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ORIGINAL RESEARCH ARTICLE

The displacement view of a multilayered HSDT plate

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ABSTRACT

This paper presents the state of displacement of a multilayered composite laminate subjected to transverse static load with varying balance, symmetric and anti-symmetric angle-ply and cross-ply staking sequences. Higher-order shear deformation theory (HSDT) is considered in the finite element formulation of nine-noded isoparametric element with seven degrees of freedom at each node. The finite element formulation is transformed into computer codes. A convergence study is carried out first to obtain the optimal mesh size for minimizing the computational time. The maximum deflection at the center of plate for both fixed and simply supported edges is verified with reported literature and a good conformity is found. An attempt has been made to observe the minimum value of maximum deflection in the laminate for attaining the maximum strength of laminate with a suitable combination of stacking sequences with a constant volume of material.

Keywords: laminated composite plate; static behavior; deflection; HSDT; FEM

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1. Introduction

Composites are superior to the conventional metal materials because of their high stiffness and strength-to-weight ratios, long durability, resistance to corrosion, and other important properties. Fibre reinforced composite materials are widely used in aerospace, automobile, nuclear, marine, biomedical and civil engineering. The use of composite materials in designing the structures provides significant decrease in structural weight, increase in payload, and increase in range, speed and durability. In the last few decades, the composite plates were investigated by different researchers. Reddy[1] developed a higher-order shear deformation theory of laminated composite plates. Exact closed form of solutions for symmetric cross-ply laminates was presented and the results were compared with three-dimensional elasticity and first-order shear deformation theory solutions. Pandya and Kant[2] presented the higher-order finite element formulation of thick composite plates for flexure under transverse loads. The discrete element chosen was a nine noded quadrilateral with nine degrees-of-freedom at each node. Results for plate deformations and stresses were compared with the closed-form and the theory of elasticity solutions by using another higher-order displacement model[3]. Pal and Ray[4] and Pal and Bhattacharyya[5] used an eight-noded isoparametric element to model the progressive failure analysis of angle-ply and cross-ply laminated composite plates under transverse static load. Pal and Bhar[6] studied the displacement of symmetric and anti-symmetric angle-ply and cross-ply laminated composite plates during its ultimate failure, subjected to transverse static load. Authors studied the free vibration analyses of isotropic and laminated composite plates to ensure the overall validity of the finite element codes. Singh et al.[7], Singh and Pal[8] and Gorai and Pal[9,10] evaluated the deflection and natural frequencies of stiffened and unstiffened isotropic and orthotropic plates with varying geometries, laminae properties and orientations of stiffener. Ahmed et al.[11] studied the static and dynamic analysis of composite laminated plate. The behavior of laminated composite plates under transverse loading was analyzed using FEM. The aforementioned literature contains little information from the viewpoint of the displacement components of laminated plate. However, it appears that the state of displacement of the laminate with varying properties of composite laminae is merely illustrated so far with the importance it deserves.

The aim of the study is to determine the deflection of a composite laminate considering HSDT for both fixed (CCCC) and simply supported (SSSS) edges. The minimum value of maximum deflection of laminate is investigated for various stacking sequences, fibre orientations and the number of layers. A computer code is developed using FORTRAN for this purpose. The code is verified by solving the problems and finally, some interesting results are presented through a parametric study.

2. Governing equations of the plate

Finite element formulation for nine noded isoparametric element is well-known in literature[2,3,12]. The HSDT plate theory is well-known and may be found elsewhere[1,12,13]. The displacement components in the plate with respect to global coordinate system (x-y-z) are assumed as

$$\left.\begin{aligned} u(x,y,z) &= u_0(x,y) + z\theta_x(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= v_0(x,y) + z\theta_y(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned}\right\} \tag{1}$$

where *u, v* and *w* are the displacements in *x, y* and *z* directions, respectively at any point. u_0, v_0 are the membrane displacements and w_0 is the transverse displacement of the mid-plane. The parameters θ_x and θ_y are the rotations of the transverse normal cross-section in the *y-z* and *x-z* planes, respectively. The parameters θ_x^* and θ_y^* are the corresponding higher-order terms. For plane stress conditions, the constitutive equation of a k^{th} orthotropic layer in local coordinate system is yielded as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^k = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} & 0 & 0 \\ Q_{21} & Q_{22} & Q_{26} & 0 & 0 \\ Q_{61} & Q_{62} & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & Q_{45} \\ 0 & 0 & 0 & Q_{54} & Q_{55} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^k \tag{2}$$

