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Article

Analysis of pedagogical content knowledge for teaching mathematics in early childhood education

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Abstract: This study investigates pedagogical content knowledge (PCK) among teachers teaching mathematics at the preschool level in Colombia, highlighting the importance of integrating mathematical knowledge with innovative and effective pedagogical strategies. Using a mixed exploratory and transactional methodology, the perceptions and practices of 82 teachers were examined, focusing on their understanding of mathematical content, pedagogical skills, and knowledge of children's cognitive development. The findings reveal a significant gap in teachers' understanding of these concepts, indicating a critical need to strengthen PCK among teachers. To this end, training should be provided to enable teachers to foster meaningful and contextualized mathematical learning in preschool students. The study suggests reviewing teacher training curricula and fostering the development of pedagogical strategies that prioritize conceptual understanding and mathematical reasoning. Additionally, it identifies critical areas for improvement and offers concrete recommendations for transforming mathematics teaching in preschool education. To enhance the quality of mathematics education, several measures are proposed: ensuring continued availability of training programs for teachers, encouraging collaboration between educators, adopting constructivist approaches, and helping teachers understand the value of mathematics learning outside the school.

Keywords: preschool education; mathematics teaching; child cognitive development; teaching methodology; mathematical competence

1. Introduction

The educational system in Colombia underwent a significant transformation with the enactment of Law 115 of 1994, which established preschool education as a fundamental part of the national education system. This law promoted a comprehensive view of education, particularly emphasizing the importance of pedagogical content knowledge (PCK) in teaching mathematics to children aged 3 to 6 years. Consequently, the role of the teacher emerged as a key element in children's cognitive and mathematical development, acting as the mediator between knowledge and the innate curiosity of young learners. Law 115 mandates at least one grade, corresponding to grade zero and known as the transition grade, for children aged 5 to 6 (Jaramillo et al., 2011).

With the recognition of preschool education as a catalyst for developing fundamental mathematical competencies, questions arise regarding teachers' preparation and pedagogical knowledge of mathematical content. Various studies, including Programme for International Student Assessment (PISA) reports, have highlighted the correlation between preschool education and superior performance in mathematical competencies (Ortega, 2023; Reséndiz, 2020). Additionally, research conducted by the Ministry of National Education (2017) revealed that children in Colombia who participated in early education programs achieved scores equivalent to approximately one additional year of schooling compared with those who did not attend these programs.

Emphasizing the need for effective coordination between the transition grade and subsequent educational levels is fundamental. Children possess the necessary potential to develop and acquire learning from early childhood. Thus, effective transition will enable them to gain mathematical competence. Competencies are intrinsically linked to the learning process, and in early stages, children build knowledge through their interactions and experiences with others (Slicker et al., 2021).

Interactions with their evolving real-world environments allow children not only to acquire new knowledge but also to reorganize previous knowledge structures and explore new possibilities (Ministry of National Education, 2016). Morales et al. (2010) emphasized that teachers play a crucial role in fostering child development and empowering capacities and talents by outlining appropriate and timely educational routes and practices based on pedagogical knowledge.

In line with this perspective, Escudero-Ávila et al. (2015) concluded that it is essential for teachers to understand both the content they teach and the underlying reasons for selecting the content, as well as to possess a deep understanding of how to approach it in the classroom. Godino (2009) and Vergara and Cofré (2014) distinguished various models to understand the knowledge teachers need for effective teaching, including Shulman's (2005) theoretical construct of PCK.

According to Shulman (2005), PCK is defined as the knowledge required to organize, represent, and adapt content to the interests and abilities of students or certain topics or problems. PCK integrates three dimensions of knowledge disciplinary, didactic, and pedagogical knowledge allowing teachers, to use reflection to guide their didactic actions and decisions in the classroom. This type of knowledge enables teachers to meet the demands of their professional practice by contextualizing their work (Marzabal et al., 2016).

Clements et al. (2004) demonstrated that preschool children are active learners of mathematics and possess informal mathematical knowledge, which should be recognized and encouraged by teachers. Baroody et al. (2006) argued that informal mathematical knowledge develops when children engage in problem-solving processes that often occur during free play. Thus, in early stages, mathematics is learned through playful activities that include representations of the real world, integrated into children's daily experiences, and formalized through symbols and letters (Lee, 2017).

