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ORIGINAL ARTICLE

# Public investment, deficit financing, and intertemporal redistribution in a long-run McGuire-Olson-Solow economy

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## ABSTRACT

We investigate the impact on intertemporal distribution caused by a change of policy from tax to deficit financing of public investment, using a simple theoretical framework which combines the one-period McGuire-Olson economy with the conventional long-run Solow economy. This theoretical framework provides a simple way to highlight some significant interdependencies between private and public investments as well as the negative impact of taxation on aggregate productivity, and to trace some possible transmission mechanisms between deficit financing policies and the long-run path of consumption per head. The main tentative (theoretical) result is that although under fairly acceptable assumptions the likely impact of a deficit financing policy is to benefit the present at the expense of the future, under equally acceptable assumptions concerning the possibility of an excessive macro private saving–investment propensity, and/or of a significant productivity loss due to the excess burden of taxation, the adverse intertemporal distributional impact of deficit financing might become negligible, or even disappear altogether.

**Keywords:** *public investment; deficit financing; productivity; excess burden; redistribution; social welfare*

## 1. Introduction

The purpose of this article can be stated in simple terms. We want to investigate whether, how, and if so why, a policy of deficit financing of public investment may affect the long-run time path of consumption per head, relative to the time path that would prevail under a policy of full tax financing.

The problem can be studied in empirical or theoretical terms, and in the latter case, under different approaches and methodologies. Our investigation is purely *theoretical*, and is carried out using the conventional methodology of the Solow theory of *long-run economic dynamics*, in its simplest version (Solow, 1956; 2000; Atkinson and Stiglitz, 1980, Lecture 8: "Taxation and debt in a growing economy", especially pp. 249–253). This means that we are placing the issue within an economy working in a state of *long-run full employment* (or con-

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-stant unemployment). The limitations of the results obtained in a context like this are well-known, and there is no need to spend time on them here. Although far removed from the complexities and instabilities of current economic developments, we regard it as nothing more and nothing less than a useful logical framework for organizing one's thinking about the problem at hand. We have no ambition to derive from such theoretical exercise any single answer and/or policy prescription of general validity in the real world, as if they were based on some kind of law of physics, and neither to claim for its logical framework any *a priori* superiority over other different logical frameworks possibly leading to different conclusions. As a matter of philosophical orientation, we believe with Solow that there are no laws of physics in economics (Solow, 2004).

The specific methodological feature of our theoretical exercise lies in the fact that we incorporate into the Solow dynamic long-run full-employment economy the 'production' properties of the static one-period full-employment economy designed by McGuire-Olson for developing their *positive theory of a government run* by rent-extracting ruling interests (McGuire and Olson, 1996). Such 'production' features consist of two elements. One is that public goods are of the public infrastructure type, indispensable for ensuring—and raising—the private economy's productivity. In other words, they are treated as public investment essential for the *very emergence* of a productive private ('commercial') economy. The other is that taxes decrease the private economy's aggregate productivity because of their intrinsic *excess burden*.

Extending the 'production' properties of the McGuire-Olson one-period economy—henceforth denoted MOE for brevity—into the Solow dynamic long-run economy is a necessary step for making the resulting economy capable of dealing with *deficit spending*. The McGuire-Olson one-period economy is by construction incapable of accommodating private investment, saving, lending, and borrowing. However, when used for the purpose of the authors' positive theory of a government run by ruling interests, there is no need to constrain it into a balanced-budget economy. The whole economy is divided into two groups of 'people', one consisting of those forming the ruling interests, the other consisting of the rest. The revenues of the ruling interests consist of tax revenues plus the (net of tax) revenue they earn in the 'commercial' part of the economy. Public expenditures are financed by the ruling interests out of their (gross) revenues, who then spend their remaining net revenues on consumption, while the rest of the population divides its revenues between taxes and consumption. Such scenario is perfectly compatible with any balance (surplus, balance, or deficit) of the public budget, because any surplus is just pocketed by the ruling interests, while any deficit is financed out of their gross revenues.

On the other hand, under the *standard normative scenario* for assessing the impact of alternative government policies, the *balanced budget becomes a necessary constraint of such one-period economy* because within a single unique time unit, saving, lending, and borrowing have no economic meaning, and thus neither the 'private' economy nor the government can save, lend, or borrow. This latter scenario is precisely the one we use for outlining the McGuire-Olson economy, before extending its features into the Solow one in order to deal with deficit spending.

There is finally one further aspect of combining the McGuire-Olson and Solow economies that we want to underline. It is a combination that allows bringing out within a simple stylized framework the cooperative nature of the *state-economy inseparability*, their necessary coexistence and *potentially virtuous* interaction centered around the positive impact of public investment on private productivity, and the close interdependence between private and public investments (Mazzucato 2015; 2018, especially Chapter 8 "Undervaluing the Public Sector").

As for the impact of deficit financing of public investment, we were initially expecting, on the basis of *common sense intuition*, that in a long-run full-employment perspective, a change of policy from tax to deficit financing would under very general conditions have the perverse effect of *benefiting the present at the expense of the future*. However, in partial contrast to such intuition, our theoretical exercise suggests that, although that proposition may indeed hold under certain acceptable conditions, under different and equally acceptable conditions, it may no longer hold.

Section 2 highlights the relevant production and consumption frontiers of the McGuire-Olson balanced-budget economy. Section 3 provides a graphic representation of the standard dynamics of primary deficit, total deficit, and debt. Section 4 translates the structural aspects of the McGuire-Olson one-period economy into the standard Solow long-run one. Section 5 summarizes formally and graphically the properties of the production and consumption frontiers, and the associated golden rule, in the benchmark long-run scenario of this McGuire-Olson-Solow economy. Section 6 analyzes the *intertemporal distributional* impact of a policy change from tax to deficit finance of public investment, under the assumption of a negligible excess burden of taxation. Section 7 re-views the previous results under the assumption of a significant such excess burden.

## 2. The production and consumption frontiers of the McGuire-Olson one-period balanced-budget economy

There are two 'production' properties of the MOE that we will extend into the Solow economy. One is the *productivity-enhancing* role of public investment, which in the MOE yields a bell-shaped consumption frontier, radically different from the conventional decreasing textbook consumption frontier of the Samuelson type. The other is the loss of potential output caused by the average tax pressure, due to the *excess burden of taxation*.

