Journal of Infrastructure, Policy and Development 2024, 8(8), 5727.
https://doi.org/10.24294/jipd.v8i8.5727

Article

Statistical and fuzzy signature-based analysis of the aggressive attitudes of a forensic population

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CITATION

Kóczy LT, Susniene D, Purvinis O, Zostautiene D. (2024). Statistical and fuzzy signature-based analysis of the aggressive attitudes of a forensic population. Journal of Infrastructure, Policy and Development. 8(8): 5727. https://doi.org/10.24294/jipd.v8i8.5727

ARTICLE INFO

Received: 9 April 2024
Accepted: 17 May 2024
Available online: 19 August 2024

Abstract: Clustering technics, like k-means and its extended version, fuzzy c-means clustering (FCM) are useful tools for identifying typical behaviours based on various attitudes and responses to well-formulated questionnaires, such as among forensic populations. As more or less standard questionnaires for analyzing aggressive attitudes do exist in the literature, the application of these clustering methods seems to be rather straightforward. Especially, fuzzy clustering may lead to new recognitions, as human behaviour and communication are full of uncertainties, which often do not have a probabilistic nature. In this paper, the cluster analysis of a closed forensic (inmate) population will be presented. The goal of this study was by applying fuzzy c-means clustering to facilitate the wider possibilities of analysis of aggressive behaviour which is treated as a heterogeneous construct resulting in two main phenotypes, premeditated and impulsive aggression. Understanding motives of aggression helps reconstruct possible events, sequences of events and scenarios related to a certain crime, and ultimately, to prevent further crimes from happening.

Keywords: questionnaires; forensic population; aggression; fuzzy signature; clustering; statistical evaluation

1. Introduction

Human aggression and violence are significant elements in criminogenic behavior, and comprehending the origins and triggering factors behind aggression could be immensely beneficial for forensic behavioral analysts (Henslin, 1999).

Aggression encompasses various concepts, making it a complex phenomenon. The challenge in defining aggression stems partly from the myriad biological, cultural, environmental, and social factors that shape how this problematic behaviour manifests (Lindsay and Anderson, 2000; Stanford et al., 2003a). It can be understood differently—as frustrating offensive reaction to insurmountable obstacles (Haden et al., 2008; Vitiello et al., 1990) as the habit of reacting with hostile actions or words to other people's actions, the physical environment (Buss and Perry, 1992; Klein Tuente et al., 2021; Tharp et al., 2011; Weinshenker and Siegel, 2002) or as the need to constantly defend oneself (Berkowitz, 2012; Haden et al., 2008). Wall Myers et al. (2018) define aggression as a physical or verbal act aimed at to inflict pain on someone else. The psychologist refers to this term as punches to the face, threats and insults, gossip and ridicule, destruction of property or even lying. The most important purpose of all these actions is to hurt the other person. Aggression is perceived in a very similar way by Berkowitz (2012). He argues that aggression is a form of behaviour that is intended to hurt someone psychologically or physically. Another formulation of aggression, almost identical to the previous ones is given by Weinshenker and Siegel

(2002): "Aggression is behaviour that hurts and damages another person or thing". In all three formulations of the term, the dominant idea is that aggression is an offensive, injurious action intended to physically harm another person or to cause him or her to experience negative emotional states. Obviously, most psychologists agree that aggression is an insolent, hostile behaviour whose main purpose is to harm another person physically or psychologically.

The absence of consensus in attempting to classify its various types accounts for the multitude of classifications present in the literature (Ramirez, 2009), although the most prevailing classification of aggression in academic research has been defined in two categories, namely, as premeditated (predatory, instrumental, callous-unemotional, proactive) and impulsive (defensive, affective, reactive, hostile) (Christopherson et al., 2013; Howell, 2014; Kockler, 2006; Klein Tuente et al., 2021; Raine et al., 1998; Silver and Yudofsky, 1991; Stanford et al., 2003b; Woodworth and Porter, 2002).

Impulsive aggression is manifested in unexpected, unusual situations, when the person is upset, does not have time to calmly consider all possible alternatives, to anticipate his or her actions the consequences, when they are highly aroused and experiencing negative emotions. The impulsive-affective aggressor reacts to provocation with instant and harmful violence (Battaglia, et al., 2021; Kockler et al., 2006; Weinshenker and Siegel, 2002; Zwets et al., 2015). The violence and coercion used are self-inflicted, i.e., the aggressor has no clear purpose other than to target the other person to cause discomfort and pain to the other person. The victim is usually chosen to be weaker, who is unable to fight back adequately. Anger consistently plays a role in hostile aggression, serving as its mediating factor (Huitema et al., 2021; Ramirez, 2009). While hostile aggression invariably involves anger as a pivotal mediating factor, instrumental or premediated aggression doesn't necessarily demand provocation or anger. Instead, it hinges on variables affecting outcome beliefs (such as calculating potential costs and benefits) and efficacy beliefs (pertaining to one's capability to execute aggression) (Bushman and Anderson, 2001). Contrary to impulsive aggression, instrumental aggression is purposeful, necessitating premeditation and planning, typically carried out with minimal autonomic arousal (Christopherson et al., 2013; Howells, 2011; Ramirez, 2009; Stanford et al., 2003b) as e.g., in instances of robbery, when a robber may assault a victim to steal something, it's not due to the robber's anger towards the victim, but rather because the aggressive behavior serves as a means to achieve the desired goal. It also can be aimed at achieving certain goals beyond direct harm to the victim. For example, adolescents often behave aggressively in order to raise their value among their peers in the eyes of their peers or to avoid possible rejection.

