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Article

New definitions of isometric latitude and the Mercator projection of the ellipsoid

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Abstract: The article discusses the interrelationships of the loxodrome or rhumb line, isometric latitude, and the Mercator projection of the rotational ellipsoid. It is shown that by applying the isometric latitude, a very simple equation of the rhumb line on the ellipsoid is obtained. The consequence of this is that the isometric latitude can be defined using the generalized geodetic longitude and not only using the geodetic latitude, as was usual until now. Since the image of the rhumb line in the plane of the Mercator projection is a straight line, the isometric latitude can also be defined using this projection. Finally, a new definition of the normal aspect of the Mercator projection of the ellipsoid is given. It is a normal aspect cylindrical projection in which the images of the rhumb line on the ellipsoid are straight lines in the plane of projection that, together with the images of the meridians in the projection, form equal angles as the rhumb line forms with the meridians on the ellipsoid. The article provides essential knowledge to all those who are interested in the use of maps in navigation. It will be useful for teachers and students studying cartography and GIS, maritime, or applied mathematics. The author uses mathematical methods, especially differential geometry. The assumption is that the readers are no strangers to mathematical cartography.

Keywords: map projection; rhumb line; isometric latitude; generalized longitude; ellipsoid

1. Introduction

In this article, we begin with the derivation of the rhumb line equation on a rotational ellipsoid with geodetic parameterization. Then, instead of the geodetic latitude, we introduce isometric latitude as a parameter. It is shown that in this way a very simple equation of the rhumb line on the ellipsoid is arrived at. It is a linear relationship between the isometric latitude and geodetic longitude, with the fact that the geodetic longitude should be taken in a generalized sense, i.e., from the interval $(-\infty, \infty)$. This, in turn, allows us to define the isometric latitude on the ellipsoid using the rhumb line and geodetic longitude.

After that, we consider the normal aspect Mercator projection of the ellipsoid in the usual way and using the isometric latitude. Then we derive the equation of the rhumb line image in that projection. This gives us the possibility of a new interpretation of the isometric latitude using the normal aspect Mercator projection of the ellipsoid. Finally, the idea appears to approach the Mercator projection in a new way. We define it as a normal aspect cylindrical projection in which the images of the rhumb line from the ellipsoid are straight lines in the plane of the projection that make the same angles with the images of the meridians in the projection as the rhumb lines with the meridians on the ellipsoid.

The results of this article are generalizations to the ellipsoid of the results published in the article that dealt with the relationship between the rhumb line, the isometric latitude, and the Mercator projection of the sphere [1].

In geography, latitude is a coordinate that determines the position of a point on the Earth's surface in the north-south direction. It is an angle that ranges from −90° at the South Pole to 90° at the North Pole, with 0° at the equator. Lines of constant latitude, or parallels, run east-west as circles parallel to the equator. Latitude and longitude are used together as a pair of coordinates to determine a location on the Earth’s ellipsoid.

There is relatively detailed cartographic literature on loxodromes, isometric latitude, and the Mercator projection [2−5].

Isometric latitude (see details in section 3) appears in conformal mappings [6,7]. For instance, isometric latitude is used in the derivation of the Gauss-Krüger projection, the normal and transverse aspects of the Mercator projection, and any other conformal map projection [8]. The name “isometric” derives from the fact that at any point on an ellipsoid, equal increments of isometric latitude and longitude lead to equal displacements of distance along the meridians and parallels. A cartographic network defined by lines of constant isometric latitude and constant longitude divides the surface of an ellipsoid into a network of squares (of different sizes). Isometric latitude is equal to zero at the equator but quickly deviates from geodetic latitude, tending to infinity at the poles.

The Mercator projection is one of the most famous map projections. Even in recent times, it has been researched and written about by many, e.g., Kawase [9], Abee [10], Lapaine and Frančula [11], Pápay [12] and Kerkovits [13]. Lapaine and Frančula [11] investigate a new variant of the Mercator projection, the web-Mercator projection. Kerkovits [13] deals with the transverse Mercator projection and the problem of secant cylinders.

The loxodrome and the Mercator projection are closely related to navigation [14−17]. The loxodrome was specially investigated by Alexander [18], Kos et al. [19,20], Elhashash [21], Petrović [22,23], Weintrit and Kopcz [24], Babaarslan and Yayli [25], Kovalchuk and Mladenov [26] and Lambrinos et al. [27].