Potential output $Q$ is an increasing function of public investment $G$, which stands for a *flow* of infrastructure goods, defined in the widest sense to include both material 'things' and immaterial 'services' (such as public funding of education and research). The cost of producing $G$ is an increasing function $h(G)$, defined as the opportunity cost of G in units of consumption $C$. Total output $Q = GDP$ is thus defined by $Q = C + h(G)$, where $C$ is the 'commercial' part and $h(G)$ the 'non-commercial' part. Tax revenues are measured by $\tau Q$, where $\tau$ is the average tax pressure.

Taxes decrease the economy's productivity according to a coefficient of tax distortion $\eta(\tau)$. In summary:

$$\begin{aligned} Q(\bullet) &= \eta(\tau)X(G) \\ &= C + h(G) \end{aligned} \tag{1}$$

$$C(\bullet) = \eta(\tau)X(G) - h(G)$$

$$T(\bullet) = \tau\eta(\tau)X(G)$$

where:

- $X(G)$ is a positive, *purely technological relationship* between $G$ and potential output $X$, with no reference to how G may be financed (voluntary saving, or non-distortionary taxes).
- $\eta(\tau)$ is a *coefficient of macro excess burden*, decreasing from 1 to 0 as $\tau$ ranges from 0 to 1, with shapes like those depicted in red in **Figure 1.** It compacts into a single macro excess burden all allocative distortions caused by the tax system in the whole economy. The thick black line $\eta(\tau) = 1 - \tau$ represents the polar shape of the worst possible tax structure, causing some sort of maximum distortion. The horizontal thick black line $\eta(\tau)$ = 1 represents an ideal tax structure causing zero distortion.

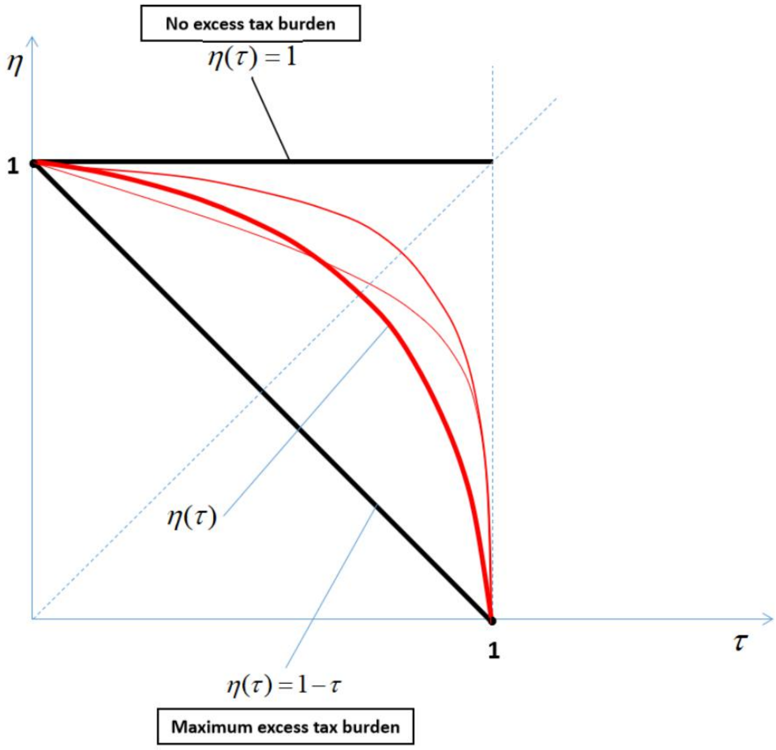


**Figure 1.** The coefficient of excess tax burden

As already mentioned, this one-period economy must by construction incorporate *a balanced-budget constraint*, which in turn yields an implicit relationship going from $\tau$ to $G$ and *vice versa*:

$$\begin{aligned} &T(\bullet) = \tau\eta(\tau)X(G) = h(G) \\ &\rightarrow G(\tau),\ \tau \in [0,1] \\ &\rightarrow \tau(G), G \in [0.\overline{G}] \end{aligned} \tag{2}$$

Inserting the implicit relationship $\tau(G)$ into the output and consumption functions yields the production (output) and *consumption frontiers* of the MOE:

$$\begin{aligned} &\hat{Q}(G) = Q(G,\tau(G)) = \eta(\tau(G))X(G) \\ &\hat{C}(G) = C(G,\tau(G)) = \eta(\tau(G))X(G) - h(G) \end{aligned} \tag{3}$$

where the hat notation indicates function composition.

These frontiers and the implicit relation $G(\tau)$ are drawn in **Figure 2.**

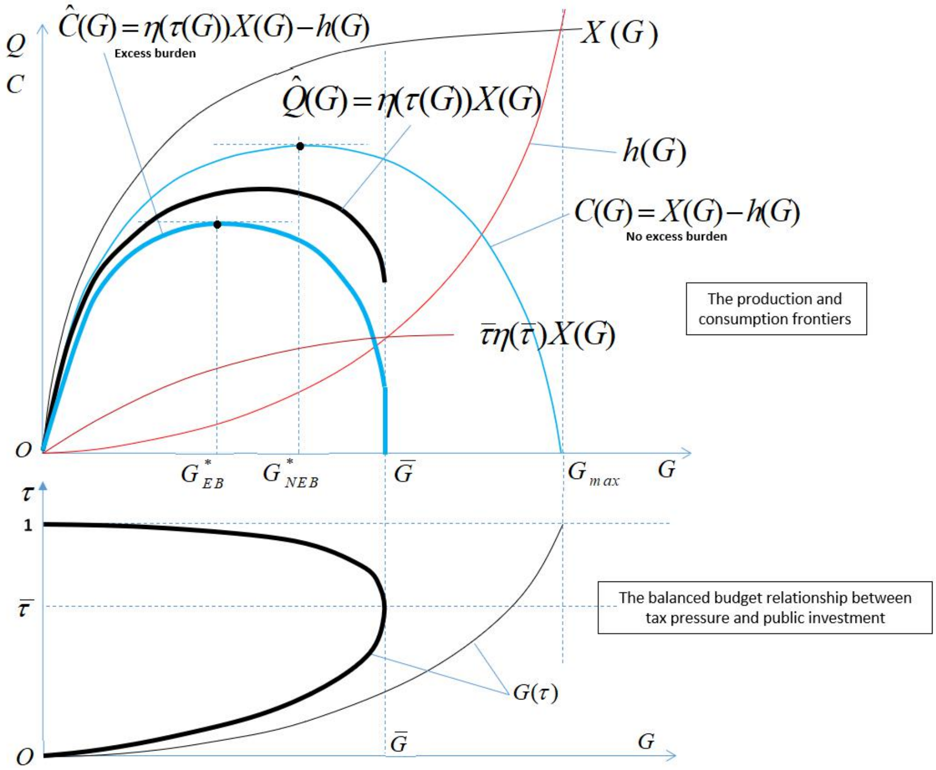


**Figure 2.** The McGuire-Olson production and consumption frontiers

Assuming a *negligible excess burden* (corresponding to the horizontal black line $\eta(\tau)$ = 1 in **Figure 1**), the balanced-budget production and consumption frontiers of the economy are drawn in **Figure 2** as the *thin black* and *blue curves*, with maximum total consumption at $G^*_{NEB}$.

