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Fuzzy Programming Approach to a Multi-Objective Fuzzy Stochastic Routing and Siting Hazardous Wastes

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ABSTRACT

The aim of the research article is not only to propose a solution procedure to solve multi-objective fuzzy stochastic programming problem by using genetic-algorithm-based fuzzy programming method, but also to apply the computational techniques for transportation of the hazardous waste materials. In this article, routing and siting problems for nuclear hazardous waste material are studied and solved. The amount of waste materials generated in the nuclear reactors follows normal distribution. The two considered objective functions are about route selection which includes minimum travel time and minimum number of houses along the way, taking the safety measures into consideration. A multi-objective fuzzy stochastic mathematical model is formulated with the above mentioned objective functions and the route selection as the constraints. The proposed solution procedure is illustrated by a numerical example and a case study.

Keywords: Fuzzy Stochastic Programming; Multi-Objective Programming; Fuzzy Programming; Genetic Algorithm

1. Introduction

Hazardous materials and wastes have created a major problem throughout the world. The planning and design of hazardous waste management system involves allocation of hazardous wastes, selection of treatment and disposal facilities, waste residues from generator to the treatment in disposal sites, and selection of the transportation routes. The shipment, dumping of hazardous materials has emerged as a critical problem for decision makers. Some of work done on it can be found in (Ardjmand *et al*.[1,2], Karadimas *et al*.[3], Samanlioglu[4], Current and Ratick[5], and Giannikos[6]). Another issue associated with transporting the hazardous materials is siting problem which may increase the transportation cost if it wasn't considered. The risk factors and the cost associated with transporting the waste materials from source to destination is of great concern for the all the countries which can be seen in (List and Mirchandani[7,8], Nema and Gupta[9], and Gomez *et al*.[10]).

These problems are handled with different planning and objectives to meet the common goal of safety and minimization of cost and time. Different methods and approaches are being used and studied to solve the problem in order to make better and accurate decision (Warmerdam and Jacobs[11], Liu[12], Chang and Davila[13]). The problem of hazardous wastes treatment demands not only optimal solution but also precise and accurate solution. One of such approach is fuzzy programming approach which is used in various fields to obtain an optimal solution. Some of the applications where the fuzzy programming approaches are used can be found in (Bit *et al*.[14], Hulsurkar *et al*.[15], Kumar *et al*.[16], Liu and Kao[17]).

In order to have optimal precise solution, the impreciseness and uncertainty which exist in various parameters need to be handled. This can be done using both the fuzzy theory Zadeh[18] and stochastic programming Charnes and Cooper[19]. The idea on the fuzz-ifying approach to multi-objective stochastic programming problem were developed by Mohan and Nguyen[20]. Recent developments in fuzzy stochastic problem can be found in (Acharya and Biswal[21], Sakawa *et al.*[22], Wang and Watada[23], Mousavi *et al.*[24], Sakawa and Matsui[25], Aiche *et al.*[26], Acharya *et al.*[27,28], Li *et al.*[29]).

Genetic Algorithm (GA) are based on the concept of the biological process of natural selection, and developed by Holland[30]. Liu[12] showed that the existing chance constrained programming models for fuzzy decision systems are essentially maximin model. Also, he analysed that fuzzy simulation based genetic algorithm was suitable for the minimax model. Liu and Iwamura[31] formulated fuzzy simulation based genetic algorithm for solving chance constrained programming models with fuzzy decision. A few researches have been done to handle fuzzy and stochastic using GA (Jana and Biswal[32,33], Dutta *et al.*[34,35]).

The article is organized as follows. Following the introduction, basic preliminaries are presented in Section 2. The mathematical model of multi-objective fuzzy stochastic problem is presented describing the alpha-cuts in Section 3. GA based fuzzy programming approach procedure is provided in Section 4. Solution procedure is presented in Section 5. A numerical example and a case study are provided in support of the proposed method in Section 6. Finally, conclusion is provided in Section 7.

2. Basic preliminaries

Definition 2.1

A fuzzy number $\widetilde{A}$ is a convex normalized fuzzy set $\widetilde{A}$ of the real line $\Re$, with membership function $\widetilde{A}$: $\Re \rightarrow [0,1]$, satisfying the following conditions:

(1) There exist unique interval J such that $\mu_A(x) = 1: x \in J$

(2) The membership function μ_A is piecewise continuous.

Definition 2.2

A fuzzy number $\widetilde{B} = (B^{(p)}, B^{(m)}, B^{(o)})$ is said to be triangular if its membership function is strictly increasing in the interval $(B^{(p)}, B^{(m)})$ and strictly decreasing in $(B^{(m)}, B^{(o)})$ and $\mu_B(B^{(m)}) = 1$, where B(m) is core, $(B^{(m)} - B^{(p)})$ is left spread and $(B^{(o)} - B^{(m)})$ is right spread of the fuzzy number $\widetilde{B}$[36].

Definition 2.3

α cut of the fuzzy number $\widetilde{A}$ is the set $x|\ \mu_A(x) \geq \alpha$ for $0 \leq \alpha < 1$ and denoted by $\widetilde{A}[\alpha]$[37].

Definition 2.4

Let $\widetilde{A} = (A^{(p)}, A^{(m)}, A^{(o)})$ and $\widetilde{B} = (B^{(p)}, B^{(m)}; B^{(o)})$ be two fuzzy numbers with α cuts $\widetilde{A}[\alpha]=[A_*, A^*]$ and $\widetilde{B}[\alpha]=[B_*, B^*]$ respectively, then $\widetilde{A} \preccurlyeq \widetilde{B}$ if and only if $A_* \leq B_*$[38].

Definition 2.5

A fuzzy random variable is a random variable whose parameter is fuzzy number. Let $\widetilde{X}$ be continuous random variable with fuzzy parameter $\widetilde{\theta}$ and $\widetilde{P}$ as fuzzy probability, then $\widetilde{X}$ is said to be continuous fuzzy random variable with density function $f(x, \theta)$, $\tilde{P}(\widetilde{X} \leq x) = \beta$, where $0 \leq \beta \leq 1$; $\tilde{\beta} = (\beta^{(p)}, \beta^{(m)}, \beta^{(o)})$, $\beta^{(o)} \geq 0$ and $\beta^{(p)} \geq 0$[39].

Definition 2.6

Let E = [c, d] be an event. Then the probability of the event E of continuous fuzzy random variable $\widetilde{X}$ is a fuzzy number whose α cut is[39]

$$\widetilde{P}[c \leq \widetilde{X} \leq d] =$$
$$[\min\{\textstyle\int_a^b f(x,\theta)dx | \theta \in \widetilde{\theta}[\alpha];\ \int_{-\infty}^{\infty} f(x,\theta) = 1\}$$
$$\max\{\textstyle\int_a^b f(x,\theta)dx | \theta \in \widetilde{\theta}[\alpha];\ \int_{-\infty}^{\infty} f(x,\theta) = 1\}]$$
$$= [\beta_*[\alpha], \beta^*[\alpha]]$$

Bounded Random Number (BRN): For any type of C compiler, the subfunction of generating pseudo random number has been given in the C library as: include < stdlib.h > and rand function rand () which produces an integer between 0 and RAND MAX, where RAND MAX is in # stdlib as $(2^{15} - 1)$. Therefore, a random number on an interval [0, 1] can be generated as:

Step 1: m = rand ()

Step 2: m $\leftarrow$ (m = RAND MAX)

Fuzzy Normal Distribution: A random variables has a fuzzy normal distribution if its probability density function (pdf) is given by:

$$f(s) = \frac{1}{\widetilde{\sigma}\sqrt{2\pi}} e^{\{-\frac{(s-\widetilde{\mu})^2}{2\widetilde{\sigma}^2}\}}, -\infty < s < \infty$$

Denote the pdf as $\widetilde{FNL}(\widetilde{\mu}, \widetilde{\sigma}^2)$, where $\widetilde{\mu}$ is the mean and $\widetilde{\sigma}^2$ is the variance

Step 1: Generate n_1 and n_2 from BRN (0,1)

Step 2: $m = [-2 \ln (n_1)]^{0.5} \operatorname{Sin}(2\pi n_2)$

Step 3: Return ($\tilde{\mu}$ + $\tilde{\sigma}$ m)

Fuzzy Exponential Distribution: A random variables has a fuzzy exponential distribution if it probability density function (pdf) is defined as

$$f(s) = \begin{cases} \frac{1}{\tilde{\beta}} e^{-\frac{s}{\tilde{\beta}}}, \text{if } s > 0 \\ 0, \quad otherwise \end{cases}$$

Denote the pdf as $\widetilde{\text{FEXP}}\,(\tilde{\beta})$, where $\tilde{\beta}$ is the mean. It can be generated as follows

Step 1: Generates from BRN (0,1)

Step 2: Return $-\tilde{\beta}\ \ln(s)$

Fuzzy Weibull Distribution: A random variables has a fuzzy Weibull distribution if its probability density function (pdf) is defined as

$$f(s) = \begin{cases} \frac{\tilde{\beta}}{\tilde{\gamma}^{\tilde{\beta}}}\, s^{\tilde{\beta}-1}\, e^{(\frac{\tilde{s}}{\tilde{\gamma}})^{\tilde{\beta}}}, \text{if } s > 0 \\ 0, \ otherwise \end{cases}$$

Denote the pdf as $\widetilde{FW}\,(\tilde{\beta}\,,\tilde{\gamma})$, where $\tilde{\beta}$ is the shape parameter and $\tilde{\gamma}$ is the scale parameter, and $\tilde{\beta}\ > 0$; $\tilde{\gamma}\ > 0$. It can be generated as follows

Step 1: Generates from EXP (1)

Step 2: Return $\tilde{\gamma}\ s^{\frac{1}{\tilde{\beta}}}$.