where the material constants are given by

$$Q_{11} = \frac{E_1}{1-\mu_{12}\mu_{21}},\ Q_{12} = \frac{\mu_{12}E_2}{1-\mu_{12}\mu_{21}},\ Q_{22} = \frac{E_2}{1-\mu_{12}\mu_{21}},\ Q_{66} = G_{12},\ Q_{44} = G_{23},\ Q_{55} = G_{13} \tag{3}$$

where E_1, E_2 are the Young modulus in the 1 and 2 directions, respectively, and G_{12}, G_{13}, $G_{23}$are the shear modulus in the 1-2, 1-3, 2-3 planes, respectively, and μ_{ij} are the Poisson's ratios. The laminated plate is usually made of several layers, where the stress-strain relationship for the orthotropic lamina with varying fiber orientations maps to the reference as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^k = \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} & 0 & 0 \\ \overline{Q_{21}} & \overline{Q_{22}} & \overline{Q_{26}} & 0 & 0 \\ \overline{Q_{61}} & \overline{Q_{62}} & \overline{Q_{66}} & 0 & 0 \\ 0 & 0 & 0 & \overline{Q_{44}} & \overline{Q_{45}} \\ 0 & 0 & 0 & \overline{Q_{54}} & \overline{Q_{55}} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^k \tag{4}$$

where $\overline{Q_{ij}}$ are transformed material constants of the k^{th} lamina[14]. The element stiffness matrix is yielded as

$$[K^e] = \int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]|J|d\xi d\eta \tag{5}$$

where $[B]$ is the strain-displacement matrix obtained by solving the Equation (1), after differentiating the shape functions of nine-noded isoparametric element for the strain components expressed in Equation (2). The Jacobian and the rigidity matrices are given below.

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \tag{6}$$

$$[D] = \begin{bmatrix} A & G & E \\ G & C & F \\ E & F & H \end{bmatrix} \ \& \ [D^S] = \begin{bmatrix} A^S & G^S \\ G^S & C^S \end{bmatrix} \tag{7}$$

where

$$\left.\begin{aligned} (A_{ij}, G_{ij}, C_{ij}, E_{ij}, F_{ij}, H_{ij}) &= \int_{-t/2}^{t/2}(1, z, z^2, z^3, z^4, z^6)Q_{ij}dz \quad i,j,= 1,2,6 \\ \& \ (A_{ij}^S, G_{ij}^S, C_{ij}^S) &= \int_{-t/2}^{t/2}(1, z, z^4)Q_{ij}dz \quad i,j,= 4,5 \end{aligned}\right\} \tag{8}$$

3. Results and discussion

3.1. Convergence study and validation

This problem is studied to select the optimal mesh division required to achieve a minimum error in the calculated displacement of an isotropic plate subjected to a uniformly distributed load of 1.0 kN/m^2. A square plate of size 1.0 m × 1.0 m and thickness of 0.01 m is considered. The material properties of plate considered are: modulus of elasticity, $E = 2.2 \times 10^5$ MPa, Poisson's ratio, $\mu = 0.3$ and mass density, $\rho = 7850$ kg/m^3. Both eight and nine noded isoparametric elements with seven degrees of freedom per node are considered to determine the maximum deflection of plate for different boundary conditions and the results are illustrated in **Figure 1**. Keeping the computational time in mind, it is observed that the results tend to converge at 8 × 8 mesh for 8-noded elements and at 5 × 5 mesh for 9-noded elements for both the edges. It is clearly seen that the results for 9 noded elements converge at lower mesh size for both the edges. Thus, the parametric study is carried out for 9 noded isoparametric elements with 5 × 5 mesh size for both the edges to minimize the computational time.