As is intuitively obvious, assuming a *non-negligible excess burden* such as the one represented by the red curves in Figure 1, the balanced-budget production and consumption frontiers of the economy shrink towards the origin. They become the *thick black* and *blue curves* drawn in the figure, with maximum total consumption at $G^*_{EB}$. Under excess burden, there is a *maximum sustainable* level $\bar{G} < G_{\max}$ of public investment because there is a *threshold* (*maximal*) *average tax pressure* $\bar{\tau}$ beyond which a further increase in $\tau$ would decrease tax revenues instead of increasing them, due to the output-reducing impact of taxation. The fact is readily represented in Figure 2 by the *dark red tax revenue function* corresponding to $\tau = \bar{\tau}$

$$T(\bullet) = \bar{\tau}\eta(\bar{\tau})X(G) \tag{4}$$

As long as $\tau < \bar{\tau}$, increases in $\tau$ rotate the tax revenue function $T(\bullet) = \tau\eta(\tau)X(G)$ anticlockwise, moving forwards its balanced-budget intersection with the cost curve $h(G)$. But when $\tau > \bar{\tau}$, further increases in $\tau$ rotate the function clockwise, moving the intersection backwards. The existence of such threshold (maximal) tax pressure $\bar{\tau}$ is a consequence of the shape of the coefficient of macro excess burden, and is readily obtained by setting the derivative over of the tax revenue function equal to zero:

$$T(\bullet)'_{\tau} = [\tau\eta(\tau)]'_{\tau} X(G) = 0$$

It is shown graphically in **Figure 3** as the intersection between the two curves $\eta(\tau)$ and $-\dfrac{\eta(\tau)}{\eta'(\tau)}$ where $\eta'(\tau) = -1$ (the intersection lies here on the $\tau$ straight line only because the $\eta(\tau)$ curve is drawn with uniform convexity for purely *geometric convenience*. With non-uniform convexities it would lie elsewhere).

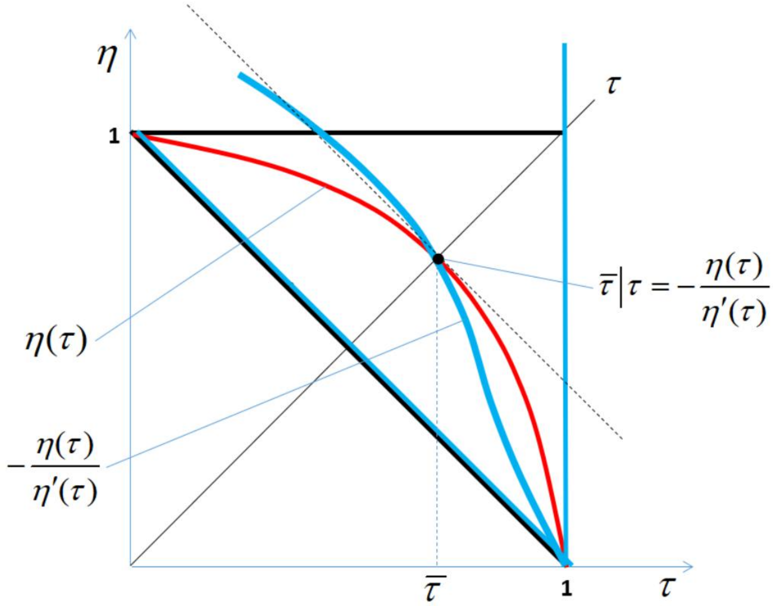


**Figure 3.** The McGuire-Olson maximal tax rate

## 3. The standard dynamics of primary deficit, total deficit, and debt

Before extending the MOE properties into the Solow economy, we review the *well-known dynamic long-run relationship between primary deficit, total deficit, and debt (in ratios to GDP):*

$$(\gamma - \tau) + r\beta = \theta = \frac{\dot{b}}{Q} = \dot{\beta} + g\beta$$

$$\rightarrow \dot{\beta} = \theta\text{-}g\beta$$

where the dot stands for time derivative, and

$\gamma = \dfrac{G}{Q}$: share of public investment in GDP

$\tau = \dfrac{T}{Q}$: average tax pressure (rate) over GDP

$(\gamma \text{ - } \tau)$: share of primary deficit in GDP

*r*: net (of taxes) real interest on debt

*b*: real debt

$\beta = \dfrac{b}{Q}$ : debt/GDP ratio

$\theta = \frac{\dot{b}}{Q}$: share of total real deficit in GDP

$g$: real growth rate (of GDP)

This is a *theory-free relationship*, in the sense that it descends from the definitions of the variables and is therefore independent from the assumed laws governing the underlying economy. However, it does impact on the underlying long-run economy because it affects the *macro aggregate saving-investment propensity*, and this is precisely the reason why we need it. The graphic counterpart of the equation is drawn in **Figure 4**. Suppose the economy starts at time $t = 0$ under a policy of *long-run budget balance* $\gamma = \tau \,\&\, \beta = 0$. Its budget position is at point $P_0$ with $\theta_0 = 0\,(\gamma = \tau)$, and under such a policy it would continue to stay there over time. Now suppose there is a *change of policy*, which for the sake of graphic simplicity we assume to consist in moving to a *full-deficit financing of* $\gamma$, i.e. in reducing $\tau$ to zero while keeping $\gamma$ unchanged. The budget position moves to point $P_1$ with $\theta_1 = \gamma\,(\tau = 0)$. By keeping the *initial primary deficit constant*, this sets in motion an increase in $\beta$ described by:

$$\dot{\beta} = (\gamma - \tau) - (g - \gamma)\beta \tag{5}$$

If $r \geq g$, the growth of $\beta$ would continue indefinitely; if $0 \leq r < g$, it would tend to some finite sstate value such as point $P_3$ if r = 0, and some point to the right of $P_3$ if $r > 0$.