3. Mathematical model of multi-objective fuzzy stochastic programming problem

The mathematical programming model for multi-objective fuzzy stochastic programming (MOFSP) problem is expressed as

$$\min Z_k = \sum_{j=1}^{n} C_j^k\, x_j\ , \ k = 1,2, \ldots ..K \qquad (3.1)$$

Subject to

$$\tilde{P}\left(\sum_{j=1}^{n} a_{ij}\, x_j \preccurlyeq \tilde{b}_i\right) \succcurlyeq \left(1 - \tilde{\beta}_i\right), i = 1,2, \ldots .,m \qquad (3.2)$$

$$x_j \ \geq 0\,, \forall\, j \qquad (3.3)$$

where β_i is the fuzzy number and $C_j^k \in \Re\,, \forall\, j,k$

Depending on the different distribution of the random variables, following cases are considered in this article.

Case 1: Let $b_i\,, i = 1,2, \ldots .,m$ are independent FRVs distributed normally. Let the FRVs b_i be denoted as $\tilde{b}_i$.

The α-cut of the probabilistic constrained defined above can be expressed as:

$$\tilde{P}\left(\sum_{j=1}^{n} a_{ij}\, x_j \preccurlyeq \tilde{b}_i\right)[\alpha], i = 1,2, \ldots .,m \qquad (3.4)$$

$$= \{P(A_i \ \leq\ b_i) | b_i \in\ \tilde{b}_i[\alpha]\}, where\ A_i\ = \sum_{j=1}^{n} a_{ij}\, x_j \qquad (3.5)$$

Using fuzzy inequality, the α-cut of the fuzzy constraints (3.4 - 3.5) is expressed as:

$$\tilde{P}\left(\sum_{j=1}^{n} a_{ij}\, x_j \preccurlyeq \tilde{b}_i\right)[\alpha] \succcurlyeq (1 - \ \beta_i[\alpha]) = \{P(A_i\ \leq\ b_{i*})\ \geq (1 - \beta_i^*)\} \qquad (3.6)$$

where

$$[b_{i*}[\alpha], b_i^*[\alpha]] \in\ b_i[\alpha]\ and\ [\beta_{i*}[\alpha], \beta_i^*[\alpha]] \in\ \beta_i[\alpha]$$

Case 2: Let $b_i\,, i = 1,2, \ldots .,m$ are independent FRVs distributed Weibull. Let the FRVs b_i be denoted as $\tilde{b}_i$.

The α-cut of the probabilistic constrained defined above can be expressed as:

$$\tilde{P}\left(\sum_{j=1}^{n} a_{ij}\, x_j \preccurlyeq \tilde{b}_i\right)[\alpha], i = 1,2, \ldots .,m \qquad (3.7)$$

$$= \{P(A_i \ \leq\ b_i) | b_i \in\ \tilde{b}_i[\alpha]\}, where\ A_i = \sum_{j=1}^{n} a_{ij}\, x_j \qquad (3.8)$$

Using fuzzy inequality, the α-cut of the fuzzy constraints (3.7 - 3.8) is expressed as:

$$\tilde{P}\left(\sum_{j=1}^{n} a_{ij}\, x_j \preccurlyeq \tilde{b}_i\right)[\alpha] \succcurlyeq (1 - \ \beta_i[\alpha]) \qquad (3.9)$$

$$= \{P(A_i\ \leq\ b_{i*}) \geq (1 - \beta_i^*)\} \qquad (3.10)$$

where

$$[b_{i*}[\alpha], b_i^*[\alpha]] \in\ b_i[\alpha]\ and\ [\beta_{i*}[\alpha], \beta_i^*[\alpha]] \in\ \beta_i[\alpha]$$

4. Fuzzy programming approach simulation based GA

The fuzzy programming simulation-based GA is design to solve the fuzzy probabilistic programming problems. A fuzzy stochastic simulation-based GA algorithm is described in Dutta *et al.*[35]. We describe the steps of the algorithm as follows:

Algorithm: Fuzzy Programming GA

P = $(x_{i1}, x_{i2}, \dots \dots, x_{in})$, $n \in N$ - Initial Population
D = $(r_{i1}, r_{i2}, \dots \dots \dots, r_{in})$, $n \in N$ -Distribution Parameter
gen = generation
x_{in}= decision variables $n \in N$
l_i= Lower bound $n \in N$
u_i= Upper bound $n \in N$
C_i = Constraints $n \in N$
x_{in}; x_{jm} = New Child $n, m \in N$
x'_{in}= Mutated Child $n \in N$
x^*_{in} = Best Solution $n \in N$
max-gen = maximum generation

Begin
{
generate D
//Generating Distribution Parameter
init P
// Initializing Population for each objective function
gen ← 0
$l_i \le x_{in} \le u_i$
// Applying bounds separately for each objective function

while $(gen \le max - gen)$ *do*
$[C_i x_{in} - r_{in}]\alpha$
// Applying _ cut separately for each objective function
$x_{in} \leftarrow select\ best$
// Applying Selection separately for each objective function
$x_{in}, x_{jm} \rightarrow x^{\#}_{in}, x^{\#}_{jm}$
//Crossover
$x_{in} \rightarrow x'_{in}$
//Mutation
Evaluate Fi
//Function value of each objective function separately
$\Pr(C_i x_{in} - r_{in})$
// Probability Criteria
if (Probability Criteria is satisfied) then
Elitism
else
goto init
$gen \rightarrow gen + 1$
end if
end while
x^*_{in}
// Ideal Solution for each objective function separately
}
End

The Flow Diagram of Fuzzy Stochastic GA is shown in **Figure 1.**

4.1 Representation and initialization

A population of potential solution $x_1, x_2, \dots \dots, x_n$ is generated and initialized. Each solution is called chromosomes can be represented as $X_p = (x_1, x_2, \dots \dots, x_n)_p$, where p = 1, 2, ……, p_size and p_size being the size of the population.

4.2 Checking constraints by the fuzzy simulation

The constraints of the model are represented as fuzzy probabilistic constraints. Consider the fuzzy probabilistic constraints

$$\tilde{P}\left(\sum_{j=1}^{n} a_{ij}\, x_j \le \tilde{b_\iota}\right)[\alpha] \succcurlyeq (1 - \beta_i[\alpha]),$$
i = 1, 2, ……., m
(4.1)

We defuzzifying the constraints using the α-cut and inequality conditions, so that the constraint reduces to

$$\{P(A_i \le b_{i*}) \succcurlyeq (1 - \beta_i^*)\}, j = 1,2, \dots\dots, n$$

as discuss before (4.2)

The above inequality can be represented by

$$\{P(A_i \le b_{i*}) \ge (1 - \beta_i^*)\}, j = 1,2, \dots\dots, n \quad (4.3)$$

$$= P\big(s_i(a_{ij}x_j, s_1) \le 0\big) \ge (1 - \beta_i^*), j = 1,2, \dots, n \quad (4.4)$$

where $s_1 = b_{i*}, i = 1,2, \dots, m$; $x_j = (x_1, x_2, \dots \dots, x_n)$ is the decision variables.

Let $N_{ij}\ (\le N), i = 1,2, \dots, m$; $j = 1,2, \dots\dots, n$ be the number of times the following relation are satisfied: $s_i(a_{ij}x_j) \le 0,\ i = 1,2, \dots, m$; $j = 1,2, \dots\dots, n$

And $s_j(a_{ij}x_j) \ge 0,\ i = 1,2, \dots, m$; $j = 1,2, \dots\dots, n$, where N is the initial population. Then, by the definition of probability, (3.2) hold, if $\frac{N_{ij}}{N} \ge (1 - \beta_i), i = 1,2 \dots \dots, m$; $j = 1,2, \dots\dots, n$.

4.3 Fitness

The value of objective function is called fitness value if the given constraints are satisfied.

4.4 Selection

Selection operator selects the best chromosomes depending on the fitness values. In this article, we choose Binary Tournament Selection (BTS) for selection process.

4.5 Crossover

Probability of crossover (pc) is assigned, and a random number is generated within [0, 1] for each pair of chromosomes. If the random number is less than (pc), then the chromosomes are selected for mating.

4.6 Mutation

Probability of mutation (pm) is assigned, and a random number is generated within (0, 1) for each chromosome. If the random number is less than (pm), the chromosome will undergo mutation.

4.7 Termination

When number of iterations becomes equal to the generation number as defined, the execution will be stopped.

5. Solution procedure

Using GA based fuzzy programming approach, we find out the ideal solution for each objective function in the following way:

Step 1: Initialize the population P_1(t), P_2(t), …., P_n(t), for each objective function (Z_1(x), Z_2 (x), ………, Z_n(x)) separately, keeping the constraints same for all the objective functions.

Step 2: Apply fuzzy stochastic GA based approach to find the ideal solution for each objective function. (See **Figure 1**)

Step 3: Construct a pay-off matrix containing the ideal solution and functional values.

Step 4: From the constructed pay-off matrix, determine the bounds for m-th objective function $Z_m(x)$; m = 1, 2, ……, n. As there are only two possible choices for the bound, either a lower bound or an upper bound. Find the best lower bound L_m^{**} and worst upper bound U_m^{*} in case an objective function being of minimization type and in case of maximization type finding the best upper bound U_m^{**} and worst lower bound L_m^{*}; $m = 1,2, \dots \dots, n$.

Step 5: Define fuzzy membership function μ_{zm} for the m-th objective function $Z_m(x)$ as:

$$\begin{cases} 1, & if\ Z_m(x) \geq U_m^{**}, \quad m = 1,2, \dots \dots, n \\ \dfrac{Z_m(x) - L_m^{*}}{U_m^{**} - L_m^{*}}, & if\ L_m^{*} < Z_m(x) < U_m^{**} \\ 0, & if\ Z_m(x) \leq L_m^{*} \end{cases} \quad (5.1)$$

Or

$$\begin{cases} 1, & if\ Z_m(x) \geq L_m^{**}, \quad m = 1,2, \dots \dots, n \\ \dfrac{U_m^{*} - Z_m(x)}{U_m^{*} - L_m^{**}}, & if\ L_m^{**} < Z_m(x) < U_m^{*} \\ 0, & if\ Z_m(x) \geq U_m^{**} \end{cases} \quad (5.2)$$

	$\mathbf{Z_1(x)}$	$\mathbf{Z_2(x)}$	…………	$\mathbf{Z_n(x)}$
	Z_{11}	Z_{12}	…………	Z_{1n}
$X_1(x)$ $X_2(x)$	Z_{21}	Z_{22}	……..	Z_{2n}
	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.				
$X_n(x)$	Z_{n1}	Z_{n2}	.	Z_{nn}

Table 1. Pay-off matrix

Step 6: (i) Formulate a single objective mathematical programming problem using maxmin operator with augmented variable λ

max: λ (5.3)

Subject to

$\lambda \leq \mu z_p(x), p = 1,2,3, \ldots \ldots R$ (5.4)

$x \in S$ (5.5)

where S is the feasible region of the MOFSP model.