The literature review indicates growing interest in the intersection of mathematical knowledge and children's everyday experiences, highlighting the importance of teachers' pedagogical practices that transcend mathematics instruction alone. However, despite increasing research on this topic (Gómez and Rodríguez, 2014), there is a noticeable gap in the literature regarding the holistic integration of mathematics teaching in early childhood education, specifically concerning the combination of the three dimensions of PCK proposed by Shulman (2005).

Given the relevance and need for robust teacher training that encompasses not only mastery of mathematical content but also a deep understanding of how to effectively teach such content to preschoolers, this study analyzes the current state of PCK in mathematics at the preschool level. The study focuses on three dimensions of PCK: children's mathematical content knowledge (CMCK), knowledge of the principles and theories underpinning the teaching of children's mathematics (CKCMD), and pedagogical knowledge of mathematical practice (PKMP) in the teaching of mathematics in early childhood.

Using a mixed methodology, this research aims to contribute to a deeper understanding of the pedagogical training of mathematics teachers, identifying strengths and opportunities for improving educational policies and enriching pedagogical practices. This methodology is adopted to fully utilize the cognitive and mathematical potential of children in their early academic training.

The results and methodological approaches of this study may serve as a reference for educators who teach mathematics in preschool settings, as well as for coordinators in different public or private institutions in Colombia and other countries. The findings can help them plan educational policies to enhance their teaching dynamics and improve the different types of knowledge involved in teaching mathematics to children.

Help educators improve teaching effectiveness by focusing on understanding children's cognitive and developmental stages, a critical factor informed by psychology and neuroscience. To enhance the quality of mathematics education, several measures are proposed: ensuring continued availability of training programs for teachers, encouraging collaboration between educators, adopting constructivist approaches, and helping teachers understand the value of mathematics learning outside the school.

2. Materials and methods

This study adopted a mixed methodological, exploratory, and cross-sectional approach, employing an integrated sequence of quantitative and qualitative techniques. Focus groups and interviews were conducted, and the Evaluation Scale for Early Childhood Mathematics Education (ESECME), developed by Ortiz-Padilla et al. (2017), was used to address the complexity of PCK in early childhood mathematics teaching.

The study sample comprised 89 teachers affiliated with educational institutions supervised by the Municipal Secretary of Education in Ciénaga, Magdalena. Through a voluntary selection process, a significant sample of 82 teachers was compiled, representing 92.1% of the target population and ensuring representative coverage of teachers' perspectives. A quota sampling strategy was applied to ensure fair representation across educational contexts, resulting in 68% of participants from the public sector and 32% from the private sector. This distribution provides a balanced view of pedagogical practices in different educational settings.

In the qualitative phase, the study emphasized deepening the understanding of the three dimensions of PCK through a participatory approach. To this end, eight focus group activities were organized, designed to foster dialogue among teachers and allow for a detailed exploration of their experiences and perceptions.

Data were collected using the ESECME. This instrument evaluates three critical dimensions of PCK: specific mathematical knowledge for early childhood education, focusing on numerical thinking; the pedagogical knowledge necessary to foster the development of mathematical processes in children, in accordance with the Curricular Guidelines of the Colombian Ministry of Education; and knowledge about child development and pedagogical theories that support effective teaching and promotion of mathematical thinking in children.

The ESECME scale included 68 items and was designed following a rigorous process of validation and adaptation based on previous instruments in the literature, such as the those by Coronata (2014) and Goldrine et al. (2015). This process involved key stages to ensure the instrument's validity and reliability, culminating in a Cronbach's alpha coefficient of 0.88, reflecting excellent internal consistency for scales of this type, as per Cervantes (2005).

The levels are indicated in **Table 1**, along with the corresponding scale parameters.

Table 1. Parameters of the evaluation scale for early childhood mathematics education for assessing teaching performance.

Percentage	Level of Knowledge
Less than 40%	Very Low level
41%–69%	Low Level
70%–80%	Intermediate Level
81%–90%	High Level
91%–100%	Very High level

Source: Ortiz et al. (2017).

The qualitative phase involved the implementation of focus group strategies, a technique that enriched dialogue and allowed for a deeper understanding of teachers' concepts, perceptions, and experiences regarding mathematics teaching at the preschool level. Essential questions guided discussions on the following aspects: a) the relevance and appropriate timing for introducing mathematical concepts in preschool education; b) teachers' personal perceptions of the nature and content of children's mathematics; c) specific mathematical content that teachers consider essential for this education level; d) knowledge and implementation of mathematical processes in the classroom; and e) skills and knowledge that teachers should possess to facilitate mathematical learning in children.