The black line going through $P_1$ and $P_3$ represents the dynamics of $\beta$ described by:

$$\dot{\beta} = \theta_1 - g\beta \tag{6}$$

which holds if the government, after moving from $\theta_0$ to $\theta_1$ at $t = 0$, decided to keep the *total real deficit constant over time*. In order to do so, it must continuously adjust (increase) $\tau$. The figure shows that if $r = g$, keeping a constant total deficit $\theta_1$ and stabilizing $\beta$ requires $\tau$ to rise back to a primary zero balance $\tau = \gamma$. If $0 < r < g$, it requires the primary deficit to be reduced, though not to zero but to some point such as $P_2$, while the initial primary deficit may remain unchanged only if $r = 0$.

Of course, the change of policy from tax to deficit financing could take the opposite form. Instead of reducing $\tau$ with a given $\gamma$, the government could increase $\gamma$ with a given $\tau$, i.e., increase public investment without tax covering. The graphic description of such a deficit policy would remain essentially the same shown in **Figure 4**, but the impact on the macro aggregate saving-investment propensity would change. Michael Hüther and Jens Südekum (2019a; 2019b), two economists who—we believe—are no friends of deficit-spending, have recently argued that if it were politically impracticable, for whatever reasons, to increase taxes, then a deficit-financed increase in public investment might after all be in in the interest of future generations, because the alternative would be to forgo the investment.

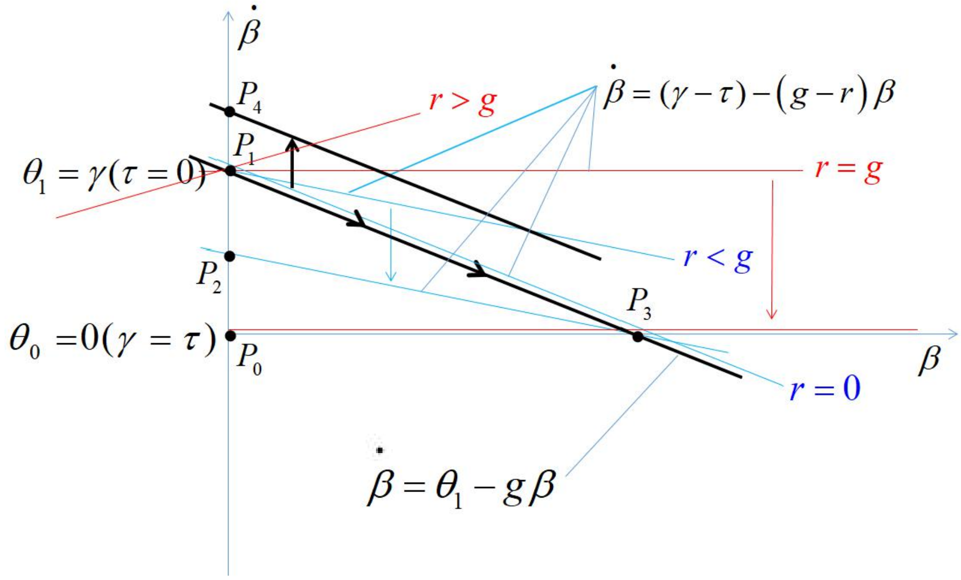


**Figure 4.** Primary deficit and debt dynamics under constancy of the real deficit

For the sake of brevity, in the present article we will not analyze the long-run aspects of this particular deficit-financing policy, but there is a point we want to make concerning its peculiar nature. We believe that in addressing the issue of the impact on intertemporal distribution of a policy change from tax to deficit financing of public expenditures, *the tax reduction approach is conceptually superior to the expenditure increase approach*, in the precise sense that it is simply *the only correct one in general terms*: one should start with some tax-financed public expenditure, and then estimate the change in intertemporal distribution caused by a move to deficit finance through *tax reduction*. To start with some tax-financed public expenditure, and then estimate the change in intertemporal distribution caused by a move to a deficit-financed *increase in public expenditure* is actually a roundabout procedure for doing the same thing done under the other approach: it is equivalent to starting with a tax-financed new public expenditure, and then estimating the distributional impact of a move to deficit finance through tax reduction. However, we are aware that this is a rather abstract argument, and we concede that *the reasons in favor of thinking in terms of a deficit-financed expenditure increase are undisputable*, and worth of further investigation.

## 4. Extending the McGuire-Olson features into the Solow long-run economy

The Solow economy is, like the MOE, a potential output one, but it has the advantage of the long-run perspective, which makes it capable of accommodating the dynamic relationships associated with deficit financing.

In the MOE, public investment increases productivity and taxes decrease it. Both effects are incorporated into its one-period output and consumption frontiers according to Eq. (1) and **Figure 2.** The simplest way to extend them into the Solow long-run economy is to multiply the standard (effective) labor-intensive production function *f*(*k*) by a '*public productivity*' *coefficient* $\varphi(\gamma)$, and by the same MOE macro excess burden coefficient $\eta(\tau)$:

$$q = \eta(\tau)\varphi(\gamma)f(k) \tag{7}$$

where,

$f(k)$ : standard (effective) labor-intensive production function

$q = \dfrac{Q}{E}$ : output per (effective) worker

$k = \dfrac{K}{E}$ : private capital per (effective) worker

$E$: number of (effective) workers

The coefficient $\eta(\tau)$ of macro excess burden has already been defined in Section 2, and drawn in Figure 1. As for the coefficient $\varphi(\gamma)$ of 'public productivity', its natural shape is something like that drawn in **Figure 5**. The MOE assumption of a positive relationship between $G$ and $X$ is extended into the Solow economy as a positive relationship between the share $\gamma$ of public investment in output and the (effective) labor-intensive production function $f(k)$. With zero public investment, the economy collapses, while as public investment increases, productivity increases, reaching some maximum when $\gamma = 1$.