Further, the same plate is considered with different edges by applying a point load of 1 kN at centre. To validate the present developed computer codes, the maximum deflection is observed and compared in **Table 1**. The results are verified with the results calculated from the mathematical expressions given by Timoshenko and Woinowsky-Krieger[15] and are found to be ok.

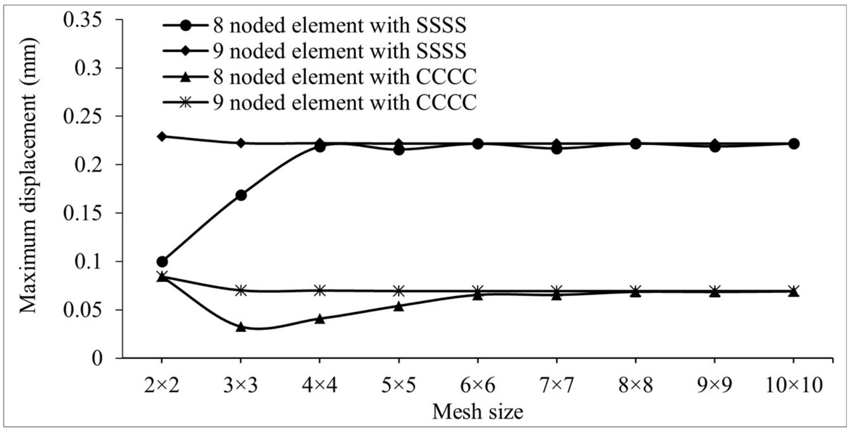

Figure 1. Maximum deflection of isotropic plate.

Table 1. Comparison of results for maximum displacement (mm).

S. No	Edge conditions	Type of load	Present result	Timoshenko and Woinowsky-Krieger [15]	Variation (%)
1	CCCC	Distributed load	0.0692	0.0688	0.58
2	CCCC	Point load	0.3075	0.3058	0.55
3	SSSS	Distributed load	0.2219	0.2271	2.28
4	SSSS	Point load	0.6342	0.6334	0.13

3.2. Parametric study

At the next natural step, an orthotropic plate subjected to transverse loading with varying balance, symmetric and anti-symmetric angle-ply and cross-ply laminated plate with different edges is studied. The material used for composite laminate is epoxy-carbon, which is available in ANSYS 15.0 workbench. The geometric and material properties of laminate are presented in **Table 2**. The thickness of plate is fixed as 10 mm to maintain a constant volume of material in the laminate.

Table 2. Properties of orthotropic plate.

Material properties	Geometric properties
Modulus of elasticity, $E_1 = 1.21 \times 10^5$ MPa Modulus of elasticity, $E_2 = 8600$ MPa Poisson's ratio, $\mu_{12} = \mu_{13} = 0.27$ Poisson's ratio, $\mu_{23} = 0.4$ Shear modulus, $G_{12} = G_{13} = 4700$ MPa Shear modulus, $G_{23} = 3100$ MPa Mass density, $\rho = 1490$ kg/m^3	[1] Size: Length = 1000 mm, Width = 1000 mm & Thickness = 10 mm [2] Staking sequence (θ =15°, 30°, 45°, 60°, 75° & 90°): (i) $(\theta/-\theta)$; balance ply (ii) (θ/θ); angle ply (iii) (0°/90°); cross ply [3] Number of layers = 2, 4, 6, 8 & 10

3.2.1. Balance and symmetric ply laminate

The balance and symmetric angle-ply and cross-ply laminated plate subjected to uniformly distributed load of 1.0 kN/m^2 is studied for both the edges. The numbers of layers in the laminate are varied from 2 to 10 in which the volume of laminate is maintained to be constant by altering the thickness of laminae. The fibre orientations with a gradual variation of θ are taken as 15°, 30°, 45°, 60° and 90°. The maximum deflection of the laminate is evaluated and the results are plotted in figures. **Figure 2** illustrates the deflection for balance angle-ply laminate with simple supported edges. **Figure 3** illustrates the deflection for symmetric angle-ply and cross-ply laminate with simple supported edges. It is found that the deflection of laminate decreases with the increase in numbers of layers, as expected. The minimum value of displacement is obtained at 10-ply laminate and for 45° fibre orientations. **Figure 4** illustrates the deflection for balance angle-ply laminate with clamped edges. The minimum value of displacement is obtained at 10 ply laminate and for 15° fibre orientations. **Figure 5** illustrates the deflection for symmetric angle-ply and cross-ply laminate with clamped edges. Whereas the minimum value of displacement is obtained at 4-ply laminate and for 15° fibre orientations, which is very surprising. The pattern of displacement view for SSSS laminate is found to be concave type and for CCCC laminate is convex type.