Petrović [22] considers the rhumb line on the rotational ellipsoid, but only gives equations without a more detailed derivation and without concrete applications. Alexander [18] mainly deals with the historical development and connection of the rhumb line with the Mercator projection. In maritime and air navigation, ships and aircraft sailing or flying along fixed compass directions travel along a rhumb line, so knowing the properties of a rhumb line is important. It is known that the normal aspect of the Mercator projection (cylindrical conformal projection) has the unique property that rhumb lines from the Earth’s ellipsoid are mapped as straight lines on the map. Tseng et al. [28] deal with solving the direct and inverse problem of navigation along the rhumb line.

The primary goal of the study and its novelty are found in the title of the article, i.e., a new definition of isometric latitude and the Mercator projection of the ellipsoid will be explained and given. Scientists from diverse fields will be able to grasp the essential aspects of the research if they know the basics of mathematics, especially differential geometry.

2. The equation of the rhumb line on the rotational ellipsoid

Let us recall that the following expressions define a rotational ellipsoid with the center at the origin of the coordinate system, the semi-major axis a, and the numerical eccentricity e:

$$x = x(\varphi, \lambda) = M \cos\varphi \cos\lambda,\ y = y(\varphi, \lambda) = M \cos\varphi \sin\lambda,$$
$$z = z(\varphi, \lambda) = N(1 - e^2)\sin\varphi \tag{1}$$
$$(\varphi, \lambda) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \times [-\pi, \pi],\ (x, y, z) \in \mathrm{R}^3$$

here, φ is the geodetic latitude, λ is the geodetic longitude,

$$M = M(\varphi) = \frac{a(1 - e^2)}{\sqrt{(1 - e^2\sin^2\varphi)^3}} \tag{2}$$

is the radius of curvature of the meridian and

$$N = N(\varphi) = \frac{a}{\sqrt{1 - e^2\sin^2\varphi}} \tag{3}$$

is the radius of curvature of the section along the first vertical. We call the mapping (1) the geodetic parametrization of the rotating ellipsoid. The coefficients of the first differential form of this mapping are:

$$E = M^2,\ F = 0,\ G = N^2\cos^2\varphi.$$

The differential expressions for any curve on the rotational ellipsoid are [29]:

$$ds^2 = M^2 d\varphi^2 + N^2\cos^2\varphi d\lambda^2 \tag{4}$$
$$\cos\alpha\, ds = M d\varphi \tag{5}$$
$$\sin\alpha\, ds = N\cos\varphi\, d\lambda \tag{6}$$
$$\tan\alpha = \frac{N\cos\varphi d\lambda}{M d\varphi}. \tag{7}$$

where α is the angle between the observed curve and the meridian. Let us agree that the angle α will have a value from the interval $[0,2\pi]$. It is the azimuth that will be measured clockwise so that the relationships shown in **Table 1** apply.

Table 1. Basic relations between geodetic latitude, geodetic longitude and azimuth.

$\varphi_1 < \varphi_2$	$\lambda_1 < \lambda_2$	$\alpha \in \left(0, \frac{\pi}{2}\right)$
$\varphi_1 > \varphi_2$	$\lambda_1 < \lambda_2$	$\alpha \in \left(\frac{\pi}{2}, \pi\right)$
$\varphi_1 > \varphi_2$	$\lambda_1 > \lambda_2$	$\alpha \in \left(\pi, \frac{3\pi}{2}\right)$
$\varphi_1 < \varphi_2$	$\lambda_1 > \lambda_2$	$\alpha \in \left(\frac{3\pi}{2}, 2\pi\right)$
$\varphi_1 = \varphi_2$	$\lambda_1 < \lambda_2$	$\alpha = \frac{\pi}{2}$
$\varphi_1 = \varphi_2$	$\lambda_1 > \lambda_2$	$\alpha = \frac{3\pi}{2}$
$\varphi_1 < \varphi_2$	$\lambda_1 = \lambda_2$	$\alpha = 0$
$\varphi_1 > \varphi_2$	$\lambda_1 = \lambda_2$	$\alpha = \pi$

Let $\alpha =$ const. The differential equation of the rhumb line on the ellipsoid is then e.g., Equation (5), and can be solved as follows:

$$\cos\alpha \int ds = \int M d\varphi,$$

which after integration gives

$$s\cos\alpha = s_m(\varphi) - s_m(\varphi_1), \tag{8}$$

where

$$s_m(\varphi) = \int_0^{\varphi} M d\varphi. \tag{9}$$

Equation (8) is the equation of the rhumb line connecting the geodetic latitude φ and the arc length s. This rhumb line passes through a point with geodetic latitude φ_1 and at that point the arc length is 0.