The questions associated with these aspects were crucial to explain the central dimension of pedagogical content knowledge specific to early childhood mathematics education (PCK_CME), which encompasses three fundamental dimensions: CMCK, CKCMD, and PKMP.

To ensure a more in-depth analysis, inductive subcategories and semantic networks were constructed to facilitate the interpretation and understanding of the study phenomenon. The implementation of a concurrence matrix based on the relations between the emerging subcategories enriched the analysis.

The use of specialized software, such as Atlas ti V7 and NVivo version 12, complemented by quantitative analysis in Excel and SPSS version 22, ensured a rigorous review of quantitative and qualitative data.

Finally, methodological rigor was maintained throughout the study, adhering strictly to informed consent protocols and ethical guidelines for information handling. This commitment to research ethics ensures the integrity and reliability of the data collected, supporting the validity of the study's findings and contributions.

3. Results

The results are summarized in three subsections, beginning with an analysis of the sociodemographic profile and educational background of the participants. Next, the results on the levels of PCK and the associations between the dimensions of the PCK_CME are presented. Finally, this section reports finding of the focus groups, providing a holistic view of the subject under study.

3.1. Participants' sociodemographic profile and training

In terms of academic qualifications, 60% of participating teachers held a degree in preschool education, while 36% had degrees in various fields of basic education, including natural sciences, Spanish language, foreign languages, and educational administration. Additionally, 2% had technical training in related areas, and another 2% had undergone training in elementary education.

Regarding age, 60% of the participating teachers were aged between 42 and 53 years, indicating considerable professional maturity. Most teachers were over 40 years old owing to the time taken for undergoing training, responding to calls for applications, and being appointed to work as a teacher. According to a study published by the Banco de la Republica, official teachers in Colombia have an average age of 47.4 years (Bonilla et al., 2018). Meanwhile, 20% of the participating teachers were 24–41 years old, and another 20% belonged to the 54–65 age category, reflecting wide generational diversity.

Almost half of the participants (47%) had extensive experience, with over 22 years in the field. Furthermore, 42% had between 10 and 21 years of experience, while 11% had between 1 and 9 years, demonstrating a broad range of experiences and pedagogical perspectives.

3.2. Levels of pedagogical knowledge in early childhood mathematics education content

Teachers were categorized based on their performance in the ESECME to present the overall results of PCK_CME and its dimensions. The results revealed that 58.5% (n = 48) of the teachers had a low level of knowledge, 32.9% (n = 27) had a medium level, 6.1% (n = 5) had a high level, and 2.4% (n = 2) had a very high level.

Regarding the mathematical content knowledge (MCK) dimension, 75.6% of the teachers were classified as having a very low or low level of MCK, 22% (n = 18) had a medium level, and 2.4% (n = 2) had a high level. The following difficulties were noted among teachers:

- 18.2% of teachers were able to identify errors in temporal correspondence when children counted numbers.
- 29.1% of teachers correctly understood that the notion of number conservation involves maintaining numerical equivalence without relying on visual correspondence.
- When presented with a task to identify the type of mathematical thinking in development, 67.3% of the teachers failed to do so adequately. Additionally, 63.6% believed that children should not be allowed to count by pointing their fingers at or touching objects at the beginning of the counting process.
- 43.6% of teachers thought that the main objective of mathematics education in early childhood is to encourage writing numbers and performing mathematical operations, reflecting limited understanding of the purpose of teaching at this stage.
- 38.2% of teachers believed that specific counting and numbering should be taught before moving toward teaching abstraction.

With regard to teachers' PKMP, as indicated in **Table 2**, the participating teachers lacked knowledge in developing mathematical processes related to problem-solving, reasoning and proof, communication, connections, and representation, showing low or very low levels of performance.

Table 2. Results in the PKMP dimension.

Level of Knowledge	Frequency	%
Very Low Level	52	63.4
Low Level	30	36.6
Total	82	100

Source: Designed by the authors.

When asked whether children should be supported in memorizing numbers and operations during mathematics teaching, 47.3% of the teachers strongly agreed, 25.5% agreed, and 10.9% partially agreed.