(ii) Similarly, using min-max operator with augmented variable λ, and formulating a single objective mathematics programming problem as:

min: λ (5.6)

Subject to

$\lambda \geq \mu z_p(x), \mathrm{p} = 1,2,3, \ldots \ldots R$ (5.7)

$x \in S$ (5.8)

where S is the feasible region of the MOFSP model.

Step 7: Finally, solve the single objective mathematical programming problem (5.3) - (5.5) or (5.6) - (5.8) using GA-based fuzzy stochastic approach to obtain the Pareto solutions.

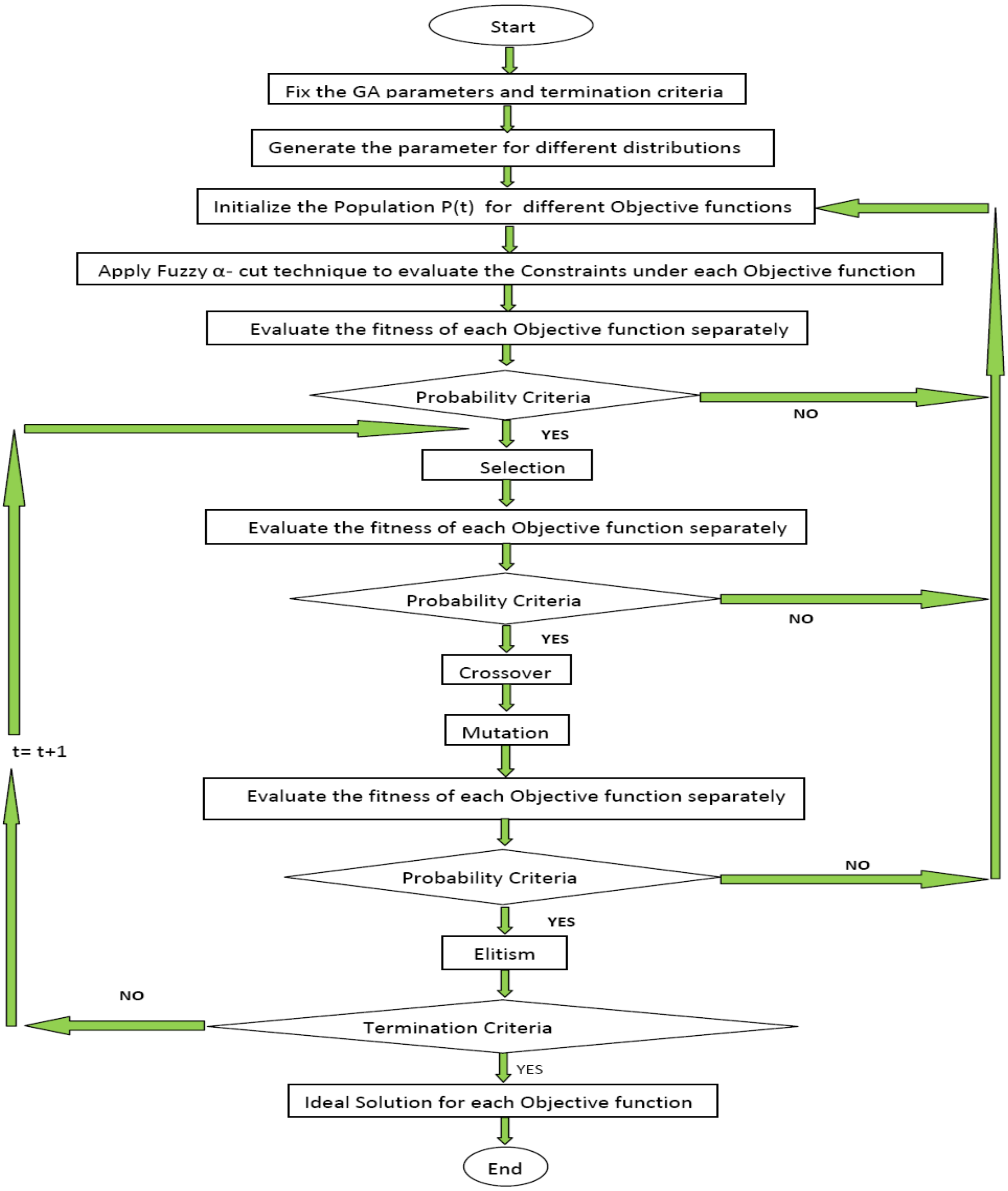

Figure 1. Flow diagram of fuzzy stochastic fuzzy programming approach GA.

6. Numerical example

Consider the following example:

$\max = 5x_1 + 7x_2$ (6.1)

$\max = 10x_1 - 2x_2$ (6.2)

$\max = 9x_1 + 8x_2$ (6.3)

Subject to

$\tilde{P}\left(2x_1 + x_2 \leq \widetilde{b_1}\right) \geqslant \widetilde{0.7}$ (6.4)

$\tilde{P}\left(x_1 + 2x_2 \leq \widetilde{b_2}\right) \geqslant \widetilde{0.60}$ (6.5)

$2x_1 + 2x_2 \leq 4$ (6.6)

$x_1, x_2 > 0$ (6.7)

where $\widetilde{b_1}$ and $\widetilde{b_2}$ follows Weibull distribution with $\widetilde{FW}(\widetilde{\beta_{b_1}}, \widetilde{\gamma_{b_1}}) = \widetilde{FW}(\tilde{4}, \tilde{2})$, $\widetilde{FW}(\widetilde{\beta_{b_2}}, \widetilde{\gamma_{b_2}}) = \widetilde{FW}(\tilde{5}, \tilde{2})$ where $\tilde{2}$ = (2/3/4) ; $\tilde{4}$ = (4/5/6); $\tilde{5}$ = (5/6/7) are fuzzy triangular numbers respectively.

After applying the steps of fuzzy programming approach, we obtain three ideal solutions as $X_1(x)$ = (0.90458; 1.08882); $Z_1(x)$ = 12.1446; $X_2(x)$ = (1.09666; 0.06076); $Z_2(x)$ =10.8451; and, $X_3(x)$ = (1.0709; 0.9253); $Z_3(x)$ = 17.0405.

	Z_1	Z_2	Z_3
X_1	12.1446	6.86816	16.85178
X_2	5.90832	10.8451	1.35602
X_3	11.8361	8.8584	17.0405

Table 2. The pay-off matrix

Use max-min operator with augmented λ the MOFSP reduces to MOFSGP problem

$\min: \lambda$ (6.8)

Subject to

$5x_1 + 7x_2 + \lambda 6.23628 \geq 12.1446$ (6.9)

$10x_1 - 2x_2 + \lambda 3.97694 \geq 10.8451$ (6.10)

$9x_1 + 8x_2 + \lambda 6.68448 \geq 17.0405$ (6.11)

$\tilde{P}\left(2x_1 + x_2 \leq \widetilde{b_1}\right) \geq \widetilde{0.70}$ (6.12)

$\tilde{P}\left(x_1 + 2x_2 \leq \widetilde{b_2}\right) \geq \widetilde{0.60}$ (6.13)

$2x_1 + 2x_2 \leq 4$ (6.14)

$x_1, x_2 > 0$ (6.15)

The proposed GA based fuzzy programming approach is coded in C++ in VB2010 professional. The population size is taken as 100. Optimum solution obtained for different values of probability of crossover, mutation and α = 0.5 over 100 generation is presented.

pc	pm	Optimum Solution X = (x_1, x_2)	Z = (Z_1, Z_2, Z_3)	λ
0.6	0.01	1.0481, 0.4921	8.6852, 9.4968, 13.3697	0.9
0.6	0.03	1.0614, 0.5415	9.0975, 9.531, 13.8846	0.5
0.6	0.05	1.05855, 0.7448	10.5064, 9.0959, 15.4854	0.9
0.6	0.08	1.0481, 0.60515	9.47655, 9.2707, 14.2741	0.9
0.7	0.01	1.06045, 0.8702	11.3937, 8.8641, 16.5056	0.5
0.7	0.03	1.0519, 0.76	10.5795, 8.999, 15.5471	0.5
0.7	0.05	1.0405, 0.88255	11.3804, 8.6399, 16.4249	0.6
0.7	0.08	1.06995, 0.68685	10.1577, 9.3258, 15.1244	0.9
0.8	0.01	1.03385, 0.6099	9.43855, 9.1187, 14.1839	0.8
0.8	0.03	1.06805, 0.4522	8.50565, 9.7761, 13.23	0.8
0.8	0.05	1.05285, 0.84265	11.1628, 8.8432, 16.2169	0.6
0.8	0.08	1.0614, 0.89395	11.5647, 8.8261,	0.6

			16.7042	
0.9	0.01	1.06805, 0.31825	7.568, 10.044, 12.1584	0.8
0.9	0.03	1.06235, 0.79135	10.8512, 9.0408, 15.892	0.5
0.9	0.05	1.06425, 0.3515	7.78175, 9.9395, 12.3903	0.9
0.9	0.08	1.0367, 0.7391	10.3572, 8.8888, 15.2431	0.6