Unlike the derivation of the rhumb line equation on the sphere, the integral on the right side in Equation (9) is an elliptic integral that appears when calculating the length of the arc of the meridian on the rotational ellipsoid and which cannot be integrated directly, but instead, developments in series or some other mathematical methods are applied. Lapaine [30] showed that for calculating the length of the arc of the meridian from the equator to the geodetic latitude φ, a suitable formula reads

$$s_m(\varphi) = A[\varphi + \sin 2\varphi(c_1 + (c_2 + (c_3 + (c_4 + c_5\cos 2\varphi)\cos 2\varphi)\cos 2\varphi)\cos 2\varphi)] + \cdots \tag{10}$$

where A, c_1, c_2, …, c_5 are corresponding coefficients.

In Equation (10), the length of the arc of the rhumb line is expressed as a function of the geodetic latitude. If it is necessary to express the geodetic latitude as a function of the length of the arc of the rhumb line, then we can use the formula that determines the geodetic latitude of a point on the meridian for which the length of the arc of the meridian from the equator to that point is known [30]:

$$\varphi(s_m) = \psi + \sin 2\psi(c_1 + (c_2 + (c_3 + (c_4 + c_5\cos 2\psi)\cos 2\psi)\cos 2\psi)\cos 2\psi) + \cdots \tag{11}$$

where c_1, c_2, …, c_5 are the corresponding coefficients and

$$\psi = \frac{s_m(\varphi)}{A},\ s_m(\varphi) = s\cos\alpha + s_m(\varphi_1). \tag{12}$$

Rhumb lines on a rotational ellipsoid are generally spiral curves that wrap around each pole an infinite number of times (**Figure 1**) and never reach it, although their length is finite. The length of the rhumb line from pole to pole is equal to the length of the arc of the meridian divided by the cosine of the angle α.

Indeed, in Equation (8) we should put $\varphi_1 = -\frac{\pi}{2}$, $\varphi = \frac{\pi}{2}$, so we get

$$s = \frac{s_m\left(\frac{\pi}{2}\right) - s_m\left(-\frac{\pi}{2}\right)}{\cos\alpha} = \frac{2s_m\left(\frac{\pi}{2}\right)}{\cos\alpha},\ \alpha \neq \frac{\pi}{2},\ \alpha \neq \frac{3\pi}{2}.$$

If we start with the differential Equation (6), we cannot integrate it immediately, but first we should express φ by means of λ or s. Therefore, we prefer to take Equation (7), which can be integrated if we write it in the form

$$d\lambda = \tan\alpha \frac{M d\varphi}{N\cos\varphi}. \tag{13}$$

After integration we get

$$\lambda = \tan\alpha\left[\ln\tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)\left(\frac{1 - e\sin\varphi}{1 + e\sin\varphi}\right)^{\frac{e}{2}}\right] + \beta. \tag{14}$$

We note that according to Equation (14) $\lambda \in (-\infty, \infty)$ for $\varphi \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Therefore, λ is a generalized geodetic longitude [29].

If we want the rhumb line to pass through a point with geodetic coordinates (φ_1, λ_1), it is necessary to take the integration constant β like this.

$$\beta = \lambda_1 - \tan\alpha\left[\ln\tan\left(\frac{\pi}{4} + \frac{\varphi_1}{2}\right)\left(\frac{1 - e\sin\varphi_1}{1 + e\sin\varphi_1}\right)^{\frac{e}{2}}\right]. \tag{15}$$

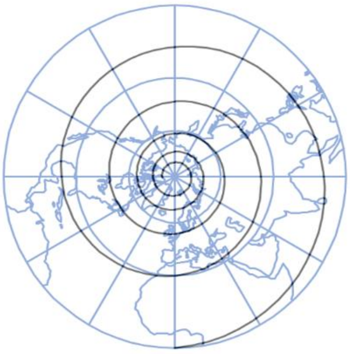

Figure 1. Rhumb line.