Results regarding the dimension of CKCMD indicate that teachers lack understanding of the theoretical foundations of teaching mathematics in early childhood and the necessary aspects of child development affected by the learning. Specifically, 68.3% of the teachers had low or very low understanding, 25.6% had a medium level of understanding, 3.7% had high level, and 2.4% had a very high level.

Analysis of the items in this dimension reveals weaknesses, particularly in teachers' responses to the following items:

- The learning of numbers is a spontaneous process that is unique to each child.
- Learning to identify and count numbers ensures that a child will be able to solve problems requiring these processes.
- From a constructivist perspective, children can build their mathematical knowledge independently, without assistance.
- Children begin their informal learning of numbers and operations only after starting school.

Teachers do not recognize the conceptual development involved in learning to identify and count numbers, which indicates a lack of understanding of the thinking processes that children undergo while learning such processes.

Figure 1 indicates the behavior of the PKC_CME variable and its three dimensions CMCK, PKMP, and CKCMD in relation to preschool teachers' performance based on their level of knowledge.

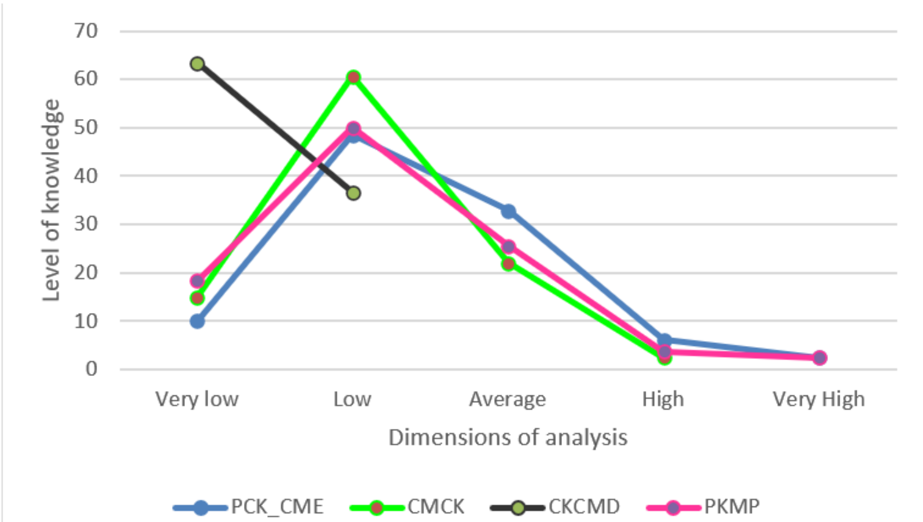

Figure 1. Level of knowledge achieved according to the dimensions of analysis.
Source: Designed by the authors.

The levels of knowledge among preschool teachers regarding both the main variable and its dimensions exhibit a similar pattern, with a predominance of low and medium levels. In the PKMP dimension, teachers show low or very low levels of knowledge, with no teacher demonstrating medium or higher levels of knowledge.

When analyzing the results obtained by teachers in the PKC_CME and correlating them with the sociodemographic variable of the institution's character, it becomes evident that the results show a consistent trend regardless of this variable. **Table 3** indicates that very high level of knowledge was achieved only by teachers from public schools.

Table 3. Level of pedagogical content knowledge for PKC_CME by institution type.

Level of Knowledge	Official %	Private %	Total %
Low Level	36.6	22.0	58.5
Intermediate Level	24.4	8.5	32.9
High Level	4.9	1.2	6.1
Very High Level	2.4	0.0	2.4
Total	68.3	31.7	100.0

Source: Designed by the authors.

Regarding the sociodemographic variable of teachers' initial training, no significant differences in performance were observed at the 5% level. However, among the teachers who obtained the best classifications, five had initial training in preschool education or early childhood education.

In examining the relation between the age of teachers and their performance in the PKC_CME, low performance levels were noted across various age groups. Medium-level performance was slightly more common among teachers aged 24 to 53, although these differences were not statistically significant at the 5% level. Similarly, no differences in average performance were found based on years of experience, as indicated by the Chi-square statistic for independence under the null hypothesis, which posits that there is no association between the variables of interest ($P > 0.05$).

3.3. Decoding pedagogical content knowledge: Qualitative analysis

For each dimension, the classification of the semantic network was analyzed based on the frequency of concepts mentioned by teachers, which were coded and grouped into three density levels according to their frequency.