Table 3. $\alpha = 0.5$

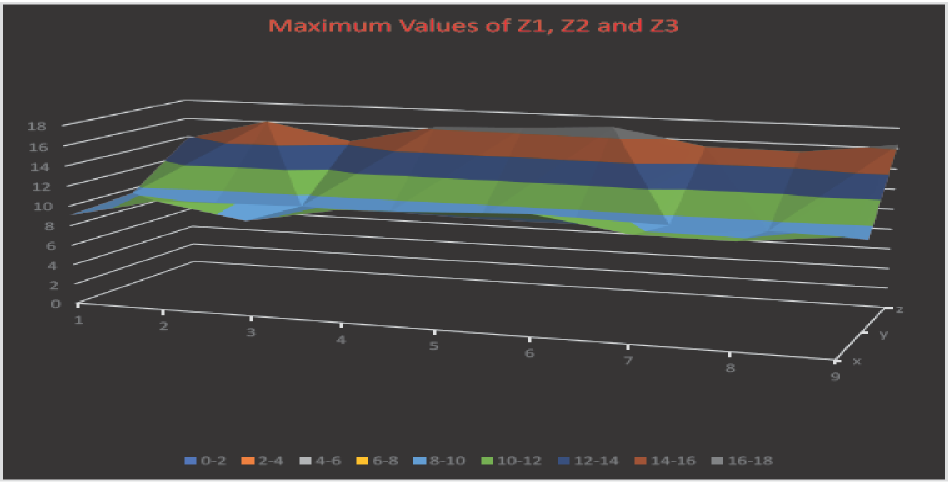

Figure 2. Maximum values of Z_1, Z_2 and Z_3

6.1 Case study

The need for the energy in the world is kept on increasing, and one of the major sources of energy is nuclear power. As a result of using nuclear as a fuel to generate energy, the waste products generated in the reactor needs to be handled and disposed very carefully, as they are hazardous to the environment and mankind. To dispose the nuclear waste in different sites away from the reactor requires shipment across a transportation network, which has become one of the major problems. To transport the nuclear waste material from the reactor to different sites, one has to look into several factors, such as environment issues, accident prone area, time taken to reach the sites, distance covered from reactor to different sites, population along the road or rail through which waste will be transported, and cost of transportation.

This problem was studied by ReVelle *et al.*[40]. Here, in this case study, we have considered minimum time required to reach different sites, and minimum number of houses along the different routes as the objective functions. One has to consider which site is open for disposal among the different available sites, and which route could be most appropriate at the same time. The two objective functions are conflicting in nature as time taken is more where number of houses are less, and vice versa. If the two objectives are not conflicting in nature then it would have become a simple multi-objective problem. The amount of the waste products generated follows fuzzy normal distribution. The pictorial representation and the information for constructing the model are given below.

Source		**Route 1/Site 1**	**Route 1/Site 2**	**Route 2/Site 1**	**Route 2/Site 2**
1	Time(hours)	5	7	13	18
	No. of Houses	900	400	3000	2000
2	Time(hours)	8	9	12	15

	No. of Houses	1800	800	2500	1000
3	Time(hours)	6	18	10	13
	No. of Houses	3000	1700	1500	800

Table 4. Model information

The mathematical model of the case study can be expressed as follows:

$$\text{Min}: Z_1 = 5x_{111} + 7x_{112} + 13x_{121} + 18x_{122} + 8x_{211} + 9x_{212} + 12x_{221} + 15x_{222} + 16x_{311} + 18x_{312} + 10x_{321} + 13x_{322} \quad (6.16)$$

$$Min: Z_2 = 900x_{111} + 400x_{112} + 3000x_{121} + 2000x_{122} + 1800x_{212} + 2500x_{221} + 1000x_{222} + 3000x_{311} + 1700x_{312} + 1500x_{321} + 800x_{322} \quad (6.17)$$

Subject to

$$\tilde{P}\left(x_{111} + x_{112} + x_{121} + x_{122} \preccurlyeq \widetilde{b_1}\right) \succcurlyeq \widetilde{0.75} \quad (6.18)$$

$$\tilde{P}\left(x_{211} + x_{212} + x_{221} + x_{222} \preccurlyeq \widetilde{b_2}\right) \succcurlyeq \widetilde{0.85} \quad (6.19)$$

$$\tilde{P}\left(x_{311} + x_{312} + x_{321} + x_{322} \preccurlyeq \widetilde{b_3}\right) \succcurlyeq \widetilde{0.70} \quad (6.20)$$

$$\textstyle\sum_{j=1}^{2} y_j = 1 \quad (6.21)$$

$$x_{ijk} \leq b_i y_j, i = 1,2,3; j = 1,2; k = 1,2 \quad (6.22)$$

$$y_j = 0 \text{ or } 1, j = 1,2 \quad (6.23)$$

where $b_1, b_2, and\ b_3$ follows normal distribution with $\widetilde{FNL}\left(\widetilde{\mu_{b_1}}, \widetilde{\sigma_{b_1}^2}\right) = \widetilde{FNL}\ (\tilde{3}, \tilde{1})$, $\widetilde{FNL}\left(\widetilde{\mu_{b_2}}, \widetilde{\sigma_{b_2}^2}\right) = \widetilde{FNL}\ (\tilde{2}, \tilde{1})$, $\widetilde{FNL}\left(\widetilde{\mu_{b_3}}, \widetilde{\sigma_{b_3}^2}\right) = \widetilde{FNL}\ (\tilde{4}, \tilde{2})$ respectively, where $\tilde{1} = (1, 2, 3)$, $\tilde{2} = (2, 3, 4)$, $\tilde{3} = (3, 4, 5), \tilde{4} = (4, 5, 6)$ are the fuzzy triangular numbers.

Applying the steps of fuzzy programming approach, we obtain two ideal solutions as $X_1(x)$ =0.7118, 0.2644, 0.1126, 0.1655, 0.5541, 0.5946, 0.2976, 0.2007, 0.1143, 0.1507, 0.1468, 0.1689, $Z_1(x)$ = 34.4201, and $X_2(x)$ = 0.69915, 0.1851, 0.146, 0.25295, 0.2938, 0.1342, 0.3204, 0.37265, 0.2157, 0.1442, 0.1962, 0.217, $Z_2(x)$ = 4817.165.

	$\mathbf{Z_1}$	$\mathbf{Z_2}$
c	34.4201	6914.185
X_2	35.0651	4817.165

Table 5. The pay-off matrix

Using min-max operator with augmented λ the MOFSP reduces to MOFSGP

$$min: \lambda \quad (6.24)$$

Subject to

$$5x_{111} + 7_{112} + 13x_{121} + 18x_{122} + 8x_{211} + 9x_{212} + 12x_{221} + 15x_{222} + 16x_{311} + 18x_{312} + 10x_{321} + 13x_{322} + 0.645\lambda \geq 35.065 \quad (6.25)$$

$$900x_{111} + 400x_{112} + 3000x_{121} + 2000x_{122} + 1800x_{212} + 2500x_{221} + 1000x_{222} + 3000x_{311} + 1700x_{312} + 1500x_{321} + 800x_{322} + 2097\lambda\ 6914.185 \quad (6.26)$$

$$\tilde{P}\left(x_{111} + x_{112} + x_{121} + x_{122} \preccurlyeq \widetilde{b_1}\right) \succcurlyeq \widetilde{0.75} \quad (6.27)$$

$$\tilde{P}\left(x_{211} + x_{212} + x_{221} + x_{222} \preccurlyeq \widetilde{b_2}\right) \succcurlyeq \widetilde{0.85} \quad (6.28)$$

$$\tilde{P}\left(x_{311} + x_{312} + x_{321} + x_{322} \preccurlyeq \widetilde{b_3}\right) \succcurlyeq \widetilde{0.70} \quad (6.29)$$

$$\textstyle\sum_{j=1}^{2} y_j = 1 \quad (6.30)$$

$$x_{ijk} \leq b_i y_j, i = 1,2,3; j = 1,2; k = 1,2 \quad (6.31)$$

$$y_j = 0 \text{ or } 1, j = 1,2 \quad (6.32)$$

The proposed GA based fuzzy programming approach is coded in C++ in VB2010 professional. The population size is taken as 100. Optimum solution Table (6-14) obtained for different values of probability of crossover (pc), mutation (pm) and α over 100 generations are presented.

pc	pm	Optimum Solution X= $x_1, x_{2,......} x_{12}$	Z= Z_1, Z_2	λ	Sites (1 or 2)
0.6	0.01	1.1725, 0.265, 0.749, 0.2133, 0.5584, 0.1282, 0.2908, 0.3076, 0.1451, 0.5796, 0.5356, 0.6214.	61.2071 8698.27	0.1	0 1 1 0 1 0
0.6	0.03	0.1803, 0.0363, 0.21, 0.1583, 0.1822, 0.208, 0.4912, 0.2266, 0.9943, 0.1825, 0.4113, 0.1649	56.9915 7126.55	0.3	1 0 1 0 0 1
0.6	0.05	0.3684, 0.0418, 0.5224, 0.7017, 0.3178, 0.6922, 0.2032, 0.106, 0.1836, 0.1429, 0.7556, 0.3057	51.3969 7230.37	0.2	0 1 0 1 0 1
0.6	0.08	1.1032, 0.4003, 0.2419, 0.1539, 0.6142, 0.1882, 0.5956, 0.2164, 0.3926, 0.155, 0.3123, 0.4091	48.7465 7385.05	0.5	1 0 0 1 0 1
0.7	0.01	1.1274, 0.0979, 0.1154, 0.65, 0.6466, 0.1942, 0.121, 0.2098, 0.4003, 0.2056, 0.1275, 0.2551	45.739 6477.31	0.3	1 0 1 0 1 0
0.7	0.03	0.375, 0.3376, 1.1087, 0.1616, 0.145, 0.2758, 0.4114, 0.2776, 0.2881, 0.3453, 0.3211, 0.2474	51.5553 8040.46	0.5	1 0 1 0 1 0
0.7	0.05	0.1847, 0.0583, 0.3717, 0.5928, 0.2872, 0.5542, 0.325, 0.3748, 0.4641, 0.3464, 0.3035, 0.2265	53.2818 7255.5	0.7	1 0 1 0 1 0
0.7	0.08	0.4421, 0.6061, 0.2419, 0.419, 0.3142, 0.3178, 0.2914, 0.1042, 0.6137, 0.4311, 0.2111, 0.1044	51.5756 6833.62	0.6	1 0 0 1 0 1
0.8	0.01	0.3112, 0.2298, 0.1891, 0.386, 0.1216, 0.349, 0.3106, 0.1234, 0.5444, 0.1528, 0.7413, 0.8348.	51.9891 6782.03	0.1	1 0 0 1 1 0
0.8	0.03	0.606, 0.7216, 0.1198, 0.4443, 0.1756, 0.466, 0.1618, 0.1156, 0.6962, 0.3409, 0.2936, 0.4674	53.198 6773.47	0.2	0 1 0 1 0 1
0.8	0.05	0.2122, 0.8382, 0.3101, 0.3618, 0.5932, 0.3946, 0.5626, 0.1846, 0.3332,	0.4982 0.1814	0.7	0 1 0 1 0 1