3. Isometric latitude and rhumb line

Taught by experience about the isometric latitude and rhumb line on a sphere [1], let us try an analogous approach on a rotating ellipsoid. In the theory of map projections, the isometric latitude q is defined by means of the geodetic latitude φ and the differential equation

$$dq = \frac{M d\varphi}{N \cos\varphi}. \tag{16}$$

The solution of the differential equation (16) is

$$q = \ln \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)\left(\frac{1 - e\sin\varphi}{1 + e\sin\varphi}\right)^{\frac{e}{2}} \tag{17}$$

with the assumption that for the integration constant we took the value that gives $q = 0$ for $\varphi = 0$. Note that $q \in (-\infty, \infty)$ for $\varphi \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

The inverse function cannot be written in its final form using elementary functions. Instead, different approximation procedures or approximate formulas are used [8].

If the conformal latitude χ is introduced as follows

$$\tan\left(\frac{\pi}{4} + \frac{\chi}{2}\right) = \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)\left(\frac{1 - e\sin\varphi}{1 + e\sin\varphi}\right)^{\frac{e}{2}} \tag{18}$$

then between it and the isometric latitude q, analogous to the relations for the sphere [1], the following relations apply:

$$\tanh q = \sin\chi,\ \sinh q = \tan\chi,\ \cosh q = \frac{1}{\cos\chi},$$
$$\tanh\frac{q}{2} = \tan\frac{\chi}{2},\ \exp(q) = \tan\left(\frac{\pi}{4} + \frac{\chi}{2}\right). \tag{19}$$

Furthermore, the differential Equation (7) written by using the isometric latitude q becomes very simple and reads

$$d\lambda = \tan\alpha\, dq. \tag{20}$$

After integration, we get the equation of the rhumb line on the ellipsoid in the form

$$\lambda = q\tan\alpha + \beta, \tag{21}$$

where β is a constant of integration. In that equation, λ is the generalized longitude or longitude in a broader sense. The corresponding value of longitude λ' from the interval $(-\pi, \pi)$ will be obtained as a remainder when dividing by 2π, i.e., by applying the Equation

$$\lambda' = \lambda - 2\pi \operatorname{sgn}(\lambda) \left[\frac{|\lambda|+\pi}{2\pi}\right], \tag{22}$$

where $\operatorname{sgn}(\lambda)$ is equal to 1, 0 or –1, according to whether λ is greater than, equal to or less than zero, and the square brackets indicate the largest integer function, i.e., $[x]$ is the largest integer that is smaller than x or equal to x.

If we want the rhumb line to pass through the point with coordinates (q_1, λ_1), it is necessary to take the integration constant β:

$$\beta = \lambda_1 - q_1 \tan\alpha \,. \tag{23}$$

Special cases

Meridians and parallels are special cases of rhumb lines (see **Table 1**). For meridians, $\alpha = k\pi$, $k = 0, 1$, and for parallels, $\alpha = \frac{\pi}{2} + k\pi$, $k = 0, 1$. Indeed, if we take $\alpha = k\pi$, $k = 0, 1$, then (5) turns into $s = \int_{\varphi_1}^{\varphi} M d\,\varphi$, and (6) into $\lambda = \lambda_1$.

For $\alpha = \frac{\pi}{2} + k\pi$, $k = 0, 1$, (5) becomes $\varphi = \varphi_1$, and the differential equation $ds = N(\varphi_1)\cos\varphi_1\, d\lambda$ gives the solution $s = N(\varphi_1)\cos\varphi_1\,(\lambda - \lambda_1)$.