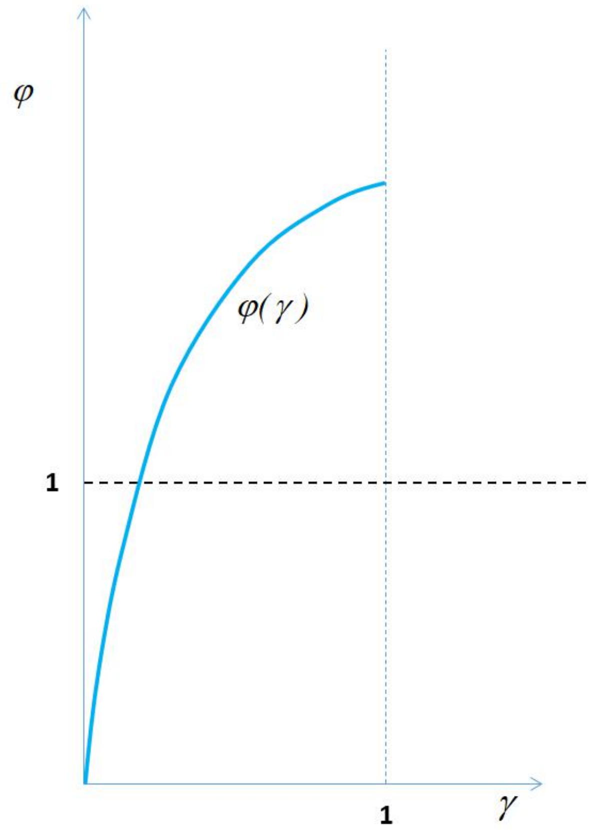


**Figure 5.** A public productivity coefficient

## 5. The benchmark long-run scenario: Balanced budget and the golden rule

Since, in our analysis of the long-run intertemporal impact of deficit financing, we take the Solow production and consumption frontiers under balanced budget as the benchmark reference, we define them here using **Figure 6**, in parallel with those described for the MOE in Figure 2. In particular, we are going to emphasize the relationship between the value of $\gamma$ and the position of the *sstate golden rule* $k_{GR}$ *along the gk line* because of the role that such position plays in *discriminating the alternative intertemporal impacts of deficit financing policies,* described in the next section.

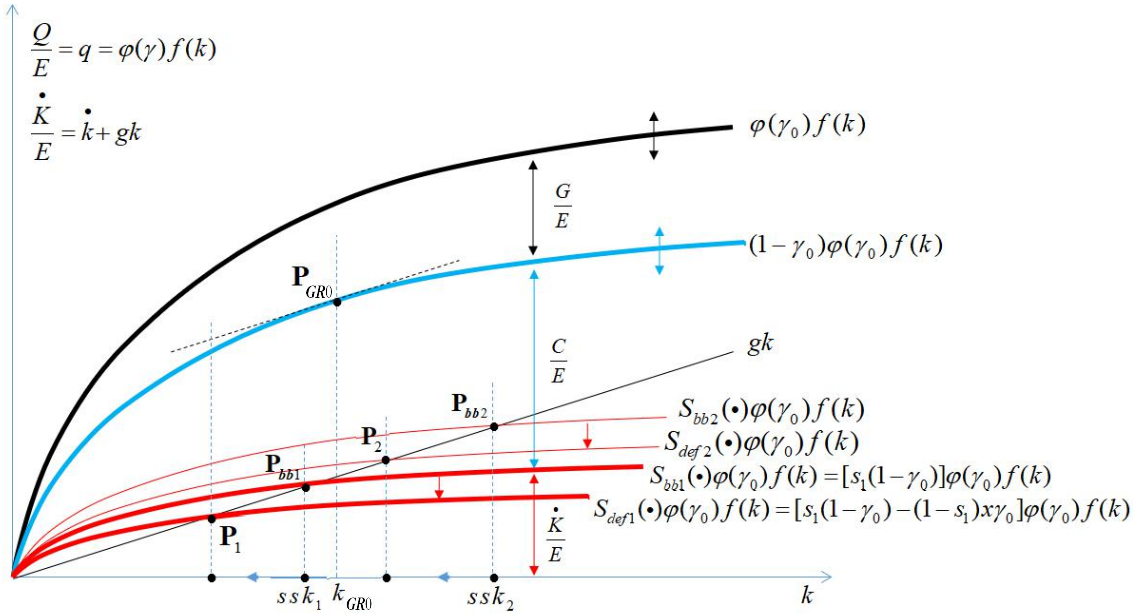


**Figure 6.** Impact on long-run consumption per (effective) worker of a change from tax to deficit financing of public investment: The benchmark case of a negligible excess tax burden

The balanced-budget condition $\gamma = \tau$ yields the output function:

$$q = \eta(\gamma)\varphi(\gamma) f(k) \tag{8}$$

which, in the case of *negligible excess burden* $\eta(\tau) = 1$, reduces to:

$$q = \varphi(\gamma) f(k) \tag{9}$$

In this latter case, as $\gamma$ rises, the production frontier (the thick black line) $\varphi(\gamma) f(k)$ keeps rotating upwards until $\gamma = 1$, while the consumption frontier (the thick blue line)

$$c = (1-\gamma)\varphi(\gamma) f(k) \tag{10}$$

rotates similarly upwards until $\gamma$ reaches some value $\gamma^*$, after which it begins to rotate downwards towards the k-axis.

For every value of $\gamma$, there is a *maximum sstate consumption per* (*effective*) *worker* $ssc_{GR}$ associated to the corresponding *golden rule sstate capital per head* $k_{GR}$. In the figure, the consumption frontier is drawn for some arbitrary $\gamma = \gamma_0$, with a golden rule *sstate* $k_{GR0}$ and a corresponding maximum sstate consumption per head $ssc_{GR0}$. As long as the increase in $\gamma$ rotates the consumption frontier upwards, the golden rule $k_{GR}$ moves to the right and the corresponding maximum consumption per head $ssc_{GR}$ rises, while beyond $\gamma^*$ the consumption frontier begins to rotate downwards, the golden rule $k_{GR}$ to move back leftwards, and the maximum consumption per head $ssc_{GR}$ to decrease. This threshold value $\gamma^*$ and its corresponding $k^*_{GR}$ and maximum $ssc^*_{GR}$ are the *Solow counterparts* of the maximum total consumption drawn for the MOE in Figure 2 at the public expenditure level $G^*_{NEB}$.

In the case of *non-negligible excess burden* the movements of the two frontiers

$$\eta(\gamma)\phi(\gamma)f(k)$$
$$(1-\gamma)\eta(\gamma)\phi(\gamma)f(k)$$

are similar, with two obvious differences: 1) at every $\gamma$ level, both frontiers lie by construction below the previous ones; and 2) there is some $\gamma$ level beyond which also the production frontier begins to rotate downwards. Since at every $\gamma$ the consumption frontier lies below the previous one, also the golden rule $k_{GR}$ lies farther to the left and the maximum consumption per head $ssc_{GR}$ is lower. In particular, under non-negligible excess burden, the values $\gamma^*$, $k^*_{GR}$, and maximum $ssc^*_{GR}$ are the counterparts of $G^*_{EB}$ in Figure 2. We notice for completeness that since in the Solow scenario, public investment is measured as a share in output, there is no equivalent in it of the maximum $\overline{G}$ drawn for the MOE in Figure 2.