In the CMCK dimension, concepts with the highest density included "geometry," "magnitudes," "shapes and figures," and "mathematical notions." The second group with lower density comprised "associated with the discipline," "cognitive process," "environment," "mathematics in context," and "enumeration." Finally, the third group with even lower densities included "preconceptions," "numerical series," "subtractions," "additions," "colors," "spatial relationships," "aligned with Basic Learning Rights," "critical thinking," and "numerical understanding." **Figure 2** illustrates the semantic network of CMCK according to density or recorded frequency (In ATLAS.Ti, density is defined as the number of times a code is represented in a document from a hermeneutic perspective), linked from predetermined expressions using the Atlas.ti software and Excel.

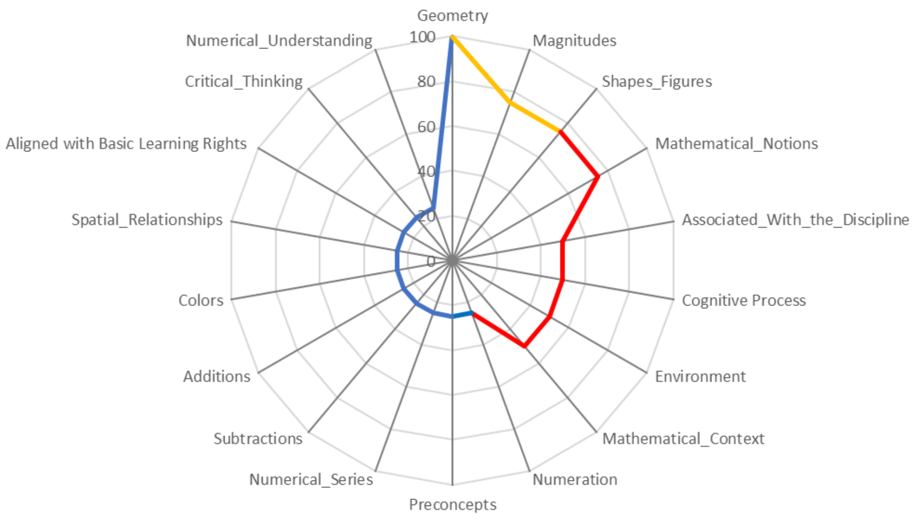

Figure 2. CMCK semantic network of preschool teachers according to density levels.

Source: Interview focus group analysis. Higher density (yellow), medium density (red), low density (blue). Designed by the authors.

The grouping of the semantic network into three density levels indicates that teachers surveyed for CMCK prioritized concepts related to geometric thinking, measurement systems, and mathematical notions. Additionally, there was a notable interest in contextualizing mathematics within the child's environment to foster cognitive processes and numerical thinking. This focus aligned with the learning objectives outlined by the Ministry of National Education and the Basic Learning Rights for the transition grade.

A similar analysis in the dimension of PKMP in early childhood education revealed that concepts with the highest density and association included "real objects," "cognitive processes," "environment," and "playful." The second group comprised "correspondence," "numerical comprehension," "critical thinking," "association," "discipline," "memory process," and "association in context." **Figure 3** illustrates the semantic network of preschool teachers in the PKMP dimension according to density.

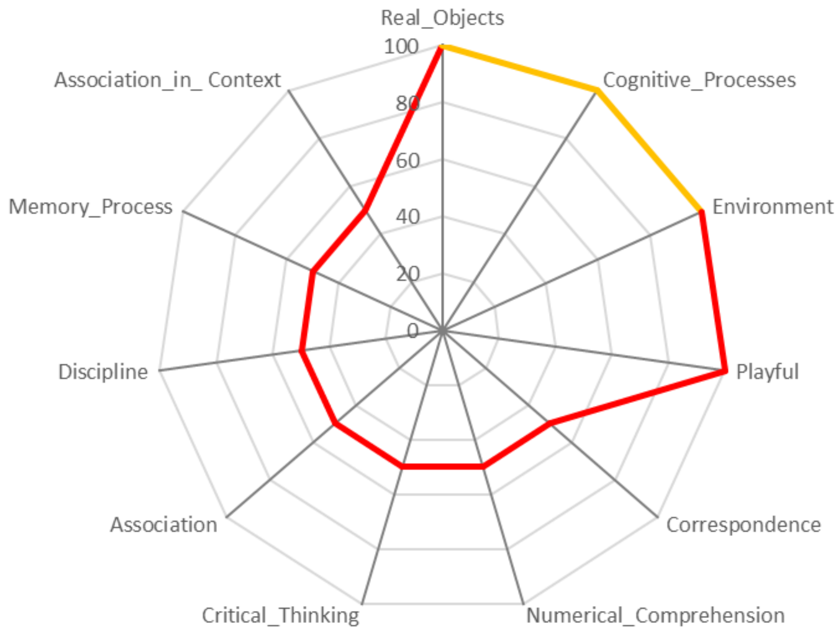

Figure 3. PKMP semantic network of preschool teachers by density levels.