		0.2045, 0.4982, 0.1814			
0.8	0.08	0.7677, 0.253, 0.1473, 0.3156, 0.2698, 0.2914, 0.5416, 0.1486, 0.2232, 0.2771, 0.1649, 0.43	47.2436 5823.34	0.7	0 1 1 0 1 0
0.9	0.01	0.7952, 0.1716, 0.7149, 0.2221, 0.5002, 0.3946, 0.2548, 0.1312, 0.1077, 0.2804, 0.1638, 0.5752	46.9333 6863.1	0.9	0 1 0 1 0 1
0.9	0.03	0.2859, 0.5246, 0.1165, 0.3739, 0.6892, 0.5686, 0.3502, 0.652, 0.3981, 0.1451, 0.1374, 0.6269	56.4649 6935.98	0	0 1 1 0 0 1
0.9	0.05	0.5785, 0.2321, 0.3277, 0.2958, 0.2692 0.3334, 0.2584, 0.436, 0.1847, 0.2628, 0.7677, 0.2243	47.5433 6353.69	0.6	1 0 1 0 1 0
0.9	0.08	0.3959, 0.3883, 0.1616, 0.1836, 0.3688, 0.1174, 0.5776, 0.6406, 0.3387, 0.3431, 0.5323, 0.2386	50.6702 6794.69	0.3	0 1 0 1 0 1

Table 6. $\alpha = 0.1$

pc	pm	Optimum Solution X= $x_1, x_{2,.......}x_{12}$	Z= Z_1, Z_2	λ	Sites (1 or 2)
0.6	0.01	0.9558, 0.066, 0.1374, 0.2089, 0.349, 0.2542, 0.154, 0.61, 0.1814, 0.5466, 0.672, 0.3211	50.5007 6281.48	0.5	1 0 1 0 1 0
0.6	0.03	0.573, 0.0099, 0.397, 0.1506, 0.5812, 0.5002, 0.5626, 0.1558, 0.1099, 0.1671, 0.9613, 0.3145	52.6006 7332.89	0.2	1 0 1 0 1 0
0.6	0.05	0.1913, 0.0264, 0.21, 0.1154, 0.505, 0.2236, 0.6946, 0.2134, 0.3013, 0.2892, 0.1352, 0.8029	45.3532 6321.97	0.3	0 1 0 1 0 1
0.6	0.08	0.4454, 0.4378, 0.1154, 0.3937, 0.2008, 0.2686, 0.1462, 0.1576, 0.1869, 1.1186, 0.4905, 0.3156	54.1536 6259.55	0.8	0 1 0 1 0 1
0.7	0.01	0.2782, 1.1417, 0.1968, 0.2463, 0.1504, 0.3052, 0.1528, 0.2932, 0.5312, 0.4322, 0.3013, 1.1351	60.6044 6668.51	0.6	1 0 0 1 0 1

0.7	0.03	0.848, 0.1683, 0.1077, 0.9503, 0.5326, 0.4894, 0.1852, 0.2566, 0.1363, 0.1066, 0.2749, 0.3222	49.6976 6384.25	0.4	1 0 1 0 1 0
0.7	0.05	0.2837, 0.0055, 0.8359, 0.1066, 0.6358, 0.373, 0.2764, 0.1654, 0.2254, 0.1011, 0.6874, 0.3926	45.8877 7470.92	0.1	0 1 0 1 0 1
0.7	0.08	0.7512, 0.4411, 0.2155, 0.144, 0.2356, 0.1822, 0.433, 0.172, 0.2188, 0.5521, 0.3222, 0.2254	56.4621 5883.28	0.5	1 0 1 0 0 1
0.8	0.01	0.32, 0.3894, 0.4322, 0.1539, 0.3574, 0.3796, 0.2962, 0.3502, 0.7644, 0.2177, 0.2628, 0.1176	48.1034 7237.43	0.7	1 0 1 0 1 0
0.8	0.03	0.8777, 0.0011, 0.1583, 0.1616, 0.5572, 0.3028, 0.1954, 0.6142, 0.2837, 0.1847, 0.4069, 0.2958	47.9282 5952.5	0.6	1 0 1 0 0 1
0.8	0.05	0.1902, 0.5225, 0.1473, 0.2892, 0.1846, 0.2122, 0.5488, 0.6772, 0.2045, 0.3343, 1.1285, 0.1044	53.7908 6909.8	0.3	1 0 1 0 1 0
0.8	0.08	0.5565, 0.3564, 0.3871, 0.1022, 0.2038, 0.2746, 0.4528, 0.1144, 0.3112, 0.4003, 1.0449, 0.1022	50.7133 7108.6	0.6	1 0 0 1 0 1
0.9	0.01	0.3277, 0.1936, 0.1209, 0.2254, 0.7024, 0.5158, 0.5968, 0.2488, 0.3937, 0.595, 0.2177, 0.3849	56.0704 7432.8	0.5	1 0 1 0 1 0
0.9	0.03	0.2837, 0.8745, 0.265, 0.177, 0.1564, 0.3082, 0.1, 0.2638, 0.5202, 0.7127, 0.1847, 0.5026	52.8856 6247.33	0.7	1 0 1 0 1 0
0.9	0.05	0.9514, 0.3817, 0.1737, 0.3101, 0.313 0.4342, 0.2746, 0.517, 0.3794, 0.177, 0.8678, 0.1231	53.6695 7105.18	0.2	1 0 1 0 1 0
0.9	0.08	0.3739, 0.4113, 0.2386, 0.1649, 0.3334, 0.3706, 0.3658, 0.1936, 0.133, 0.2199, 0.6137, 0.386	46.082 5558.24	0.7	0 1 1 0 0 1

Table 7. $\alpha = 0.2$

pc	pm	Optimum Solution $X=x_1, x_2, \ldots x_{12}$	$Z= Z_1, Z_2$	λ	Sites (1 or 2)
0.6	0.01	0.1308, 0.4455, 0.1253, 0.1132, 0.208, 0.184, 0.2836, 0.1582, 0.9448, 0.3156, 0.5015, 0.1099	43.7765 6498.11	0.8	1 0 1 0 1 0
0.6	0.03	0.3464, 0.088, 0.6808, 0.2166, 0.3298, 0.2014, 0.1288, 0.3688, 0.2155, 0.6566, 0.32, 0.6291	53.2709 7014.12	0	0 1 0 1 0 1
0.6	0.05	1.0526, 0.0858, 0.2254, 0.1198, 0.1012, 0.5056, 0.3382, 0.1726, 0.2155, 1.1593, 0.2848, 0.2716	56.0141 6766.35	0.2	1 0 1 0 1 0
0.6	0.08	0.3321, 0.0968, 0.8942, 0.2078, 0.4282, 0.3472, 0.1642, 0.2644, 0.3651, 0.5092, 0.3508, 0.1319	50.4198 7751.89	0.2	0 1 0 1 0 1
0.7	0.01	0.177, 0.5357, 0.3046, 0.1363, 0.586, 0.1804, 0.2488, 0.2254, 0.2309, 0.7479, 0.5158, 0.5334	52.9751 6771.05	0.7	1 0 1 0 1 0
0.7	0.03	0.6599, 0.4378, 0.2012, 0.3739, 0.5782, 0.3754, 0.2986, 0.2128, 0.2606, 0.5114, 0.2694, 0.1077	47.9582 6562.25	0.2	1 0 1 0 1 0
0.7	0.05	0.3992, 0.1782, 0.188, 0.3376, 0.1768, 0.1882, 0.6106, 0.4648, 0.6027, 0.2001, 0.1946, 0.188	55.4298 6729.05	0.3	0 1 0 1 0 1
0.7	0.08	0.2672, 0.5158, 0.4806, 0.155, 0.6814, 0.4438, 0.409, 0.1288, 0.3849, 0.2012, 0.1418, 0.6049	49.3315 7124.82	0.7	0 1 0 1 1 0
0.8	0.01	0.4245, 0.0561, 1.0834, 0.1253, 0.1954, 0.2038, 0.112, 0.538, 0.3178, 0.3607, 0.1594, 0.7336	54.3744 7630.62	0.2	0 1 0 1 0 1
0.8	0.03	0.6412, 0.5665, 0.6995, 0.177, 0.1102, 0.4582, 0.4102, 0.1522, 0.4454, 0.5301, 0.1088, 0.3057	53.3921 7643.93	0.1	0 1 0 1 0 1
0.8	0.05	0.947, 0.4301, 0.1176, 0.2056, 0.2404, 0.1582, 0.4258, 0.1276, 0.2804, 0.2045, 0.6225, 0.3541	45.1457 5948.4	0.8	1 0 0 1 0 1
0.8	0.08	0.8018, 0.8338, 0.5015, 0.6258, 0.3118, 0.2992, 0.637, 0.1378, 0.1517, 0.1297, 0.2452, 0.1011	51.0558 7466.41	0	0 1 0 1 0 1
0.9	0.01	0.661, 1.0043, 0.1891, 0.2518, 0.5632, 0.4132, 0.3334, 0.6826, 0.2441, 0.1319, 0.122, 0.859	58.4568 6754.67	0.3	0 1 0 1 0 1
0.9	0.03	0.9327, 0.9394, 0.1737, 0.419, 0.4468, 0.4942, 0.2392, 0.2662, 0.3035, 0.3002, 0.1154, 0.2188	50.183 6407.07	0.8	1 0 1 0 1 0
0.9	0.05	0.5598, 0.1903, 0.3959, 0.1407, 0.4108, 0.7006, 0.2332, 0.5506, 0.1022, 0.353, 0.2936, 0.771	53.4078 6446.46	0.6	1 0 1 0 1 0
0.9	0.08	0.6786, 0.6193, 0.331, 0.1792, 0.3754, 0.1924, 0.2362, 0.1144, 0.2837, 0.6214, 0.8293, 0.7226	57.9531 7473.91	0.1	1 0 1 0 1 0