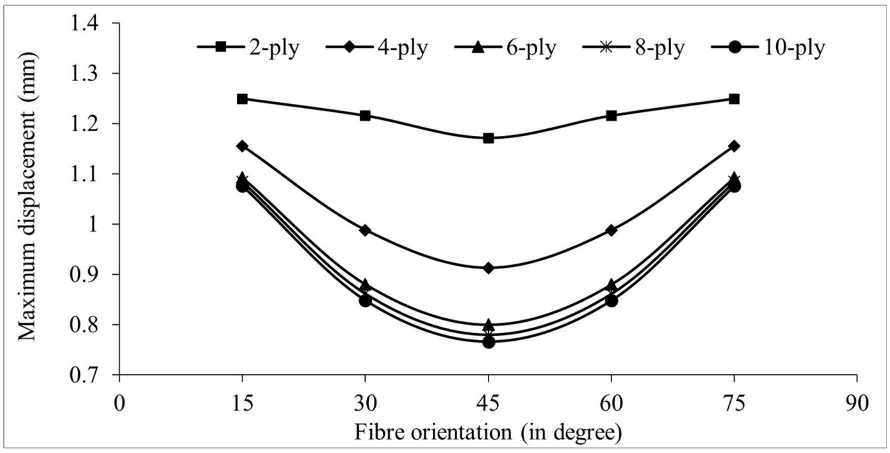

Figure 2. Maximum deflection in balanced angle ply laminates (SSSS).

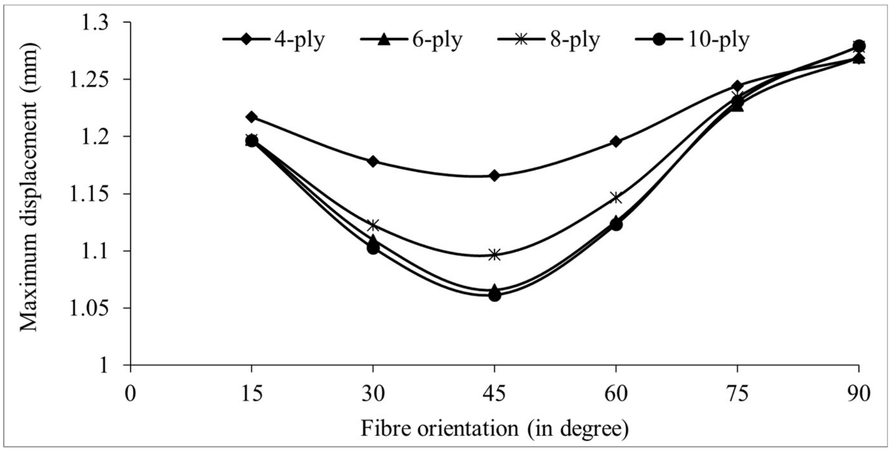

Figure 3. Maximum deflection in symmetric angle ply and cross ply laminates (SSSS).