This study aims by applying fuzzy c-means clustering to facilitate the wider possibilities of analysis of aggressive behaviour, which is treated as a heterogeneous construct resulting in the two main phenotypes mentioned above. By applying the Impulsive/Premeditated Aggression Scale (IPAS), we demonstrate a new methodology for modelling and evaluating the replies of people who are tend to aggressive behaviour, in our case inmates. Understanding motives of aggression helps reconstruct possible events, sequences of events and scenarios related to the crime, and ultimately, prevent further crimes happening.

2. The method applied

2.1. Participants (demographics) of the study

The study participants were 47 inmates of total 145 of the Panevezys Correction House for Women voluntarily filling up the questionnaire presented to them. Therefore, the sample covered 32.4% of population. The Panevezys correction house is the only women correction in Lithuania, thus it was physically impossible to increase the size of the sample.

The questionnaire targeted to reveal the reason for aggression and its components. It is worth pointing out that by certain respondents some questions were left unanswered. Therefore, the number of answers to different questions varied from 35 to 47.

A significant part of the respondents consisted of 40–54 years old married or divorced women, who had secondary or unfinished secondary education, had permanent jobs, have children and live in a city rather than in a village.

2.2. The applied procedure and data analysis (IPAS)

We applied the Impulsive/Premeditated Aggression Scale (IPAS), which we adapted to the investigation of aggressive behaviour in our study. The content of IPAS scales was developed by Stanford et al. (2003a) and consisted of questions testing both types of aggression. The original design of the scales was based on the prior research and scales aimed at distinguishing between individuals exhibiting impulsive and premeditated aggression and included State-Trait Anger Expression Inventory (STAXI) devised by Spielberger (1996), Barratt Impulsiveness Scale (BIS-11) devised by Patton et al. (1995), Buss-Perry Aggression Questionnaire (BPAQ) devised by Buss and Perry (1992), Lifetime History of Aggression (LHA) devised by Coccaro et al. (1997), and Eysenck Personality Questionnaire (EPQ) devised by Eysenck and Eysenck (1976).

In our research, we employ this scale to assess aggressive behaviour, where impulsive aggression refers to an immediate, uncontrolled aggressive reaction triggered by provocation whereas premeditated aggression denotes a deliberate, consciously planned aggressive action that is not spontaneous or influenced by agitation. It is an 18-item instrument used to assess the individual's motivation and behavioural control during aggressive acts. Of the 18 items, 10 (from 1 to 10) focus on premeditated aggression, and 8 (from 11 to 18) on impulsive aggression characteristics. Some examples of questions are: "When I was angry, I reacted without thinking," and, "I planned when and where my anger was expressed." The items are scored on a 5-point Likert scale (5_strongly agree, 1_strongly disagree), which can be transformed in a rather straightforward way into degrees of truth (fuzzy membership degrees), assuming that the mapping is linear.

Various forms of aggression were studied, and an empirical analysis instrument (aggression scale/questionnaire) was developed in order to offer conceptual clarity both in the classification and the research of aggression.

Questionnaire evaluation is a complex task that requires dealing with the subjectivity and uncertainty inherent in the data obtained from such replies. We gave an overview of questionnaire analysis where subjectivity and uncertainty, further potential interdependence of the semantics of the questions and replies may be assumed (Kóczy et al., 2020). In that case, for employee attitudes, a rather well-developed structure had been already set up in the literature. This study proposes an even more complex approach, where in addition to the methods applied there, namely, statistical analysis and fuzzy signature construction and evaluation, a fuzzy c-means-based clustering method is proposed, as one suitable for analyzing questionnaires with related (partially) unclear answers, where even the categories of the answers (and answering persons) are unknown. The applicability of this complex modeling approach is demonstrated by a real case study, dealing with the analysis of aggressive criminal behaviour, based on real data collected in the penitentiary mentioned above.

In order to find the factors that determine the questions, information obtained from the answers will be used, namely, by applying factor analysis, which allows further evaluation and finding the assumed reasons for aggressiveness.

In addition to the mathematical structure, there is also a psychometric aspect of the questions and answers. The use of the fuzzy model can be incorporated into psychometric research as a tool to capture and accurately reflect the diversity, subjectivity, imprecision, and potential intended misleading inherent in human responses to such questionnaires. It's crucial to highlight that the absence of suitable statistical methods for analyzing such subjective and imprecise responses has been a significant hurdle in the literature thus far, wherever a purely statistical approach has been attempted.

This study also aims to create an opportunity to use an instrument of empirical analysis of aggression for criminologists. The ability to classify and investigate aggression will also lead to better outcomes in the assessment of aggressive inmates' behaviour.

To evaluate the validity of results the following approach was used. The standard error

$$\mathrm{SE} = \frac{s}{\sqrt{n}}$$

of the average of scores to the question I feel my actions were necessary to get what I wanted were corrected with the finite population correction factor

$$\mathrm{fpc} = \frac{N - n}{N}$$

where s is standard deviation and $n = 35$ is the volume of the sample and $N = 145$ is the total population of women inmates in Lithuania (Lavrakas, 2008).

This way we got the corrected error

$$\mathrm{SE}' = \mathrm{SE}\ \mathrm{fpc} = \frac{1.41}{\sqrt{35}} \frac{145 - 35}{145} = 0.18$$

of the mean $\bar{x} = 2.20$.