Table 8. $\alpha = 0.3$

pc	pm	Optimum Solution $X=x_1, x_2, \ldots x_{12}$	$Z= Z_1, Z_2$	λ	Sites (1 or 2)
0.6	0.01	0.2408, 0.8051, 0.2034, 0.1396, 0.1498, 0.1234, 0.4558, 0.1096, 0.3552, 0.2903, 0.5411, 0.1902	40.2115 5568.54	0.8	1 0 0 1 1 0
0.6	0.03	0.166, 0.8788, 0.43, 0.4663, 0.2026, 0.3736, 0.4186, 0.6328, 0.1187, 0.2012, 0.2518, 0.2001	51.1035 6302.3	0.5	1 0 0 1 0 1

0.6	0.05	0.4784, 0.1562, 0.7215, 0.2078, 0.1768, 0.4132, 0.349, 0.163, 0.4498, 0.1242, 0.1044, 0.2309	41.8496 6659.3	0.7	1 0 1 0 1 0
0.6	0.08	0.3618, 0.7029, 0.441, 0.1286, 0.544, 0.187, 0.397, 0.2692, 0.2232, 0.5785, 0.4465, 0.5257	54.8974 7320.84	0.3	1 0 1 0 1 0
0.7	0.01	0.3585, 0.0671, 0.3662, 0.111, 0.2284, 0.3496, 0.4732, 0.1414, 0.2012, 0.7215, 0.3167, 0.166	50.214 6130.18	0.9	0 1 0 1 1 0
0.7	0.03	0.452, 0.7689, 0.3332, 0.155, 0.1618, 0.5398, 0.5278, 0.4162, 0.43, 0.2122, 0.4498, 0.1957	51.2348 6964.74	0.3	0 1 0 1 0 1
0.7	0.05	1.1549, 0.5863, 0.1099, 0.7919, 0.6982, 0.2278, 0.2308, 0.6472, 0.1033, 0.188, 0.1924, 0.7446	62.3155 7364.41	0.5	1 0 1 0 1 0
0.7	0.08	0.1473, 0.0484, 0.5345, 0.4971, 0.3454, 0.289, 0.163, 0.2062, 0.1132, 0.6742, 0.1583, 0.8667	54.1817 6632.8	0.7	0 1 0 1 0 1
0.8	0.01	0.2232, 0.0385, 0.1264, 0.2727, 0.1168, 0.6154, 0.6868, 0.2548, 0.5345, 0.6368, 0.2628, 0.5785	58.785 7360.45	0.4	1 0 1 0 1 0
0.8	0.03	0.4487, 0.3332, 0.1055, 0.1231, 0.6286, 0.5614, 0.1378, 0.5536, 0.3871, 0.122, 0.1803, 1.0141	51.5781 6028.94	0.5	1 0 1 0 0 1
0.8	0.05	0.9943, 0.0814, 0.419, 0.2826, 0.1258, 0.3934, 0.148, 0.304, 0.3508, 0.1275, 0.3662, 0.4124	52.759 6122.03	0.9	1 0 1 0 0 1
0.8	0.08	0.2617, 1.0086, 0.3134, 0.3926, 0.13, 0.2158, 0.2686, 0.238, 0.3211, 0.5499, 0.6467, 0.1099	52.2166 6636.61	0.4	0 1 0 1 1 0
0.9	0.01	0.3827, 0.3938, 0.3893, 0.1902, 0.1576, 0.412, 0.1786, 0.2458, 0.3937, 0.9976, 0.9734, 0.4619	63.9483 8062.47	0.4	1 0 1 0 1 0
0.9	0.03	0.265, 0.4487, 0.5257,	53.1761 7035.98	0	1 0 1

		0.5312, 0.259, 0.6136, 0.3802, 0.5974, 0.221, 0.111, 0.309, 0.1979			0 1 0
0.9	0.05	0.5015, 0.1441, 0.2661, 0.375, 0.6286, 0.1702, 0.2128, 0.574, 0.3761, 0.1968, 0.3508, 0.3409	53.1439 6696.9	0.5	1 0 1 0 0 1
0.9	0.08	0.6995, 1.0857, 0.2573, 0.254, 0.127, 0.2656, 0.1624, 0.196, 0.7589, 0.1352, 0.1803, 0.7611	53.5828 6772.68	0.4	0 1 0 1 0 1

Table 9. $\alpha = 0.4$

pc	**pm**	**Optimum Solution X=$x_1, x_2, \ldots x_{12}$**	**Z= Z_1, Z_2**	**λ**	**Sites (1 or 2)**
0.6	0.01	1.1318, 0.2321, 0.1319, 0.3915, 0.5338, 0.1564, 0.307, 0.19, 0.2287, 0.2353, 0.7842, 1.0053	59.4923 7402.7	0.7	1 0 1 0 1 0
0.6	0.03	0.4245, 0.6424, 0.2584, 0.1011, 0.6694, 0.4246, 0.2956, 0.1252, 0.3233, 0.2694, 0.1143, 0.2331	47.2941 5817.72	0.7	1 0 1 0 1 0
0.6	0.05	0.4146, 0.9636, 0.1286, 0.1803, 0.4216, 0.6472, 0.2872, 0.4714, 0.1825, 0.1935, 0.3112, 0.7303	53.1254 5899.18	0.8	0 1 0 1 0 1
0.6	0.08	0.3706, 0.2178, 0.5048, 0.298, 0.241, 0.3304, 0.7084, 0.532, 0.1286, 0.1121, 0.7347, 0.2936	51.9256 7445.48	0.1	0 1 0 1 0 1
0.7	0.01	0.7952, 0.7194, 0.5675, 0.672, 0.6898, 0.4798, 0.1612, 0.1978, 0.1011, 0.1495, 0.2782, 0.3123	54.3738 7500.81	0.4	0 1 0 1 0 1
0.7	0.03	0.2892, 0.0693, 0.133, 0.1088, 0.3946, 0.2932, 0.6412, 0.5068, 0.1264, 0.3772, 0.6709, 0.3761	47.1208 6286.91	0.6	1 0 1 0 1 0
0.7	0.05	0.2595, 0.5962, 0.3046, 0.2837, 0.7138, 0.4528, 0.1456, 0.1252, 0.1209, 0.1946, 0.7699, 0.4432	55.2625 6300.86	0.6	0 1 0 1 1 0
0.7	0.08	0.3354, 0.1254, 0.1638, 0.6082, 0.109, 0.4624,	57.4572 6892.68	0.9	1 0 1 0 1 0

		0.2788, 0.1942, 0.7028, 0.1407, 0.5015, 0.3354			
0.8	0.01	0.6665, 0.4235, 0.1374, 0.5004, 0.2554, 0.2902, 0.1042, 0.226, 0.1495, 0.2133, 0.8084, 0.6698	58.3371 5929.11	0.9	1 0 1 0 1 0
0.8	0.03	0.3695, 0.6358, 0.3827, 0.2155, 0.1996, 0.5146, 0.2794, 0.1816, 0.7897, 0.3134, 0.1176, 0.6621	55.6803 7425.15	0.1	0 1 0 1 0 1
0.8	0.05	0.7853, 0.0154, 0.5136, 0.1341, 0.3328, 0.3484, 0.2248, 0.1372, 0.5994, 0.4168, 0.1517, 0.4608	60.2807 7213.84	0.1	1 0 0 1 0 1
0.8	0.08	0.7237, 0.0506, 0.1044, 0.188, 0.3664, 0.3976, 0.2632, 0.3844, 0.188, 1.068, 0.8557, 0.1451	56.8232 7160	0.5	1 0 1 0 1 0
0.9	0.01	0.2815, 0.2914, 0.1671, 0.4278, 0.244, 0.1354, 0.1816, 0.7036, 0.7523, 0.54, 0.4003, 0.2177	57.8137 7381.44	0.5	0 1 1 0 0 1
0.9	0.03	0.7006, 1.0142, 0.4773, 0.3651, 0.1636, 0.5434, 0.6652, 0.3274, 0.1814, 0.1231, 0.1022, 0.2584	51.9713 7031.41	0.1	0 1 0 1 0 1
0.9	0.05	0.518, 0.0462, 0.2375, 0.6621, 0.13, 0.2296, 0.178, 0.1366, 0.3981, 0.5917, 0.2166, 0.3871	49.4286 6355.43	0.7	0 1 0 1 0 1
0.9	0.08	0.3585, 0.7084, 0.5961, 0.4036, 0.1684, 0.2668, 0.1336, 0.1366, 0.3596, 0.3552, 0.5235, 0.4377	56.3198 7010.8	0.9	0 1 1 0 1 0