Source: Interview focus group analysis. Higher density (yellow), medium density (red), low density (blue). Designed by the authors.

The semantic network for PKMP was categorized into two density groups. First, teachers emphasized practical approaches to developing mathematical processes, focusing on play and real-life objects from the child's environment to enhance cognitive development. Second, they expressed interest in fostering memorization processes and numerical comprehension within the discipline of mathematics.

Regarding the dimension of CKCMD, the semantic network revealed a first group of concepts that included "mathematical notions," mathematics in context," and "associated with the discipline." The second group, ranked by density, included "formal knowledge," "correspondence," "preconceptions," "enumeration," "association," "association in context," and "aligned with Basic Learning Rights." **Figure 4** depicts the CKCMD semantic network based on these density levels.

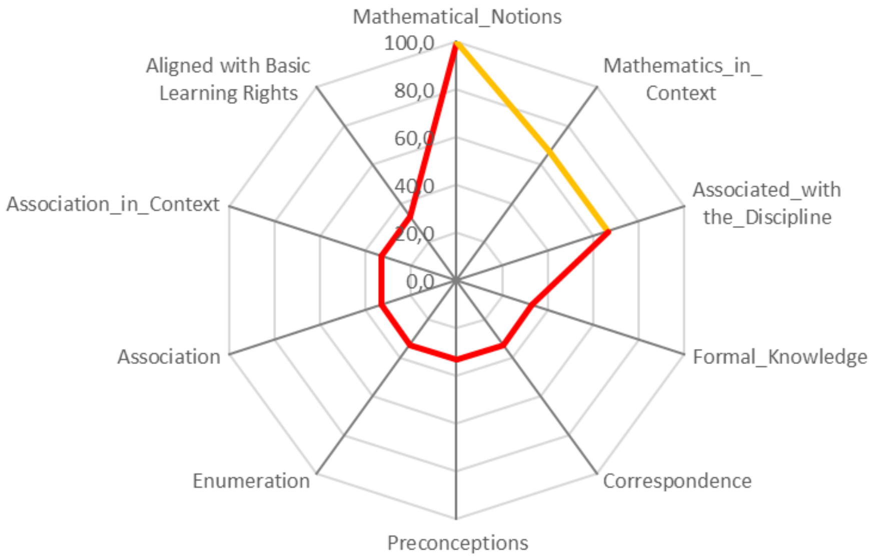

Figure 4. CKCMD semantic network of preschool teachers based on density levels.
Source: Interview focus group analysis. Higher density (yellow), medium density (red), low density (blue). Designed by the authors.

The teachers emphasized that mathematical notions associated with the discipline should be taught in a contextualized manner to facilitate progression toward acquiring formal mathematical knowledge. This approach aligns with constructivist learning theories and the curricular guidelines for teaching mathematics. In the case of enumeration tasks, teachers indicated that they began with students' preconceptions and connected these to children's real-world environments.

Table 4 presents the density of the dimensions in terms of the associated concepts that determine the semantic network. The dimension with the highest density was CMCK, representing 46% of associations, followed by PKMP with 28% associations and CKCMD with 26% associations. Notably, the CMCK category exhibited a higher density than the CKCMD category, indicating a larger number of emerging concepts related to mathematical processes and children's evolutionary recognition and learning.

Table 4. Percentage of density of associated concepts according to dimensions.

Categories	Density (%)
Children's mathematical content knowledge (CMCK)	46%
Pedagogical knowledge of mathematical practice (PKMP)	28%
Knowledge of the principles and theories underpinning the teaching of children's mathematics (CKCMD)	26%

Source: Designed by the authors.