2.3. Descriptive statistics

A major part of the respondents to questions Q1 through Q10 concerning premeditated aggression answered that they "totally disagree" or "neither agree nor disagree" (see **Table 1**).

Table 1. Statistics of the answers.

	Min	Max	Average		Min	Max	Average
Premeditated aggression (PM)	1	5	1.9	Impulsive aggression (IA)	1	5	3.0
Q1. I feel my actions were necessary to get what I wanted	1	5	2.2	Q11. When angry I reacted without thinking	1	5	3.0
Q2. I think the other person deserved what happened to them during some of the incidents	1	4	1.9	Q12. Anything could have set me off prior to the incidents	1	5	2.1
Q3. The acts were a "release" and I felt better afterwards	1	4	1.6	Q13. I felt I lost control of my temper during the acts	1	5	2.8
Q4. I felt my outbursts were justified	1	5	1.8	Q14. I became agitated or emotionally upset prior to the acts	1	5	2.9
Q5. Prior to the incidents I knew an altercation was going to occur	1	5	2.0	Q15. Prior to the incidents I knew an altercation was going to occur	1	5	3.3
Q6. I wanted some of the incidents to occur	1	5	1.6	Q16. I typically felt guilty after the aggressive acts	1	5	3.5
Q7. The acts led to power over others or improved social status for me	1	5	2.1	Q17. I usually can't recall the details of the incidents well	1	5	2.8
Q8. Some of the acts were attempts at revenge	1	1	1.7	Q18. I feel that I have hurt others with my actions, I have done wrong	1	5	3.8
Q9. I understood the consequences of the acts before I acted	1	5	2.5	Age	21	61	41.9
Q10. I was in control during the aggressive acts	1	5	2.5				

Also, most of the respondents marked their choice about impulsive aggression (questions Q11 through Q18) ranging from "totally disagree" to "disagree" or "neither agree nor disagree". This obvious contradiction is the base for the assumption that the respondents intentionally replied in a way that, according to their opinion, corresponded to the expectations of the "official world" (authorities, psychologists, the persons presenting the questionnaires).

Nevertheless, there were several respondents who "agreed" or "totally agreed" with statements about their premeditated aggression or impulsive aggression.

The education of respondents was the following: 11% of the inmates had higher education, 4% incomplete higher education. 5% special secondary education, 42% secondary education, 27% incomplete secondary education, and 11% vocational education.

To reveal the structure of the responses, and through this, to classify the respondents, some well-established classical clustering approaches were applied.

In the future, it may be subject of further research how the actual responses may be transformed into a distribution that better complies with the reality.

2.4. Application of K-means clustering

It is purposeless to analyse each respondent individually. Therefore, we employed cluster analysis to split respondents into groups-each group containing more or less similar answers about their aggressiveness. To represent properties of each group the so-called prototypes (centroids) are computed. Prototypes represent properties of each cluster and enable to make conclusions about aggressiveness of the entire group.

First, a classical clustering approach (ignoring the uncertainty aspects) was applied to the responses to separate premeditated aggression, impulsive aggression, and both types of aggression mixed together.

We applied the K-means clustering method and used Orange data mining software to find the so-called prototypes for each cluster (Clatworthy et al., 2005; Witten, 2011). This program at first assigns clusters randomly and then updates with further iterations. How many times the iterations run from random initial position depends on the result with the lowest within-cluster sum of squares (see the Orange documentation). Therefore, the Euclidean distance is employed.

The number of clusters is selected by an algorithm using the Silhouette score (see the Orange documentation).

The K-means algorithm develops so-called prototypes (centroids) which are the records containing averages of each cluster members' properties (features). "Properties" in our case are responses to the questions. The respondents are the always assigned to the closest prototype based on their respective answers. The software distinguished three clusters to find groups of respondents with high, medium, and low aggressiveness.

The largest cluster of premeditated aggression contains 60% of respondents (**Table 2**). Their average aggression level equals 1.4. These respondents disagree that they acted in a premeditatedly aggressive way. That means that respondents' premeditated aggression is low.

Twelve respondents (34%) neither agreed nor disagreed with the assumed premeditated aggressiveness. Cluster N# 3 consists of only two respondents' replies therefore, this cluster is not statistically important.

Table 2. Premeditated aggression.

	Cluster N# 1	**Cluster N# 2**	**Cluster N# 3**
Number of respondents	12 (34%)	21 (60%)	2 (6%)
Q1. I feel my actions were necessary to get what I wanted	3.6	1.5	1
Q2. I think the other person deserved what happened to them during some of the incidents	2.9	1.4	1
Q3. The acts were a "release" and I felt better afterwards	2.7	1.0	1
Q4. I felt my outbursts were justified	3.0	1.1	1.5
Q5. Prior to the incidents I knew an altercation was going to occur	2.8	1.7	1
Q6. I wanted some of the incidents to occur	2.3	1.0	3.5
Q7. The acts led to power over others or improved social status for me	3.3	1.2	4
Q8. Some of the acts were attempts at revenge	2.7	1.1	1.5
Q9. I understood the consequences of the acts before I acted	2.5	2.2	5
Q10. I was in control during the aggressive acts	2.9	2.1	3.5
Average	2.9	1.4	2.3

Let us consider the largest cluster and analyse IA. This cluster contains 46% of respondents who neither agree nor disagree about their impulsive aggression (**Table 3**). Thirteen (37%) respondents agree that their aggressive actions have been impulsive and only 17% of them strongly disagree that they acted in an impulsive way.