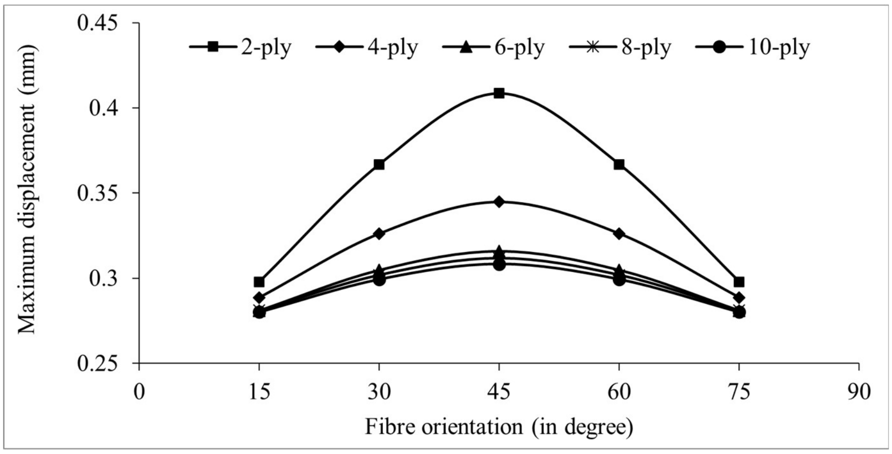

Figure 4. Maximum deflection in balanced angle ply laminates (CCCC).

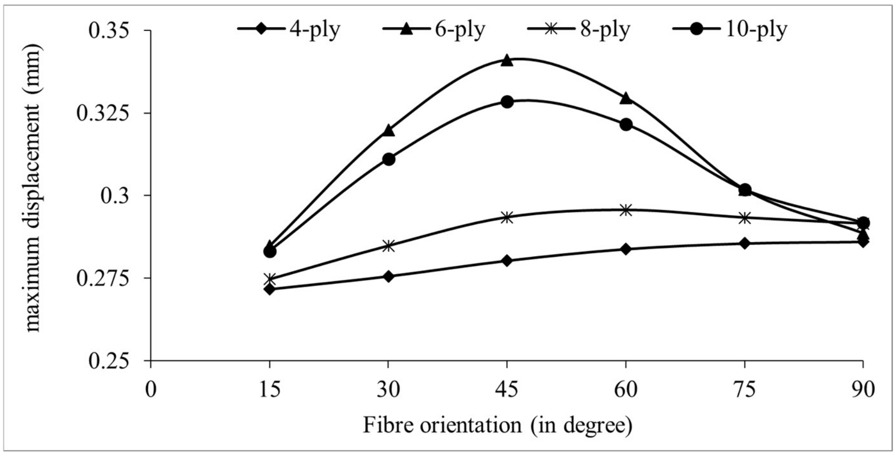

Figure 5. Maximum deflection in symmetric angle ply and cross ply laminates (CCCC).

3.2.2. Unbalance and anti-symmetric ply laminate

As the natural step forward, next to balance and symmetric fibre orientations, the unbalance and anti-symmetric angle-ply and cross-ply laminated plate is studied now. The maximum deflection of the laminate is obtained for the similar conditions as mentioned above and the obtained results are plotted in figures. **Figure 6** illustrates the deflection for unbalance angle-ply laminate with simple supported edges. **Figure 7** illustrates the deflection for anti-symmetric angle-ply and cross-ply laminate with simple supported edges. The minimum value of displacement is obtained at 10-ply laminate and for 45° fibre orientations. **Figure 8** illustrates the deflection for unbalance angle-ply laminate with clamped edges. **Figure 9** illustrates the deflection for anti-symmetric angle-ply and cross-ply laminate with clamped edges. The minimum value of displacement is obtained at 10-ply laminate and for 15° fibre orientations. The similar pattern of displacement view is found in this case.

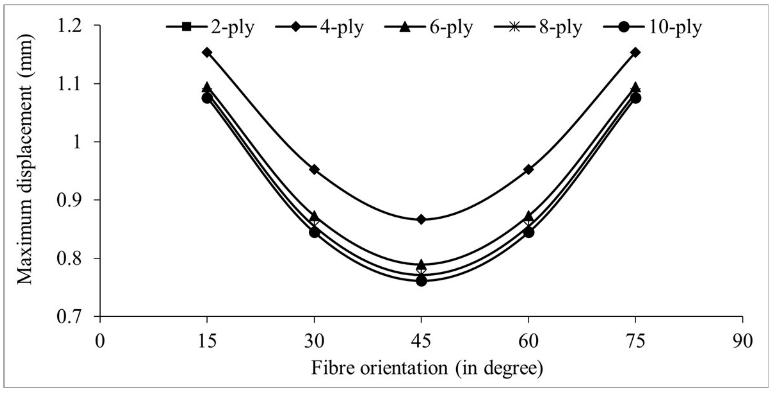

Figure 6. Maximum deflection in unbalanced angle ply laminates (SSSS).