## 6. The intertemporal impact of a change from tax to deficit financing of public investment

We trace the impact of a policy change, in the *benchmark case of a negligible excess burden* $\eta(\tau)=1$. Assume some given level $\gamma_0$ of public investment, and consider the state of this economy at some initial time $t = 0$ under the balanced-budget condition $\tau=\gamma_0 \,\&\, \beta=0$.

The *macro aggregate saving-investment propensity* $S_{bb1}(\bullet)$ of this long-run economy is equal to the *macro private saving propensity* $s[(1-\tau)+r\beta]$, minus the GDP *share of total real deficit* $[(\gamma_0-\tau)+r\beta]=\theta$:

$$S_{bb1}(\cdot)=\{s_1[(1-\tau)+r\beta]-[(\gamma_0-\tau)+r\beta]\} \qquad (11)$$
$$=s_1(1-\gamma_0)$$

where,

$s_1$ is the *macro private saving propensity* out of disposable income

$S_{bb1}$ is the *macro aggregate saving-investment propensity*

The intersection of the s*aving-investment function* with the *gk* line yields the sstate capital:

$$\begin{aligned}\dot{k} &= S_{bb1}(\cdot)\varphi(\gamma_0)f(k)-gk\\ &= s_1(1-\gamma_0)\varphi(\gamma_0)f(k)-gk\\ \dot{k} &= 0 \to s_1(1-\gamma_0)\varphi(\gamma_0)f(k)=gk\\ &\to ssk_1\end{aligned} \tag{12}$$

This sstate of the economy is shown in **Figure 6** at point **P***bb1*, with sstate capital per (effective) head $ssk_1$ and sstate consumption per (effective) head $ssc_1$ equal to the vertical distance between the consumption frontier and point **P***bb1*.

Assume now that at time $t$ = 0, the government changes its financing policy from full tax financing to *deficit financing* through the following *tax reduction*:

$$\gamma-\tau = x\gamma \to \tau = (1-x)\gamma \tag{13}$$

where $x \in [0,1]$ is the *deficit-financed share* of $\gamma$. At $t$ = 0, this yields the new *macro aggregate saving-investment propensity*:

$$S_{def1}(\bullet) = s_1(1-\gamma)-(1-s_1)x\gamma \tag{14}$$

which *rotates the macro saving-investment function* clockwise to:

$$S_{def1}(\cdot)\phi(\gamma_0)f(k) = [s_1(1-\gamma)(1-s_1)x\gamma]\phi(\gamma_0)f(k) \tag{15}$$

shifting the sstate capital to the left.

It is true that over time, $\beta$ becomes positive and keeps rising, but if we assume a government policy of keeping $\theta$ constant over time (either by *institutional constraint* or by choice), then we see that also the macro aggregate saving-investment propensity $S_{def1}(\cdot)$ remains unchanged over time through the continuous increase in $\tau$ required to keep the constancy over time of $\theta$ in spite of the increase over time in $\beta$.

It is trivial to check that the relationship between $d\tau$ and $d\beta$, which ensures the maintenance over time of a constant $\theta$, ensures at the same time the constancy of $S_{def1}(\cdot)$.

Start with:

$$\begin{aligned}&\theta = [(\gamma-\tau)+r\beta]\\ &d\theta = \theta'_\tau d\tau + \theta'_\beta d\beta = 0\\ &\to d\tau = rd\beta\end{aligned} \tag{10}$$

and then substitute $d\tau = rd\beta$ into $dS$:

$$\begin{aligned}&S(\bullet) = \{s[1-\tau]+r\beta]-[(\gamma-\pi)+r\beta]\}\\ &\to dS = S'_\tau d\tau + S'_\beta d\beta\end{aligned} \tag{17}$$

$$= (1-s)(d\tau - rd\beta)$$

$$=0$$

The continuous adjustment of the primary deficit ensuring a constant total real deficit over time has already been illustrated graphically in Section 3 using Figure 4.

With this we see that under a constant real deficit, the new lower initial macro aggregate saving-investment propensity remains constant over time, leading the economy to the new *ssk* and *ssc* corresponding to point $\mathbf{P}_1$ in Figure 6.

The downward rotation of the macro saving-investment function determines the intertemporal impact of a move from tax to deficit financing. This is shown graphically in **Figure 7**. The policy change at time $t = 0$ causes an *adverse intertemporal redistribution* from the future to the present: over an initial time interval, the *long-run path of consumption per* (*effective*) *worker* $c_1(t)$ *increases*, relative to the level $ssc_{bb1}$ that would have been maintained under a balanced budget, but then, and for ever after, it *decreases towards the new lower sstate* $ssc_1$.

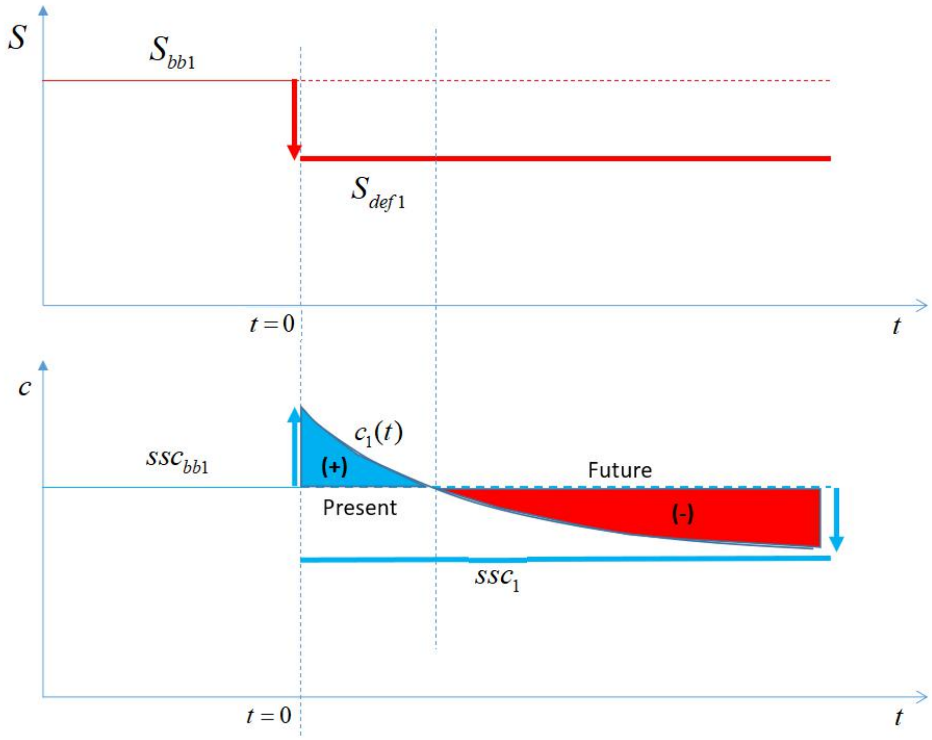


**Figure 7.** Insufficient private investment (dynamic backward inefficiency). Reducing the macro saving-investment propensity when long-run private investment is too low (to the left of the golden rule) increases consumption per (effective) worker today but decreases it tomorrow. (See Figure 6)