Table 3. Impulsive aggression.

	Cluster N# 1	Cluster N# 2	Cluster N# 3
Number of respondents	13 (37%)	6 (17%)	16 (46%)
Q11. When angry I reacted without thinking	4.5	1.0	2.7
Q12.Anything could have set me off prior to the incidents	2.5	1.0	2.3
Q13. I felt I lost control of my temper during the acts	4.2	1.0	2.5
Q14. I was confused during the acts	3.7	1.0	3.1
Q15. I consider the acts to have been impulsive	4.8	1.0	3.1
Q16. I typically felt guilty after the aggressive acts	4.5	1.2	3.4
Q17. I usually can’t recall the details of the incidents well	4.1	1.0	2.3
Q18. I feel some of the incidents went too far	4.9	1.8	3.6
Average	4.2	1.1	2.9

Clustering of both types of aggression (premeditated and impulsive) together revealed that the largest cluster contains 43% of respondents whose answer equals 2.6, i.e., between disagree and neither agree nor disagree and disagree (**Table 4**). The next cluster contains 34% of respondents who marked the option “neither agree nor disagree”. And finally, eight respondents (23%) strongly disagree with their motivation as impulsive aggression.

Table 4. Centers of clusters when clustering both types of aggression (PM and IA) together.

	Cluster N# 1	Cluster N# 2	Cluster N# 3
Number of respondents	8 (23%)	12 (34%)	15 (43%)
Q1. I feel my actions were necessary to get what I wanted	1.5	3.6	1.5
Q2. I think the other person deserved what happened to them during some of the incidents	1.1	2.9	1.5
Q3. The acts were a “release” and I felt better afterwards	1.0	2.7	1.0
Q4. I felt my outbursts were justified	1.3	3.0	1.1
Q5. Prior to the incidents I knew an altercation was going to occur	1.1	2.8	1.9
Q6. I wanted some of the incidents to occur	1.0	2.3	1.3
Q7. The acts led to power over others or improved social status for me	1.0	3.3	1.7
Q8. Some of the acts were attempts at revenge	1.0	2.7	1.2
Q9. I understood the consequences of the acts before I acted	1.4	2.5	3.1
Q10. I was in control during the aggressive acts	2.0	2.9	2.4
Q11. When angry I reacted without thinking	1.4	3.7	3.5
Q12. Anything could have set me off prior to the incidents	1.1	3.1	1.9
Q13. I felt I lost control of my temper during the acts	1.0	3.4	3.4
Q14. I was confused during the acts	1.3	3.3	3.7
Q15. I consider the acts to have been impulsive	1.5	3.1	4.6
Q16. I typically felt guilty after the aggressive acts	2.0	3.5	4.2
Q17. I usually can’t recall the details of the incidents well	1.0	3.0	3.5
Q18. I feel some of the incidents went too far	2.1	4.0	4.5
Average	1.3	3.1	2.6

2.5. Clustering by fuzzy c-means

Clustering by the k-means method has a strong drawback. It does not allow even partial overlapping of the clusters and does not leave space to any uncertainty or gradual/partial truth.

Fuzzy c-means clustering extends the concept of the k-means method by introducing fuzziness degrees in the cluster assignment of objects, such that fuzzy memberships express the degree of belongingness of objects to clusters (Bezdek et al., 1984; Chen and Honda, 2020).

We applied standard fuzzy c-means clustering (FCM) algorithm to the answers to all questions about aggression. However, FCM requires prior knowledge about the number of clusters in the data (Bezdek et al., 1984; Spaans et al., 2017; Zanaty, 2012). The K-means clustering approach discussed above revealed that the responses contain three (crisp, i.e., clearly separated) clusters. The results obtained by applying FCM with assuming three clusters are given in **Table 5**, where membership degrees describing the dependence of each respondent to clusters are included.

Table 5. Centres of the fuzzy clusters.

	c1	**c2**	**c3**
Q1	1.6	1.6	3.0
Q2	1.5	1.2	2.7
Q3	1.2	1.0	2.5
Q4	1.3	1.3	2.7
Q5	2.0	1.3	2.7
Q6	1.2	1.0	2.5
Q7	1.8	1.2	2.9
Q8	1.2	1.1	2.6
Q9	2.6	1.6	2.9
Q10	2.2	1.9	3.0
Q11	4.0	1.4	3.3
Q12	2.1	1.2	2.8
Q13	3.7	1.2	3.1
Q14	3.6	1.4	3.2
Q15	4.5	1.6	3.2
Q16	4.4	1.8	3.3
Q17	3.6	1.2	3.0
Q18	4.8	2.2	3.5
Max	4.8	2.2	3.5
Min	1.2	1.0	2.5
Average	2.7	1.4	2.9

It can be seen from **Table 5** that cluster c1 consists of high scoring answers to questions Q11 through Q18 about impulsive aggression. Nevertheless, there are no respondents who belong to this fuzzy cluster with a membership degree higher than 0.77 (**Table 6**). This raises the question of normalization which may transform the cluster membership function into a more interpretable one. The strong sub-normality of the responses may have two psychological explanations: The tendency to “satisfy” the expectations of the “official” world, or the inadequateness of the formulation of the questions. Further research, especially, on more data sets may reveal an answer to this question.