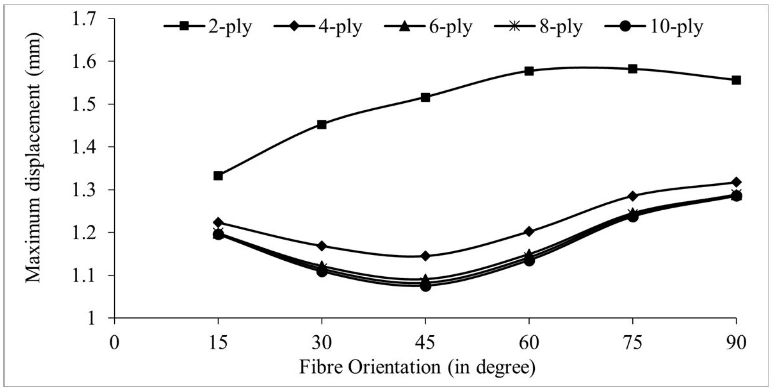

Figure 7. Maximum deflection in anti-symmetric angle ply and cross ply laminates (SSSS).

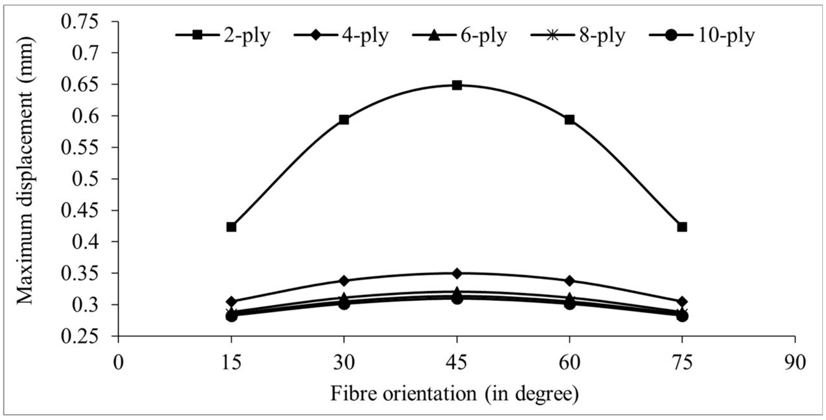

Figure 8. Maximum deflection in unbalanced angle ply laminates (CCCC).

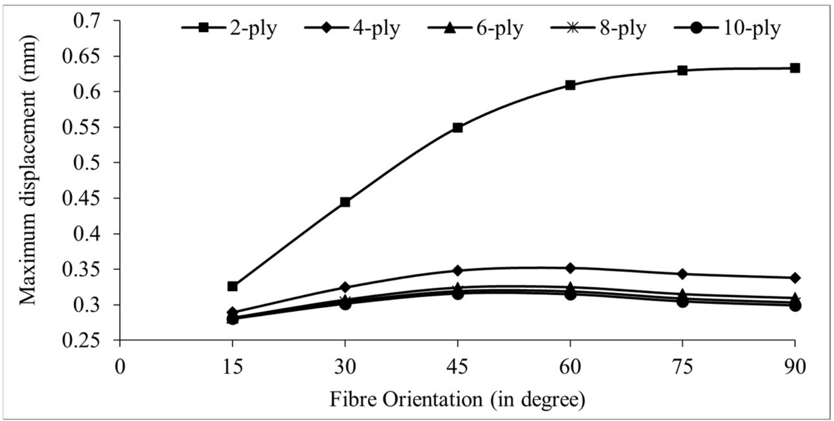

Figure 9. Maximum deflection in anti-symmetric angle ply and cross ply laminates (CCCC).

3.2.3. Displacement view

When all the results are pooled together from **Figures 2–9**, the minimal displacement value can be interpreted. **Figure 10** shows the minimum value of maximum deflection of multilayered laminate for simple supported edges. As expected, the minimum value of displacement is obtained at 10-ply SSSS laminate for unbalance angle-ply fibre orientations. **Figure 11** shows the minimum value of maximum deflection of multilayered laminate for clamped edges. However, the minimum value of displacement is obtained for 4-ply CCCC laminate for symmetric angle-ply fibre orientations.