However, within the theoretical framework of the Solow long-run economy, this is *not the end of the story*. As is clear from Figure 6, the intertemporal impact of deficit financing depends on the state of the balanced-budget economy relative to *the golden rule concerning private investment*. The adverse intertemporal impact described above holds insofar as the state of the balanced-budget economy lies to the *left of the golden rule*, $ssk_1 < k_{GR0}$ in Figure 6. In other words, since in the long-run, deficit financing decreases the macro aggregate saving-investment propensity, if under balanced budget the economy is *already investing too little*, under deficit financing it will invest even less, and this will cause an adverse intertemporal redistribution.

Suppose the economy's macro aggregate saving-investment propensity under a balanced budget were higher than $S_{bb1}$, say:

$$\begin{aligned} S_{bb2}(\bullet) &= \{s_2[(1-\tau)+r\beta]-[(\gamma_0-\tau)+r\beta]\} \\ &= s_2(1-\gamma_0) \\ &> S_{bb1}(\bullet) \end{aligned} \tag{18}$$

with intersection point $P_{bb2}$ with the *gk* line, and $ssk_2 > k_{GR0}$, to the *right of the golden rule*. The downward rotation of the macro saving-investment function caused by deficit financing would of course still shift leftwards the ssk level (from $\mathbf{P}_{bb2}$ to $\mathbf{P}_2$ in the figure), but since this time the '*starting*' *level of private investment is too high*, the intertemporal impact of deficit financing would no longer be adverse. This is shown graphically in **Figure 8**. The policy change at time $t = 0$ causes the long-run path of consumption per (effective) worker $c_2(t)$ to increase both in the present *and in the future* for ever after, relative to the level $ssc_{bb2}$ that would have been maintained under a balanced budget.

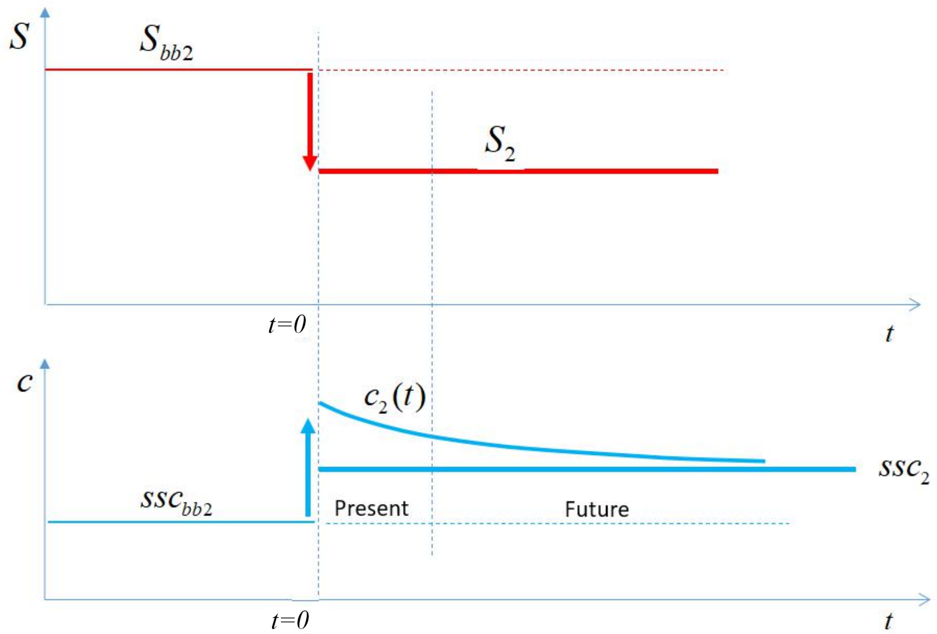


**Figure 8.** Excessive private investment (dynamic forward inefficiency). Reducing the macro saving-investment propensity when long-run private investment is too high (to the right of the golden rule) increases consumption per (effective) worker both today and tomorrow. (See Figure 6)

Two further remarks to complete the picture:

1) The balanced-budget economy's position relative to the golden-rule level of private investment, and thus the *adverse/non-adverse intertemporal impact of deficit financing*, depends on the economy's general conditions concerning the *share of public investment* and the *macro private saving propensity*. In particular we see that, whatever the value of the macro private propensity to

save *s*, as long as the public investment share $\gamma$ is below $\gamma^*$ (the value $\gamma$ beyond which its increase begins to rotate downwards the consumption frontier), the higher (or lower) $\gamma$ is, the further to the right (or left) lies the golden-rule level $k_{GR}$, and thus the more (or less) likely becomes the adverse intertemporal impact of deficit financing. At the same time, whatever the value of $\gamma$, the lower (or higher) *s* is, the further to the left (or right) lies the corresponding *ssk*, and thus again the more (or less) likely becomes the adverse intertemporal impact. In short, the *adverse intertemporal impact* of deficit financing is all the more likely *the higher* $\gamma$ *is for any given s*, and *the lower s is for any given* $\gamma$. In general, considering the enormous impact of government-funded fundamental research on long-term innovation and productivity (Mazzucato, 2015; 2018, Chapter 8), and the relatively low macro private saving propensity in mature economies, the present theoretical framework suggests that one should regard an *adverse intertemporal impact of deficit financing as a fairly likely state of affairs.*

2) Of course, if the government policy were not a constant real deficit, but a *constant primary deficit*, then the macro aggregate saving-investment propensity would tend to remain constant only in the special case of $r = 0$. With $r > 0$, it would continue to decrease over time, causing the *ssk* to shift further leftwards and the *adverse intertemporal impact to increase in size*.