The central part of cluster c2 consists of low scores for both types of aggression. Five respondents (14%) belong to this cluster with membership degrees higher than 0.8. This fact confirms the above assumption as these responses are the most “neutral” ones, which may be the mostly “liked” by the questioning persons in the mind of the inmates.

Table 6. Summary of membership degrees.

	c1	**c2**	**c3**
Number of respondents with membership degrees greater than 0.80	0	5	5
Max	0.77	0.93	0.89
Min	0.05	0.06	0.03
Average	0.34	0.37	0.29

The central region of cluster c3 contains aggressiveness scores between 2.5 and 3.5, i.e., medium level aggressiveness. Also here, these five respondents (14%) belong to this cluster with membership degrees higher than 0.8. One respondent has high membership degree in c3. This respondent can be characterized as having the strongest degree of the medium level aggressiveness.

The membership degrees of all respondents are given in **Figure 1**.

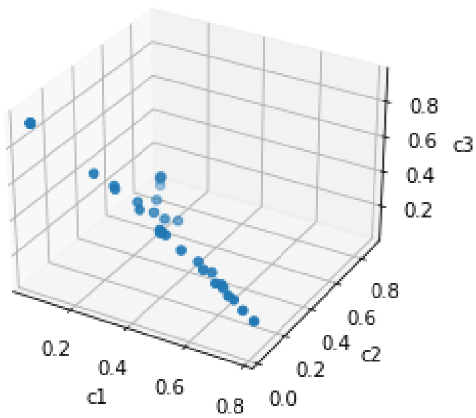

Figure 1. Membership degrees of respondents.

These results of soft clustering by FCM thus do not contradict with the conclusions drawn from the k-means (hard) clustering. Both revealed there were no clusters whose members classified themselves as high in both types of aggression (Even though the respondents had a tendency to diminish their aggressive motivations in general).

3. Evaluation of the aggression by fuzzy signatures

3.1. The concept of fuzzy signatures

Fuzzy sets were introduced by Zadeh (1965). Fuzzy sets are the tool that enables the formal description of imprecise statements, concepts and relations. Fuzzy sets are suitable for describing the population of subjective and uncertain answers, such as the answers on a given scale (e.g., the Likert scale using 1 … 5 in this study).

The definition of fuzzy set A is

$$A = \{X, \mu A\},\ \mu A: X \to [0, 1]$$

where μA is the membership function of set A, a subset of the universal set (or simply, universe) X. The main idea of fuzzy set is the recognition that truth may be partial, somewhere between "true" and "false", which is applicable also for the statement "x is member of set A", in the sense that any element x within the universal set X may be only partially belonging to the set A. To express this partial (gradual) truth, the function μA assigns to every element $x \in X$, a value from the unit interval [0, 1], which defines the degree of x belonging to A. To use partial truth values is very convenient in everyday life, when statements like "The weather is nice" or "He is an elderly person" may be only true for a weather situation or a particular man bear a degree of uncertainty. Obviously, the problem originates from the fact that in natural languages most concepts are defined only in a way that leaves the borders of the concept unsure, "fuzzy".

Later, it appeared that more sophisticated applications often require extensions of the original fuzzy set concept, as most phenomena have multiple features, "dimensions", which independently may have uncertain, gradual truth type descriptions. While working on a certain industrial project, it turned out that it was rather useful to introduce the concept of vector-valued fuzzy sets. Kóczy (1980), where the degree of belonging to a fuzzy set had to be characterized by several independent features of equal importance or relevance

$$A = \{X, \mu A\},\ \mu A: X \to [0, 1]^n.$$

In the above example mentioning "nice weather", the amount of sunshine, the temperature itself, the strength of the wind, the humidity of the air, etc. together form an adequate description of the weather situation.

The concept of vector-valued fuzzy sets was even further extended to the idea of Fuzzy Signatures (FSig) and Fuzzy Signature Sets (Vamos et al., 1999; Wong et al., 2003), because it was realized that often these multiple features are not entirely independent among themselves, and may form groups and sub-groups that can be arranged in a hierarchical structure, vectors within the vectors, going down to variable depth:

$$A = \{X, \mu A\},\ \mu A: X \to [m]^n,\ m = \{\begin{matrix}[0, 1] \\ m_i\end{matrix},$$

where m_i are defined in a similar recursive way as m itself. The depth of the recursion may be arbitrary, but finite, although in most real life problems it is just "a few". In the previous simple example, the wind and the sunshine definitely influence the humidity, but of course, the weather history in the area is also an important factor, as a recent rain definitely raises the percentual contents of water in the air. This small example points out the complexity of establishing a proper and adequate fuzzy signature graph structure.

For instance, a nested vector of this recursion of depth 3 may be the following:

$$m = \begin{bmatrix} \begin{bmatrix} m_{11} \\ m_{12} \end{bmatrix} \\ \begin{bmatrix} m_{21} \\ m_{22} \\ \begin{bmatrix} m_{231} \\ m_{232} \end{bmatrix} \end{bmatrix} \\ m_3 \end{bmatrix},$$

where

$$m_2 = \begin{bmatrix} m_{21} \\ m_{22} \\ m_{23} \end{bmatrix}$$

is a nested component, and one of its components,

$$m_{23} = \begin{bmatrix} m_{231} \\ m_{232} \end{bmatrix}$$

is one more level nested.

