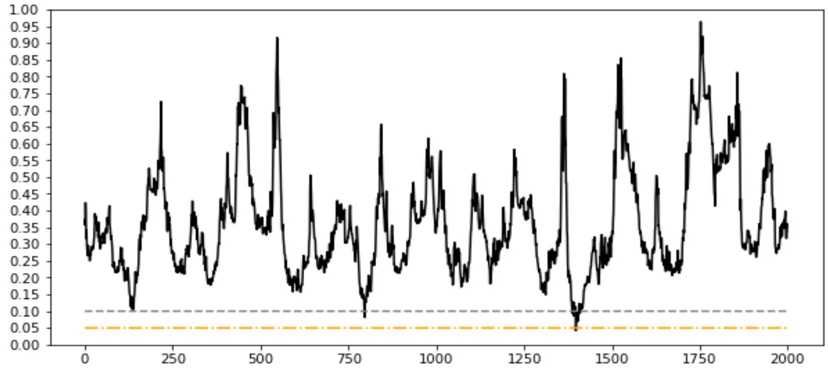

Figure 10. *p* values of the AQT of index DAX in 100 time period.

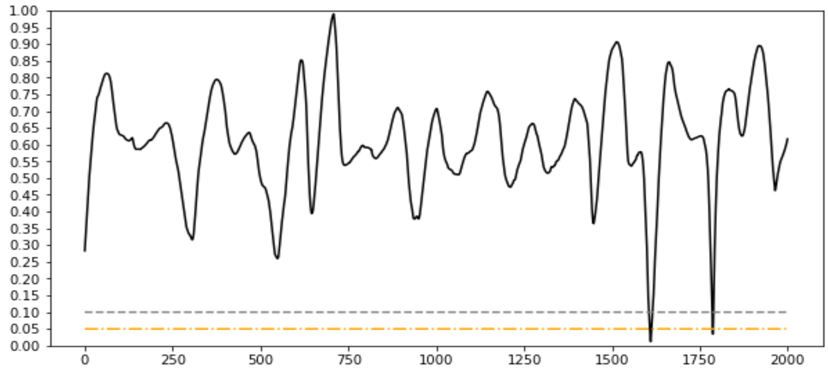

Figure 11. p values of the GST of index DAX in 100 time period.

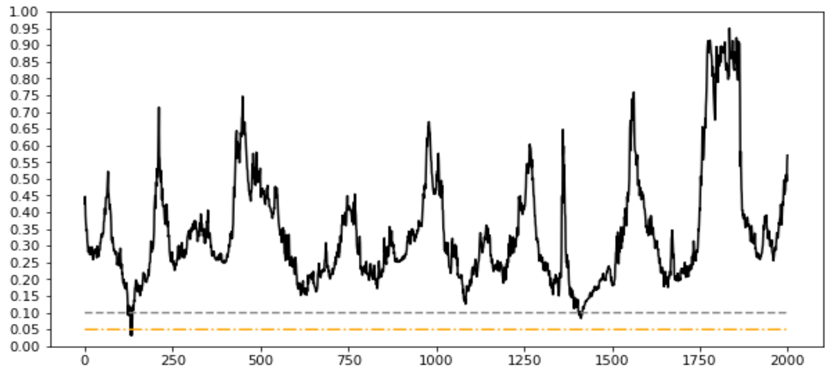

Figure 12. *p* values of the AQT of index DAX in 150 time period.

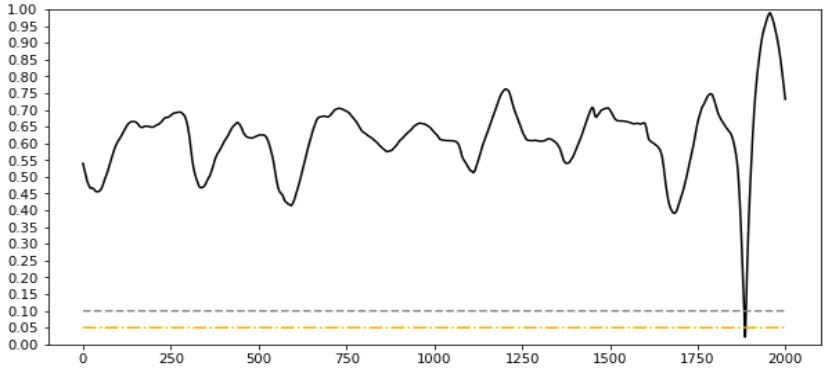

Figure 13. *p* values of the GST of index DAX in 150 time period.

6. Conclusions, practical implementations and further ideas of research

When analyzing the higher-order moments, it was clearly shown that the moments fix the presence of deviations and extreme values in the distribution of the series. Also, the calculations and graphical constructions made it possible to make sure that with an increase in the order of the moment, its sensitivity to outliers increases.

Results can be used in investment funds and central banks activity.

Further analysis in the following areas may be promising stages of future research: 1) substantiate the importance of developing highly sensitive econometric indicators; 2) consider the concept of superposition of moments, the econometric value of superpositions of moments of traditional orders; 3) develop a method for assessing lag (time delay) and difference (change in the central trend) sensitivity to economic changes; 4) conduct a comparative analysis of the approved indicators (which of them determines the crisis earlier, is sensitive enough and does not give false positives); 5) to come to conclusions that traditional moments are not enough and it is necessary to develop their own indicators based on traditional, but more sensitive and accurate ones. This task is important and relevant at the moment.

Author contributions: Conceptualization, NBAY; methodology, DS; software, GA; writing—original draft preparation, AM; visualization, MU. All authors have read and agreed to the published version of the manuscript.

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