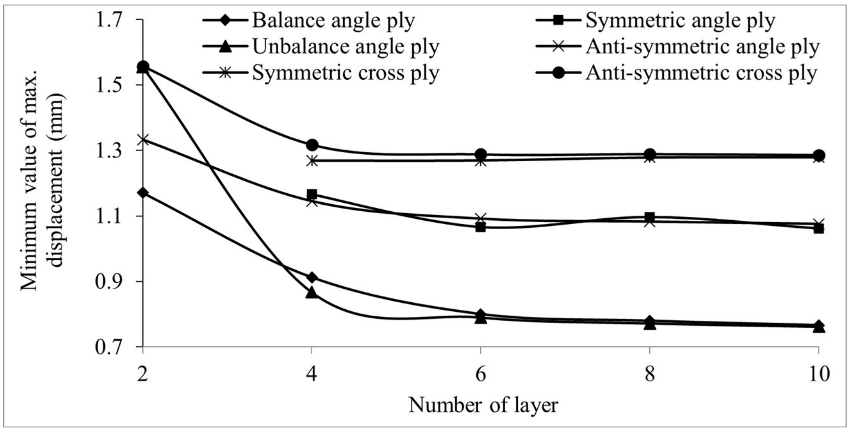

Figure 10. Minimum value of maximum displacement of laminates (SSSS).

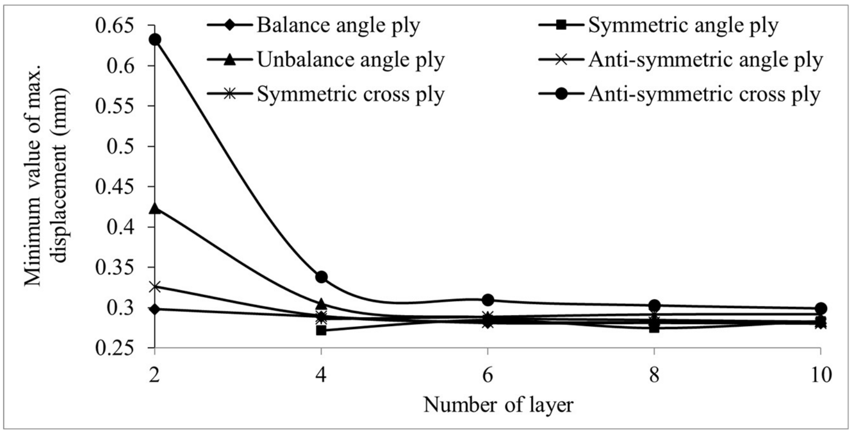

Figure 11. Minimum value of maximum displacement of laminates (CCCC).

4. Conclusions

The aim of the investigation is to find out the maximum strength of a laminate with va suitable combinations of staking sequences, fibre orientations and number of layers. In general, the laminate gives small deflection has more strength. In view of the above, the following conclusions can be drawn from the results obtained.

- In case of symmetric and antisymmetric angle-ply fibre orientations, the minimum value of displacement is obtained at 45° fibre orientation for simple supported edges laminate. If the number of ply increases the displacement value decreases.
- The minimum value of displacement is obtained at 15° or 75° fibre orientation for clamped edges laminate.
- In case of symmetric and anti-symmetric angle-ply laminate, the symmetric ply laminate gives more strength. However, the similar observation is found in the cross-ply laminate.
- In case of simple supported edges laminate, the maximum strength is obtained for unbalance angly-ply laminate. Whereas in clamped edges laminate, the maximum strength is obtained for symmetric angly-ply laminate.
- Angle-ply laminate shows the minimum value of displacement when compared to cross-ply laminate.
- Four layers symmetric cross-ply laminate gives higher strength than the more numbers of layers when all the edges of laminate are clamped.
- These usuful results may enable the engineer to select the fibre orientations and numbers of layers in the laminate productively.

Conflict of interest

The author declares no conflict of interest.

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