A FSig set is defined as a fuzzy set where the membership degrees are FSig values:

$$A^{\mathrm{FSig}} = \left\{X, \mu_A^{\mathrm{FSig}}\right\}, \mu_A^{\mathrm{FSig}}: A^{\mathrm{FSig}} \to M^{\mathrm{FSig}},$$

where M^{FSig} stands for the predefined set of fuzzy signatures for a given problem. There is one more important element of this type of multi-level description of uncertain features: how are the individual components connected with each other, within the sub-groups, and the sub-groups within the groups, and finally, all groups together the aggregate the descriptors in the root of the tree. This was defined by the assignment of a fuzzy logic/set operation to each node of the graph that is not a leaf, operations called aggregation in the literature. Aggregations are very general, there are only three axiomatic properties requested, namely that they are monotonic in terms of each argument, and further, that if all arguments are "false" then the result is "false" too, and if all operands assume "true", the result is also "true". The membership degrees at the leaves (the actual feature components) belong to the interval [0, 1], and by executing the aggregations in the intermediate nodes, each of them will be also signed a similar fuzzy membership degree.

In the above example, it means that the tree structure defined by the graph of m above is associated with the aggregation set

$$\{a_0; a_2, a_{23}\},$$

so that

$$m = m_1 a_0 m_2 a_0, m_1 = m_{11} a_1 m_{12},$$

where

$$m_2 = m_{21} a_2 m_{22} a_2 (m_{231} a_{23} m_{232}).$$

It is not necessary that all fuzzy signatures are identical in the structure (which consists of the tree graph and the aggregations associated to all intermediate nodes, including the root), but a more general approach necessitates further mathematical consideration on how to deal with (partially) differently structured signatures (Kóczy et al., 2021). In this application this mathematical approach is, however, not needed, as the FSig structure proposed for the questionnaires will be uniform, matching the uniformity of the questionnaires themselves, and all respondents have given answers to all questions, thus there are no missing branches of the model graph.

3.2. The application of fuzzy signatures for the data set

The fuzzy signature approach and graduate aggregation into nodes are based on similarities of the semantics of some questions. Answers to questions assigned to different leaf nodes may be similar, but vice versa, the answers to questions to corresponding to the same node may not be correlated. These similarities and differences depend on the respondents' character traits and beliefs.

To be able to represent the questions and answers by a fuzzy signature model (Kóczy et al., 2020), we transformed the responses given in the integer interval $\{1, \ldots, 5\}$ in a linear way into the continuous unit interval [0, 1]

$$f: \{1, \ldots, 5\} \rightarrow [0, 1]$$

where $f(x) = (x - 1)/4$.

This way, we get membership function values which express the degree of agreement of the responding inmate for the given statement. Thus, the value 0 means that the respondent totally disagrees, while the value 1 means full agreement with the statement.

After this simple transformation of the scale, further manipulations of the membership degrees in the questionnaires, especially, the execution of the fuzzy aggregations in the internal nodes of the fuzzy signature trees and other fuzzy operations become possible.

The leaves (Q1 through Q18) of the fuzzy signature of the questionnaire were arranged into six sub-trees with aggregations in the sub-root nodes based on expert evaluation of the questions' given in the above mentioned literature and semantic similarities (see **Figure 2**):

Q1, Q3, Q4—"be guided by feelings in aggressive behaviour", where a weighted arithmetic means aggregation was proposed, namely,

$k1$Q1 + $k3$Q3 + $k4$Q4 = Guided by feelings, $k1$, $k2$, $k3 \geq 0$, $k1 + k2 + k3 = 1$,

and similarly,

Q2, Q7, Q8—"revenge and power over somebody",

Q5, Q6, Q9, Q10—"intentional, premeditated, hostile and deliberate actions",

Q11, Q14, Q15—"spontaneous actions, acting without consideration",

Q12, Q13—"lack of action control, explosive behaviour",

Q16, Q17; Q18—"feelings afterwards, after the aggressive behaviour".

The degrees belonging to the first three nodes were aggregated into a single membership degree in the node Premeditated aggression and the latter three into a membership degree assigned to the node Impulsive aggression. Then, these two nodes of Premeditated aggression and Impulsive aggression were aggregated into the root node Aggression, which revealed the overall degree of membership expressing the grade of aggressiveness for each respondent.

Then, the membership degrees of Q1 through Q18 were approximately restored from the root node Aggression using expressions.

$$\widetilde{Q_i} = l_i \text{Aggression}, \; i = 1, 2, ..., 18.$$

To find the weights k_i and l_i the square error

$$\mathrm{SE} = \sum_{i=1}^{18} (Q_i - \widetilde{Q}_i)^2$$

between actual membership degrees of Q_i and approximate restored values $\widetilde{Q}_i$ was minimized using the generalized reduced gradient (GRG) method, and a simple evolutionary method alternately (Petrowski and Ben-Hamida, 2017). This approach of combining evolutionary and gradient based approximate optimization is usually referred to as memetic heuristics (Moscato, 1989). Application of this approach showed that the SE for one answer per respondent was equal to 0.05. This testifies a good reliability for both types of aggregation and of the weights determined.