Table 10. $\alpha = 0.5$

pc	**pm**	**Optimum Solution $X=x_1, x_2, \ldots x_{12}$**	**Z= Z_1, Z_2**	$\boldsymbol{\lambda}$	**Sites (1 or 2)**
0.6	0.01	0.1319, 1.0263, 0.1847, 0.3068, 0.2806, 0.1318, 0.3022, 0.1792, 0.1319, 0.5444, 0.2903, 0.3255	44.7828 5259.41	0.8	1 0 1 0 1 0
0.6	0.03	0.188, 0.2475, 0.2859, 0.1066, 0.1378, 0.4846, 0.286, 0.2566, 0.5807, 0.9096, 0.2287, 0.496	55.4518 6974.69	0.5	1 0 1 0 1 0

0.6	0.05	0.3607, 0.6611, 0.2606, 0.4674, 0.1666, 0.2878, 0.2248, 0.1012, 0.144, 0.7567, 0.7446, 0.8282	60.508 6996.84	0	0 1 0 1 0 1
0.6	0.08	0.8667, 0.011, 0.3233, 0.1, 0.4162, 0.4006, 0.3022, 0.4864, 0.3827, 0.1572, 0.4641, 0.9811	54.6189 7162.24	0.8	0 1 0 1 0 1
0.7	0.01	0.914, 0.297, 0.4025, 0.3871, 0.5044, 0.2836, 0.1384, 0.1696, 0.2793, 0.2056, 0.2606, 0.3277	44.6774 6413.98	0.6	0 1 0 1 0 1
0.7	0.03	0.3805, 0.8084, 0.1451, 0.32, 0.1756, 0.3976, 0.1288, 0.4372, 0.4179, 0.1242, 0.4212, 0.2881	45.1737 5461.59	0.9	0 1 1 0 1 0
0.7	0.05	0.2166, 0.4147, 0.4201, 0.1066, 0.1606, 0.277, 0.1978, 0.3556, 0.199, 1.2132, 0.2551, 0.3849	55.4277 6545.11	0.2	1 0 1 0 1 0
0.7	0.08	0.2529, 0.5939, 0.2529, 0.287, 0.4558, 0.5848, 0.226, 0.3148, 0.5565, 0.1484, 0.3992, 0.4256	51.3191 6827.01	0.3	0 1 0 1 1 0
0.8	0.01	0.1528, 0.6281, 0.5257, 0.5004, 0.5308, 0.5116, 0.2782, 0.643, 0.1913, 0.1814, 0.1891, 0.1055	52.4247 6920.21	0.3	1 0 1 0 1 0
0.8	0.03	0.2023, 0.5082, 0.2507, 0.2529, 0.271, 0.3982, 0.1552, 0.2548, 0.5917, 0.1836, 0.4168, 0.3189	51.4945 6066.54	0.9	0 1 0 1 1 0
0.8	0.05	0.3486, 0.2199, 0.2584, 0.5477, 0.148, 0.4234, 0.3634, 0.6172, 0.1495, 0.3288, 0.4333, 0.6632	56.3785 6591.09	0.5	1 0 1 0 0 1
0.8	0.08	0.4091, 0.2475, 0.3673, 0.1726, 0.2218, 0.478, 0.3784, 0.1702, 0.7105, 0.2001, 0.1517, 0.4036	48.5855 6836.25	0.1	0 1 0 1 0 1
0.9	0.01	0.8898, 0.0781, 0.1957, 0.6148, 0.3442, 0.3304, 0.124, 0.3832, 0.7138, 0.1638, 0.2254, 0.4234	53.6968 7322.52	0.2	0 1 0 1 0 1

0.9	0.03	0.6863, 0.7314, 0.3607, 0.3431, 0.262, 0.331, 0.2716, 0.5176, 0.4047, 0.1814, 0.2562, 0.3783	52.7347 6820.95	0.7	0 1 0 1 0 1
0.9	0.05	1.0691, 0.1914, 0.1132, 0.2023, 0.1564, 0.1576, 0.4474, 0.2446, 0.6819, 0.2463, 0.2595, 0.2749	54.297 6636.51	0.4	1 0 0 1 0 1
0.9	0.08	0.4322, 0.386, 0.4014, 0.1176, 0.5782, 0.3412, 0.2896, 0.6586, 0.1396, 0.276, 0.9833, 0.122	51.8692 7139.65	0.4	0 1 0 1 1 0

Table 11. $\alpha = 0.6$

pc	**pm**	**Optimum Solution $X = x_1, x_2, \ldots, x_{12}$**	**Z= Z_1, Z_2**	**λ**	**Sites (1 or 2)**
0.6	0.01	0.2958, 0.3388, 0.8348, 0.2034, 0.4564, 0.2344, 0.1024, 0.1486, 0.2881, 0.2639, 0.43, 1.134	55.9846 7591.71	0.6	1 0 1 0 1 0
0.6	0.03	0.6324, 0.6358, 0.21, 0.1341, 0.1972, 0.2704, 0.1552, 0.2884, 0.7831, 0.232, 0.4179, 0.3992	49.0302 6659.27	0.5	1 0 1 0 1 0
0.6	0.05	0.1495, 0.3201, 0.1836, 0.6918, 0.2152, 0.283, 0.385, 0.13, 0.1528, 0.1935, 0.4432, 0.5994	46.818 5834.92	0.7	1 0 1 0 1 0
0.6	0.08	0.3552, 0.2651, 0.3915, 0.331, 0.1564, 0.1606, 0.4114, 0.1846, 0.1605, 0.2793, 0.2331, 0.9261	47.0473 5932.16	0.9	1 0 1 0 1 0
0.7	0.01	0.1231, 0.9141, 0.1363, 0.1011, 0.1438, 0.337, 0.3652, 0.3094, 0.4564, 0.375, 0.3068, 0.5059	47.5098 5709.99	0.9	0 1 0 1 0 1
0.7	0.03	0.188, 0.671, 0.1825, 0.6775, 0.7114, 0.283, 0.2146, 0.1186, 0.1297, 0.5598, 0.3706, 0.1737	50.9126 6537.74	0.6	0 1 0 1 0 1
0.7	0.05	0.7226, 0.5456, 0.1, 0.1913, 0.1618, 0.3622, 0.2416, 0.4246, 0.166, 0.683, 1.0581, 0.3035	55.4745 6649.83	0.2	1 0 1 0 1 0
0.7	0.08	1.0526, 0.506, 0.3816, 0.4157, 0.1648, 0.3088,	54.1981 6434.47	0.3	0 1 1 0 1

		0.3604, 0.4558, 0.1858, 0.2034, 0.1165, 0.4069			0
0.8	0.01	0.276, 0.1122, 0.1231, 1.0845, 0.1198, 0.1006, 0.1438, 0.337, 0.5862, 0.2023, 0.4564, 0.375	54.4371 6911.36	0.2	1 0 1 0 1 0
0.8	0.03	0.4784, 0.3982, 0.111, 1.0031, 0.466, 0.2158, 0.1714, 0.109, 0.2859, 0.5301, 0.1121, 0.397	54.4384 6722.6	0.9	0 1 0 1 0 1
0.8	0.05	0.9932, 0.2728, 0.2122, 0.4421, 0.4606, 0.1006, 0.2482, 0.2794, 0.1858, 0.2023, 0.4487, 0.1473	51.322 6034.41	0.6	0 1 0 1 1 0
0.8	0.08	0.6379, 0.1792, 0.1836, 0.5774, 0.1756, 0.5326, 0.2014, 0.3034, 0.331, 0.584, 0.1319, 0.6654	56.1671 6616.42	0.6	1 0 1 0 0 1
0.9	0.01	0.4212, 0.5511, 0.1704, 0.3211, 0.2704, 0.2278, 0.391, 0.154, 0.1924, 1.1274, 0.254, 0.1165	52.6002 6521.36	0.3	1 0 1 0 1 0
0.9	0.03	0.1286, 0.737, 0.1792, 1.1384, 0.307, 0.3346, 0.3442, 0.4198, 0.2837, 0.1484, 0.1462, 0.2881	56.9353 6878.68	0.2	0 1 0 1 0 1
0.9	0.05	0.5917, 0.2662, 0.1814, 0.4806, 0.319, 0.3916, 0.232, 0.3076, 0.2903, 0.3277, 1.2055, 0.2771	55.506 7377.41	0.6	0 1 0 1 0 1
0.9	0.08	0.6038, 0.9064, 0.2397, 0.2177, 0.1492, 0.1186, 0.4546, 0.1996, 0.3651, 0.1407, 0.5961, 0.6137	49.422 6479.62	0.3	1 0 1 0 1 0

Table 12. $\alpha = 0.7$

pc	**pm**	**Optimum Solution X=$x_1, x_2, \ldots x_{12}$**	**Z= Z_1, Z_2**	**λ**	**Sites (1 or 2)**
0.6	0.01	0.2067, 0.198, 0.1638, 0.1814, 0.178, 0.2662, 0.5302, 0.6052, 0.2727, 0.7798, 0.3233, 0.4784	54.9261 6594.92	0.5	0 1 0 1 0 1
0.6	0.03	0.2573, 0.9195, 0.4058, 0.386, 0.2638, 0.5218, 0.2902, 0.226, 0.2221,	51.5267 6130.15	0.8	0 1 0 1 0 1