Figure 5 illustrates the matrix of concurrences among the CMCK, PKMP, and CKCMD dimensions. Concurrences represent the simultaneous occurrence of concepts across two or more dimensions. The highest number of overlaps (6) occurred between the CMCK and CKCMD dimensions, particularly in the concepts of "mathematical notions," "preconceptions," "numerical understanding," "associated with the discipline," "enumeration," and "aligned with Basic Learning Rights." Between the CMCK and PKMP dimensions, four concurrences were recorded: "environment," "cognitive process," "mathematics in context," and "critical thinking."

Finally, between the PKMP and CKCMD dimensions, three concurrences were found: “association,” “correspondence,” and “association with context.”

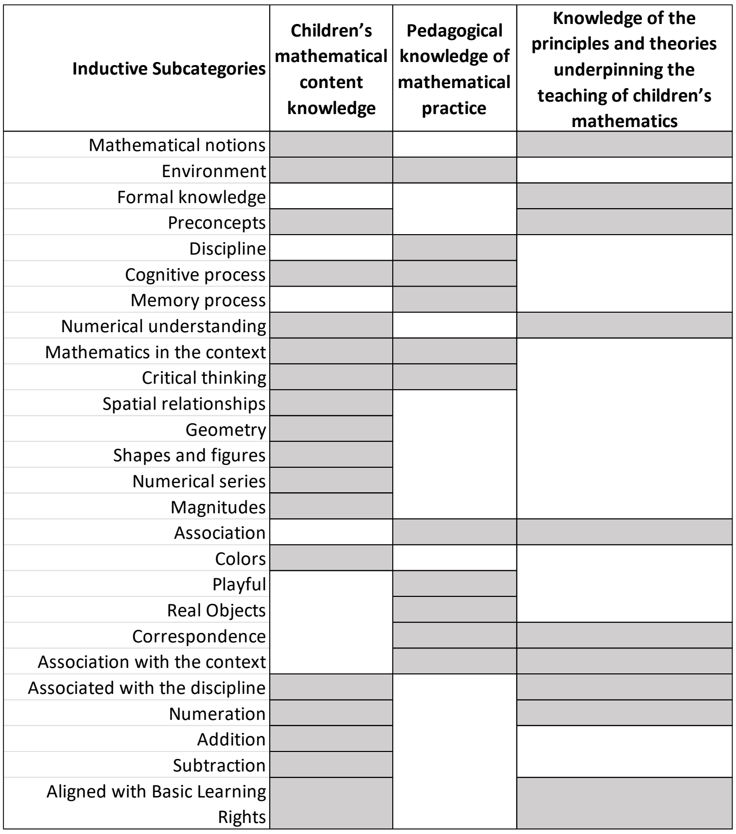

Figure 5. Matrix of concurrences between the parent categories and the inductive subcategories. Shading indicates concurrence.

Source: Analysis of focus group of preschool teachers. Designed by the authors.

Concurrences between the CMCK and CKCMD dimensions were related to the mathematical content and type of numerical thinking, whereas those between CMCK and PKMP emphasized the processes and development stages that children undergo to grasp mathematical concepts. Finally, the concurrences between PKMP and CKCMD revolved around the association of concepts and contextualization for further learning.

Teachers are expected to demonstrate concurrences across three dimensions in the following concepts: formal knowledge, environment, preconceptions, and cognitive processes. These concepts serve as organizing elements that bridge the gap among content, pedagogical process, and children’s learning.

4. Discussion

In terms of performance, teachers exhibit low levels of pedagogical knowledge specific to teaching mathematics in early childhood education, suggesting an urgent need to strengthen both initial and continuous training in this area. This issue has been explored in other areas and levels of training (Rico, 2004), where guidelines suggest incorporating infants themselves as mediators of their own cognitive development, stimulating an opportunity–propensity model (Byrnes et al., 2019).

Regardless of their academic background, age, or years of experience, teachers exhibit limited understanding of mathematical content, processes, and the theories that underpin teaching mathematics in early childhood. According to Simpson and Linder (2014), this problem persists in different contexts where teacher training lacks an adequate approach focused on comprehension, as evidenced by research in contexts such as Venezuela (Lugo et al., 2019; Romero, 2019), in Chile (Friz et al., 2009). This deficiency hampers the development of mathematical skills in young children.

The ESECME not only quantifies the level of teachers' pedagogical knowledge of children's mathematics development but also breaks this knowledge down by dimensions, facilitating detailed interpretation of teachers' performance. The results obtained were classified into performance levels using a pre-established scale, allowing for an accurate assessment of the current state of PCK in preschool mathematics education.