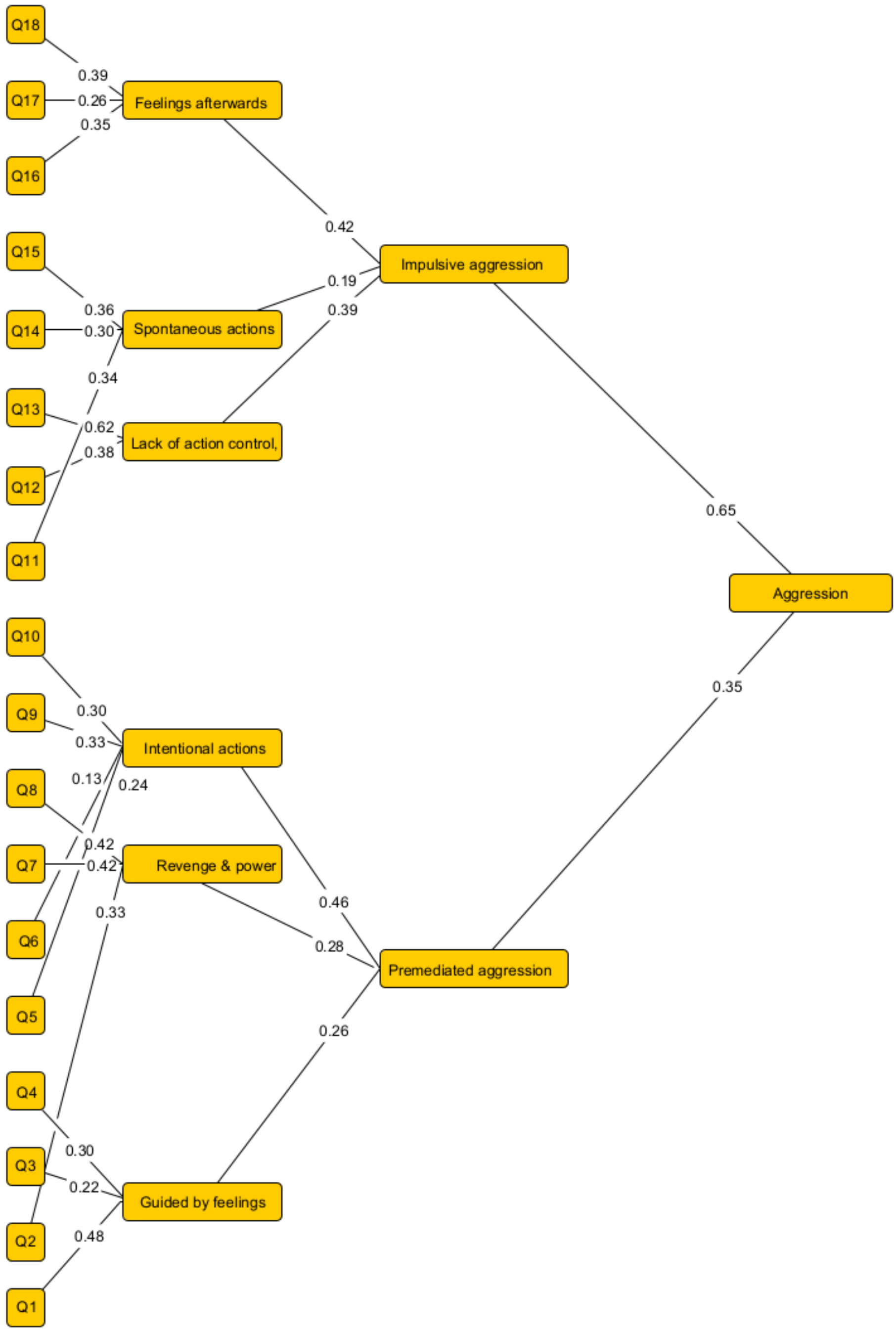

Figure 2. The Fuzzy signature structure with example degrees at the leaves and the weights k_i in the arithmetic means in the aggregations.

The resulting aggregation weights are given in **Figure 2**, which reveals the structure of aggressiveness. For instance, the impulsive aggressiveness weight equals to 0.65, while the premeditated aggressiveness weight is 0.35. This tells that impulsive aggressiveness dominates over premeditated aggressiveness in the respondents' aggressive behaviour—at least, if the answers may be considered frank.

On the other hand, the overall membership degree of aggressiveness is not high, the average value equals to 0.43. This raises the question how sincere the answers were, and how much the inmates wanted to conform with what they assumed was expected from them. It is worth to point it out here that we worked only with answers from inmates who voluntarily participated in the survey and thus, had a certain positive attitude.

4. Principal component analysis of the aggression

The computation of correlation between answers to questions revealed that all answers to the questions Q1–Q18 were linearly significantly correlated with at least one other question at level $p < 0.05$. These correlations suggest that correlated responses to different questions may be driven by common latent factors. Principal component analysis (PCA), which in a broader sense is recognized as an exploratory factor analysis was performed to reveal these factors.

PCA uses an orthogonal transformation to convert correlated variables into a set of values of linearly uncorrelated factors called principal components.

To reveal latent factors, we carried out PCA to explain at least 80% of the data variance, that is, the total factor variance should contain 80% of the total response variance (Loehlin and Beaujean, 2017). The lowest number of factors with which this variation threshold can be reached turned out to be five factors (**Table 7**).

The coefficients given in **Table 7** are called factor loadings. Loadings close to 1 or −1 mean that the latent factor significantly determines the answers to the question. Generally, if answers related to specific factor are positively correlated with each other, all the loadings are positive. Negative loadings indicate that correlations between some answers are negative.

It can be seen from **Table 7** that questions Q11 through Q18, partially excluding Q12, concerning impulsive aggression are well targeted and they reveal the same concept, namely, impulsiveness.