		0.2947, 0.2122, 0.2606			
0.6	0.05	0.694, 0.2486, 0.2716, 1.1087, 0.1288, 0.2752, 0.1438, 0.1426, 0.1605, 0.2925, 0.3508, 0.3651	52.1567 6507.37	0.9	1 0 1 0 1 0
0.6	0.08	0.1638, 0.5192, 0.1143, 0.3574, 0.121, 0.4504, 0.433, 0.3898, 0.3145, 0.1924, 0.3662, 0.5554	47.8145 5727.42	0.9	0 1 0 1 1 0
0.7	0.01	0.727, 0.3146, 0.2485, 0.1088, 0.4852, 0.2458, 0.1852, 0.5506, 0.2045, 0.5708, 0.7391, 0.4058	53.8141 6843.99	0.6	1 0 1 0 1 0
0.7	0.03	0.1308, 0.6699, 0.2991, 0.4003, 0.1504, 0.6106, 0.3286, 0.1204, 0.1781, 0.5983, 0.1616, 0.3783	56.1935 5888.29	0.5	1 0 1 0 1 0
0.7	0.05	0.7941, 0.1364, 0.4256, 0.3981, 0.5122, 0.4654, 0.2878, 0.229, 0.188, 0.2683, 0.1, 0.5708	53.4879 6716.21	0.8	1 0 1 0 1 0
0.7	0.08	0.4971, 0.5863, 0.1308, 0.5125, 0.3706, 0.1756, 0.1858, 0.1654, 0.5719, 0.3365, 0.7182, 0.3706	53.978 7198.3	0.2	0 1 0 1 0 1
0.8	0.01	0.2122, 0.8272, 0.1297, 0.5664, 0.2218, 0.214, 0.2284, 0.4786, 0.1484, 0.5356, 0.2859, 0.7545	57.0356 6051.97	0.5	1 0 1 0 1 0
0.8	0.03	0.2221, 0.0836, 0.199, 0.452, 0.3538, 0.661, 0.1204, 0.2464, 0.1077, 0.804, 0.1682, 0.7072	53.4097 5955.33	0.7	1 0 1 0 1 0
0.8	0.05	0.4949, 0.8404, 0.1165, 0.4498, 0.3076, 0.1246, 0.3616, 0.568, 0.3398, 0.2199, 0.2419, 0.4806	52.4714 6296.59	0.5	0 1 0 1 0 1
0.8	0.08	0.5829, 0.5698, 0.2958, 0.1198, 0.1738, 0.4252, 0.3382, 0.4378, 0.1484, 0.3112, 0.9943, 0.6225	54.759 6779.52	0.1	0 1 0 1 0 1
0.9	0.01	1.1054, 0.3003, 0.2419, 0.1539, 0.6142, 0.187, 0.595, 0.2158, 0.1121, 0.1561, 0.3134, 0.5499	61.6598 6634.89	0.9	0 1 1 0 1 0

0.9	0.03	1.1153, 0.4807, 0.122, 0.9734, 0.1996, 0.2092, 0.2362, 0.3922, 0.2496, 0.2617, 0.3453, 0.8788	63.8272 7432.87	0	1 0 1 0 1 0
0.9	0.05	0.9646, 0.0396, 0.3662, 0.6797, 0.115, 0.1786, 0.235, 0.3562, 0.4949, 0.3112, 0.3453, 0.5477	56.8789 7605.41	0.8	1 0 1 0 1 0
0.9	0.08	0.5279, 0.133, 0.1187, 0.4355, 0.5314, 0.484, 0.4342, 0.355, 0.1407, 0.3233, 0.386, 0.21	46.7558 6258.34	0.9	0 1 0 1 1 0

Table 13. $\alpha = 0.8$

pc	pm	Optimum Solution $X = x_1, x_2, \ldots, x_{12}$	Z= Z_1, Z_2	λ	Sites (1 or 2)
0.6	0.01	0.5422, 0.1947, 0.5576, 0.2133, 0.2674, 0.1066, 0.5878, 0.334, 0.4355, 0.1319, 0.2958, 0.3002	57.4254 7260.85	0.6	0 1 0 1 0 1
0.6	0.03	0.9745, 0.3102, 0.1616, 0.3233, 0.127, 0.4906, 0.6634, 0.388, 0.2837, 0.1693, 0.8381, 0.3552	54.7615 7480.33	0.1	1 0 1 0 1 0
0.6	0.05	0.287, 0.5258, 0.177, 0.1836, 0.3646, 0.5608, 0.175, 0.496, 0.4476, 0.2991, 0.1957, 0.9437	54.9959 6305.02	0.8	1 0 1 0 1 0
0.6	0.08	0.7336, 0.4323, 0.4454, 0.155, 0.3898, 0.1474, 0.526, 0.1996, 0.1451, 0.2287, 0.5125, 0.1517	42.5606 6527.72	0.9	1 0 0 1 0 1
0.7	0.01	0.5455, 0.9922, 0.32, 0.1418, 0.3544, 0.262, 0.1876, 0.1012, 0.3618, 0.3519, 0.1363, 0.364	43.5657 5728.43	0.7	1 0 1 0 1 0
0.7	0.03	0.5312, 0.8657, 0.2584, 0.4234, 0.1888, 0.3526, 0.4066, 0.1696, 0.1121, 0.2342, 0.3453, 0.3046	48.7659 5753.99	0.8	0 1 0 1 0 1
0.7	0.05	1.0977, 0.4257, 0.1924, 0.2771, 0.2272, 0.424, 0.4996, 0.1036, 0.3486, 0.2298, 0.87, 0.6577	56.1043 7657.99	0.1	0 1 0 1 0 1
0.7	0.08	0.3255, 0.1947, 0.1539,	52.1845 6773.66	0.4	1 0 1

		0.3607, 0.1264, 0.3982, 0.3652, 0.5092, 0.6368, 0.3046, 0.3453, 0.3816			0 1 0
0.8	0.01	0.1968, 0.3696, 0.1935, 0.4861, 0.3634, 0.2704, 0.208, 0.646, 0.3321, 0.9008, 0.3442, 0.1825	59.7058 7104.06	0.7	0 1 0 1 0 1
0.8	0.03	0.342, 0.1518, 0.8557, 0.2276, 0.2716, 0.1576, 0.2428, 0.4108, 0.1517, 0.507, 0.2144, 0.2529	47.6452 6864.5	0.3	1 0 1 0 1 0
0.8	0.05	0.1407, 0.3839, 0.1814, 0.1825, 0.2002, 0.1978, 0.3244, 0.6772, 0.2023, 0.8898, 0.4718, 0.7864	60.661 6652.57	0.7	0 1 0 1 0 1
0.8	0.08	0.5576, 0.1375, 0.4344, 0.4553, 0.118, 0.4612, 0.2998, 0.1468, 0.3651, 0.4245, 0.4762, 0.1143	48.218 6870.99	0.5	1 0 1 0 1 0
0.9	0.01	0.5576, 0.165, 0.1132, 0.3585, 0.4168, 0.148, 0.1846, 0.4312, 0.5246, 0.7963, 0.7127, 0.2012	57.6868 7543.3	0.9	1 0 1 0 1 0
0.9	0.03	0.419, 0.5301, 0.188, 1.1989, 0.1282, 0.178, 0.3034, 0.145, 0.5026, 0.2859, 0.1264, 0.5455	59.8166 7447.43	0.8	1 0 1 0 1 0
0.9	0.05	0.738, 0.4356, 0.3156, 0.2298, 0.6556, 0.1078, 0.1948, 0.4108, 0.4509, 0.1814, 0.1792, 0.8304	52.7598 7003.16	0.7	1 0 1 0 1 0
0.9	0.08	0.1869, 0.7304, 0.2001, 0.5158, 0.6676, 0.436, 0.1036, 0.3574, 0.3596, 0.1913, 0.5884, 0.2507	57.8918 6752.07	0.7	1 0 1 0 0 1

Table 14. $\alpha = 0.9$

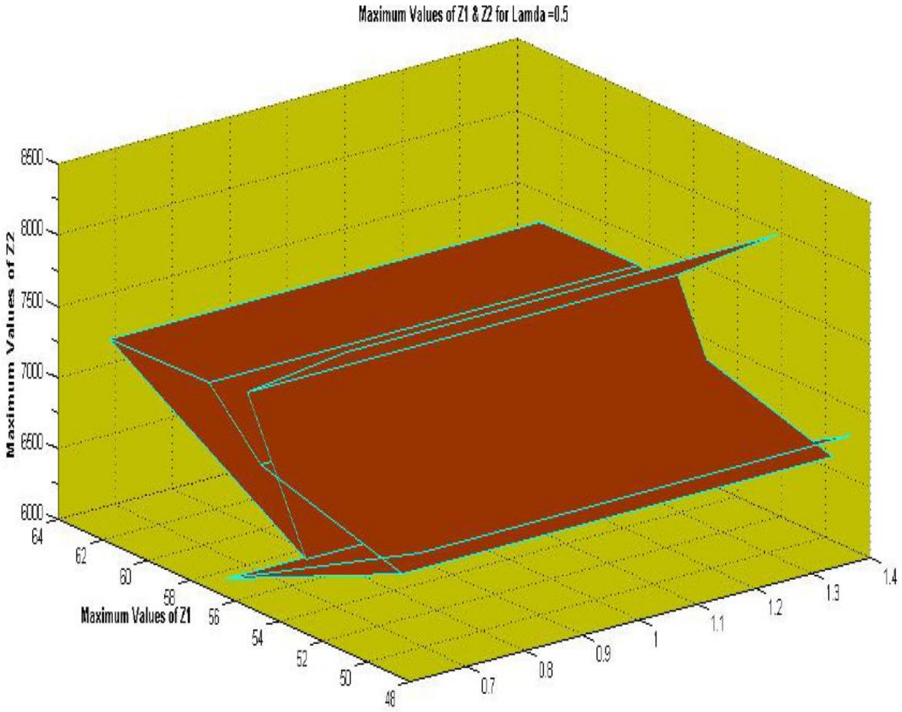

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Figure 3. Maximum values of Z1 and Z2 for $\lambda = 0.5$.

7. Conclusion and results

In this article, the multi-objective fuzzy stochastic programming problem for hazardous wastes have been handled using GA based fuzzy programming approach. The bounds of the problem and case study are described as fuzzy Weibull distribution and fuzzy normal distribution, and the confidence levels are treated as fuzzy number. The **Table 6**-**14** show the optimum Pareto's solutions for different values. The **Figure 2** and **3** present the graphical representation of the optimum solutions for α = 0.5. One of the major advantages of the procedure is that the computational time is greatly reduced to solve the main problem. Another advantage is that it provides the decision maker to have alternative Pareto solutions which readily help them in making appropriate decision regarding the problem. The results of the numerical example and case study showed that the above procedure is very efficient. The number of variables in the problem increases as we moved to some large and complex problems which put the limitation to the proposed methods. The extension of the proposed research model can be made to problems of transportation, transhipment, solid transportation, inventory, etc.

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