As for CMCK, teachers present low levels of performance, which hinders their ability to help children develop mathematical thinking. This result aligns with the findings of Goldrine et al. (2015), who identified insufficient knowledge among prospective teachers for teaching children's mathematics.

A contrast between theoretical knowledge and practice is evident. The results reveal discrepancies between what teachers know theoretically and what they apply in practice, indicating the need for training strategies to improve the connection between acquired knowledge and its effective implementation in the classroom.

In the qualitative analysis, a discourse more coherent with what is expected according to the guidelines of the Ministry of National Education regarding content knowledge is observed in teachers' responses; however, this coherence contradicts the results of the quantitative analysis. Thus, there is a gap between what teachers articulate in their discourse and what they express in their actual teaching practices.

A conceptual association can be observed in the concurrence of terms used by preschool teachers between the dimensions of CMCK and PKMP without visualizing differences between the "what" and "how" of teaching. This situation may be justified by the interaction among these elements (Ball et al., 2008).

In terms of CKCMD, many teachers believe that children spontaneously acquire numerical knowledge, reflecting limited interpretation of constructivism. In addition, children's informal mathematical thinking is not adequately valued as an essential pillar in the construction of formal thinking. Instead, the idea prevails that it is the school that provides the first significant experiences for the development of this knowledge.

We consider the following measures appropriate both to improve teachers' understanding and educational outcomes. Permanent teacher qualification spaces should be generated to allow teachers to develop aspects of the scale in which they lack proficiency, such as knowledge about cognitive and maturational development in childhood according to updated findings in psychology and neuroscience. Additionally, spaces should be created to promote exchange between teachers from different institutions, where they can learn from each other's experiences and adopt new classroom practices. It is also important to promote more constructivist approaches in teacher training in mathematics. Finally, teachers should understand the value of children's learning of mathematics outside the school.

The insufficient integration of elements that allow teachers to build, apply, and revise their concepts and strategies within the teaching–learning process is highlighted in his study. This deficiency reveals the importance of promoting greater understanding and practical skills among teachers to improve teaching dynamics, an issue that is even more challenging today given the gaps resulting from remote learning encouraged during the pandemic (Ballesteros and Gómez, 2022).

It is important to emphasize the need for continuous and ongoing training for teachers, particularly the acquisition of PKC_CME and the development of an effective pedagogical approach that promotes critical mathematical thinking and problem-solving from an early age.

The research suggests the need for a more integrated vision of the PCK, which includes a deep understanding of the child's developmental stage, their characteristics, and how pedagogy can motivate and facilitate initial access to the world of mathematics.

Similarly, the importance of the role of new technologies in mathematics teaching should be highlighted. Alsina (2020) states that the STEAM approach facilitates the learning of mathematics by integrating it in an interdisciplinary context; thus, mathematical content is no longer presented as isolated content and becomes an essential part of meaningful processes that combine theory and practice. This approach is enhanced with technological tools, which transform abstract concepts into tangible experiences, helping children understand mathematical content. In addition, it encourages active learning, curiosity, and problem-solving.

5. Conclusion

The article highlights the critical importance of improving pedagogical training and practice in early childhood mathematics, suggesting that a more integrated and continuous approach to teacher training can significantly contribute to children's mathematical development from an early age.

The research provides concrete evidence of the need to improve teacher training strategies, which is especially relevant in the context of current educational challenges, such as the transition to digital education and the need for more personalized teaching methods.

The findings of this study should be applied to develop more effective teacher training programs and positively influence student outcomes in early mathematics education.

This study contributes to innovation and research applicability in the current educational context by adopting a novel approach in examining PCK and other knowledge areas of early childhood mathematics teachers. This research addresses an underexplored area. In terms of educational theory and pedagogical practice, the article deepens the understanding of PCK, CMCK, and PKMP and their impact on the teaching of mathematics at early stages.

This theoretical contribution is complemented by clear practical implications, proposing specific recommendations to improve pedagogical practices, thus connecting research and application in real educational contexts.

The study not only identifies areas for improvement in teacher training but also proposes a way forward, suggesting future research and possible educational interventions. It is recommended that a call to action be developed for researchers, educators, and education policymakers, emphasizing the importance of adopting a more holistic approach to teacher education that considers the triangulation of PCK, CMCK, and PKMP as essential components.

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