While answers to questions Q1 through Q10 about premeditated aggression depend on four other factors. Q3 through Q10 may actually be called general premeditated aggression indicators. Factor 3 determines for the most part answers to Q1 and Q2. Factor 4 and Factor 5 coincides mostly with Q9 and Q10 only, respectively. This means that answers to the questions Q9 and Q10 have little correlation with other answers. Hence, the questions Q9 and Q10 were quite independent.

Table 7. Latent factors of aggression and loadings.

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5
Q1. I feel my actions were necessary to get what I wanted	0.02	0.26	**0.82**	0.34	−0.07
Q2. I think the other person deserved what happened to them during some of the incidents	−0.03	0.38	**0.86**	0.03	0.14
Q3. The acts were a "release" and I felt better afterwards	−0.04	**0.77**	0.35	0.12	−0.11
Q4. I felt my outbursts were justified	−0.05	**0.64**	0.32	0.54	−0.30
Q5. Prior to the incidents I knew an altercation was going to occur	0.18	**0.54**	0.44	−0.20	−0.12
Q6. I wanted some of the incidents to occur	−0.01	**0.78**	0.03	0.03	0.47
Q7. The acts led to power over others or improved social status for me	0.23	**0.75**	0.05	0.42	−0.06
Q8. Some of the acts were attempts at revenge	−0.06	**0.76**	0.43	0.05	0.17
Q9. I understood the consequences of the acts before I acted	0.15	0.05	0.05	0.18	**0.90**
Q10. I was in control during the aggressive acts	−0.09	0.16	0.13	**0.81**	0.39
Q11. When angry I reacted without thinking	**0.81**	0.32	0.10	−0.14	−0.10
Q12. Anything could have set me off prior to the incidents	0.47	**0.62**	0.08	−0.05	0.07
Q13. I felt I lost control of my temper during the acts	**0.84**	0.37	−0.15	0.03	−0.06
Q14. I was confused during the acts	**0.63**	−0.02	0.52	−0.23	0.30
Q15. I consider the acts to have been impulsive	**0.82**	−0.22	0.15	−0.21	0.22
Q16. I typically felt guilty after the aggressive acts	**0.82**	−0.22	0.09	0.31	0.08
Q17. I usually can't recall the details of the incidents well	**0.87**	0.22	−0.15	−0.21	0.10
Q18. I feel some of the incidents went too far	**0.79**	0.00	0.00	0.14	−0.02

5. Conclusions and discussion

The objective of this study was to improve the understanding of aggression by examining a simplified categorization that is as succinct as possible, reducing redundancy in measurements. We proposed that aggression manifests in two distinct primary phenotypes, despite significant semantic similarities. While authors may use different terms, these differences essentially signify overlapping concepts. Our study approves that IPAS provides a simple technique by which the manifestation of the aggressive behaviour might be revealed. Our findings are consistent with prior studies mentioned above in our research validating the IA and PM scales of the IPAS in other samples.

We tried to categorize aggressive individuals into two predominant categories, the results of our study supporting previous findings other researchers (Kockler et al., 2006; Ramirez, 2009; Standford et al., 2003b) disclosing that the majority of individuals displaying aggression often exhibit a combination of both impulsive and premeditated aggressive traits, falling into a "mixed" group. Therefore, it is recommended to classify aggressive behaviour as either primarily impulsive or primarily premeditated in its nature.

Clustering by k-means of both types of aggression together, i.e., the answers to the entire questionnaire revealed that the largest cluster contains 43% of respondents, whose answer equals to values close to 2.6, i.e., between "disagree" and "neither agree nor disagree". Other two clusters represent respondents who 'disagree" and "neither agree nor strongly disagree" concerning impulsive aggression in their behaviour.

Applying fuzzy c-means clustering enabled to reveal graduality expressed by fuzzy degrees in the respondents' belonging to the clusters. It appeared that here were no respondents who belong to the fuzzy cluster of high impulsive aggressiveness with membership degree higher than 0.77. The other two clusters, as in the k-means case, represent respondents with medium and low aggression.

The fuzzy signature approach revealed clearly that in the respondents' aggressive behaviour. impulsive aggressiveness dominates over premeditated aggressiveness, but even so, the overall membership degree of aggressiveness is not high, the average value of aggressiveness degree equaling to 0.43.

Application of factor analysis showed that five latent factors of aggressive behaviour exist.

While the IPAS demonstrates robust reliability and validity, it is important to acknowledge several limitations that should be taken into account when interpreting the current data:

- The sample included respondents only from a single and rather small Correction House—the only one existing in Lithuania—and the respondents consisted only of women due to the particularity of this institution.
- Since the sample comprised solely of women, it is possible that these findings may not be extended to aggressive men, thus limiting the generalizability of the results.
- As participants relied on their recollection of past aggressive behaviours they were involved in, it is possible that they encountered challenges in accurately recalling the details of these incidents. And one more point with recalling could be that during the past period they recalled, both types of aggression where experienced.
- Finally, it should be remarked that the degree of aggressiveness, especially, of premeditated aggressiveness turned out less than what could be expected—an obvious consequence of the answers being "smoothed", partly adapted to the expectations of the authorities.

Author contributions: Conceptualization, LTK and DS; methodology, DS; software, OP; validation, DS, OP and DZ; formal analysis, LTK; investigation, DZ; resources, DS; data curation, DZ and OP; writing—original draft preparation, DS and OP; writing—review and editing, LTK and DZ; visualization, OP; supervision, LTK; project administration, LTK; funding acquisition, DZ. All authors have read and agreed to the published version of the manuscript.

Conflict of interest: The authors declare no conflict of interest.

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