## 7. The case of excess burden

The intertemporal impact of deficit financing in the case of excess burden can be envisaged using **Figure 9**, where the production and consumption frontiers are redrawn according to Eq. (7), taking the excess burden into account:

$$\begin{gathered} q = \eta(\tau)\varphi(\gamma)f(k) \\ c = (1-\gamma)\eta(\tau)\varphi(\gamma)f(k) \end{gathered} \tag{19}$$

which under long-run balanced budget become:

$$\begin{gathered} q = \eta(\gamma)\varphi(\gamma)f(k) \\ c = (1-\gamma)\eta(\gamma)\varphi(\gamma)f(k) \end{gathered} \tag{20}$$

Consider again the state of this economy at some initial time $t = 0$ under the balanced-budget condition $\tau = \gamma_0 \,\&\, \beta = 0$, and suppose the government changes the policy to deficit financing by reducing $\tau$ below $\gamma$. As has already been shown in the preceding section, this reduces the macro aggregate saving-investment propensity from $S_{bb}(\bullet)$ to $S_{def}(\bullet)$. But this time, we are assuming the reduction in $\tau$ to increase the economy's productivity. On the other hand, if the government wants to keep the total real deficit constant, it cannot keep the initial tax reduction unchanged. It must raise back $\tau$, at least to some extent. The result of this reasoning is relatively simple. Suppose that at the current balanced-budget tax pressure $\tau = \gamma_0$, the *excess burden is significant*, and let $\tau_1$ be the *final* (*reduced*) *tax pressure* required to keep the total real deficit constant. If $\tau_1$ turns out to be *close to its initial balanced-budget level* because of a high *r* and/or a low *g*, then there will be *no durable significant increase in productivity*, the new ssk moves to point **P**, and the intertemporal impact of deficit financing remains described by a change in the long-run path of current consumption per (effective) worker $c(t)$ similar to that drawn in Figure 7, albeit in a framework of balanced-budget production and consumption frontiers '*shrunk*' *by the excess burden assumption*. Instead, if $\tau_1$ can remain *significantly lower than its initial balanced-budget level* thanks to a low *r* and/or a high *g*, then the production and consumption frontiers, as well as the macro saving-investment function, rotate upwards, the new *ssk* moves to point $\mathbf{P_1}$, and the long-run path of current consumption per (effective) worker $c(t)$ shifts upwards both in the present and in the future, rising to something like *the* $c_1(t)$ *path drawn in* **Figure 10**, *reducing or possibly eliminating altogether the adverse intertemporal impact of deficit financing.*

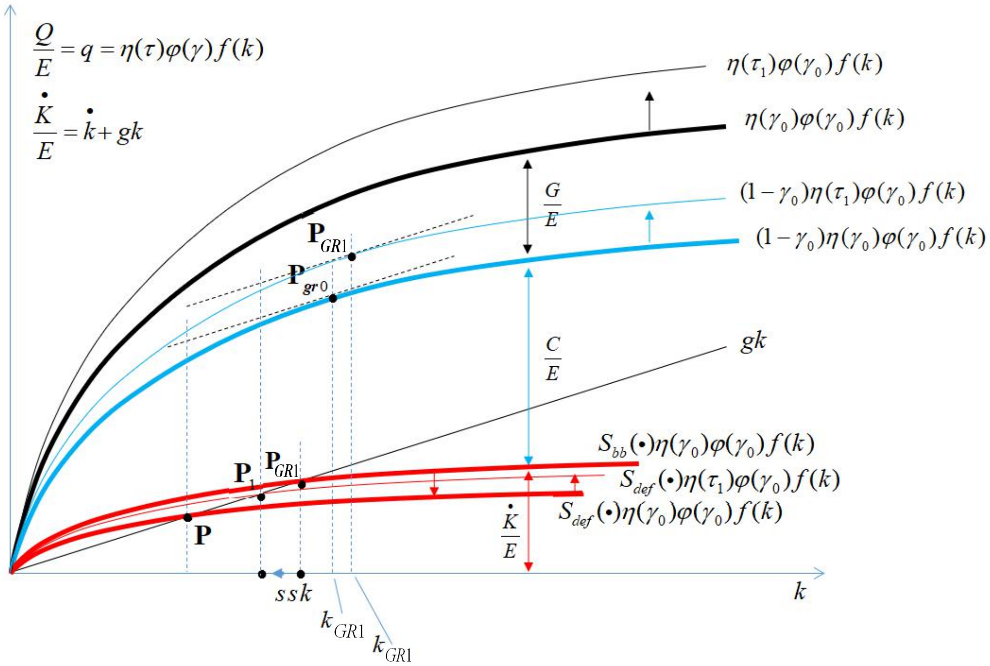


**Figure 9.** The case of excess tax burden

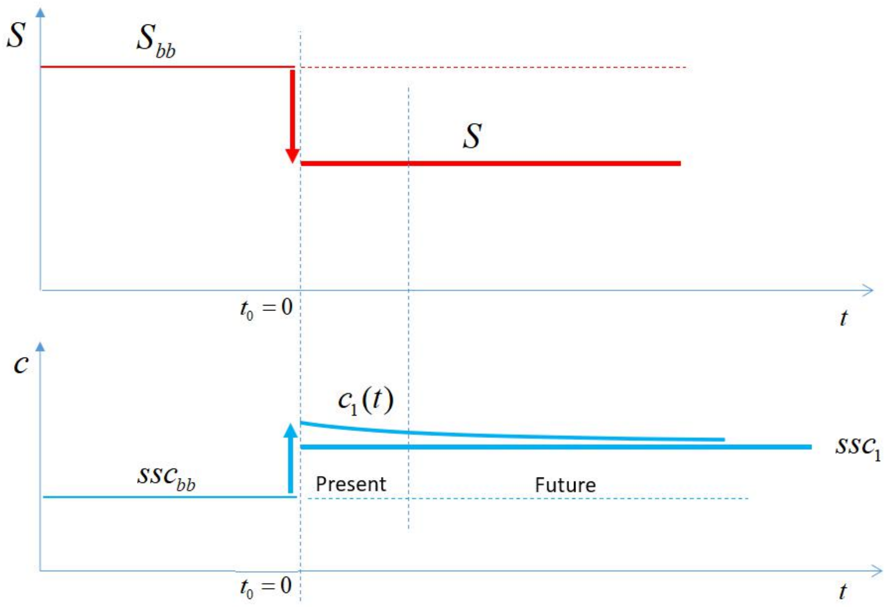


**Figure 10.** The case of excess tax burden. Impact on intertemporal consumption per (effective) worker of a reduction in tax pressure. (See Figure 9)

In short, if the economy is assumed to be—under a *balanced budget*—subject to *a significant degree of productivity loss due to excessive taxation*, and if interest and *growth* conditions allow a significant durable tax reduction to be compatible with the maintenance of a *stable total real deficit*, then the adverse *intertemporal distributional impact of a policy change from tax to deficit financing of public investment might become negligible, or even be substituted by a long-run improvement in consumption per (effective) worker both in the present and in the future*.

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