4. Mercator projection of a rotational ellipsoid

The Mercator projection is a conformal cylindrical projection. This means that the basic equations of the normal aspect projection are

$$x = a\lambda,\ y = y(\varphi), \tag{24}$$

where φ and λ are geodetic latitude and longitude, respectively, a is a constant (it does not have to be the semi-axis of the ellipsoid!), while $y(\varphi)$ is a function that should be determined so that the projection is conformal. Let us suppose that the rotational ellipsoid given by Equation (1) should be conformally mapped into the plane according to Equation (24). The condition for this mapping to be conformal reads [2]:

$$h = k, \tag{25}$$

where *h* and *k* are local linear scale factors along the meridian and parallel, respectively. From the expression [2]

$$h = \frac{dy}{Md\varphi},\ k = \frac{a}{N\cos\varphi}, \tag{26}$$

it follows according to Equation (25)

$$\frac{dy}{Md\varphi} = \frac{a}{N\cos\varphi}, \tag{27}$$

i.e.,

$$dy = a\frac{Md\varphi}{N\cos\varphi}, \tag{28}$$

and from there

$$y = a\int\frac{Md\varphi}{N\cos\varphi} + K = a\ln\tan\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)\left(\frac{1-e\sin\varphi}{1+e\sin\varphi}\right)^{\frac{e}{2}} + K, \tag{29}$$

where *K* is the constant of integration. The constants *a* and *K* can be chosen in different ways. For example, if we set the conditions $\varphi = 0$ and $y = 0$, it follows that $K = 0$. Finally, the normal aspect Mercator projection of the ellipsoid is given by

$$x = a\lambda,\ y = a\ln\tan\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)\left(\frac{1-e\sin\varphi}{1+e\sin\varphi}\right)^{\frac{e}{2}}. \tag{30}$$

At the end, let us note that the equations of the Mercator projection (30) can be written in a very simple form using the isometric latitude q (17)

$$x = a\lambda, \, y = aq. \tag{31}$$

Rhumb line in the normal aspect Mercator projection of the ellipsoid

The equation of the rhumb line on the ellipsoid is Equation (21). If we substitute Equation (21) in Equation (31), we will get the equation of the image of the rhumb line in the Mercator projection

$$x = a(\tan\alpha\, q + \beta), \;\; y = aq. \tag{32}$$

Equation (32) represent the straight line equation in parametric notation. The parameter is the isometric latitude q. By eliminating that parameter, we can obtain the equation of the straight line in an explicit, implicit, or other form.

From Equation (21) we can get

$$q = \cot\alpha\,(\lambda - \beta) \tag{33}$$

and then from Equation (31)

$$x = a\lambda, \;\; y = a\cot\alpha\,(\lambda - \beta). \tag{34}$$

Equations (34) again represent the straight line equation in parametric notation. The parameter is now the generalized longitude λ, $\lambda \in (-\infty, \infty)$. If we need an ordinary geodetic longitude, we can get it using Equation (22). In a similar way, we could write the equation of the rhumb line in the Mercator projection plane parameterized by latitude φ or arc length s.

Figure 2 shows the rhumb line in the normal aspect Mercator projection with constants $\alpha = 75° = \frac{5\pi}{12}, \beta = 0$.

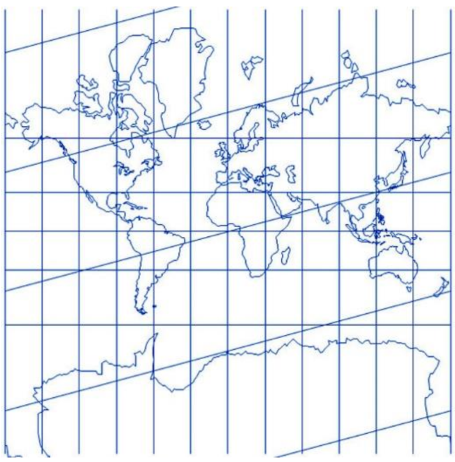

Figure 2. Rhumb line in the normal aspect Mercator projection.

Although the geometric interpretations of latitude and longitude on the ellipsoid as well as geocentric and reduced latitude are well known, a similar interpretation of isometric latitude is not easy to find. For example, Heck [31] says in his famous monograph: "While the geographic latitude φ can be given the geometric meaning of the direction of the normal on the surface, the numerical values of the isometric latitude cannot be clearly interpreted." A connection between rhumb line and isometric latitude was observed, and on this basis a new, very simple definition of isometric latitude was given. This definition is further generalized to the rotational ellipsoid [32]. This is also proof that the isometric latitude, contrary to Heck's claim, can be clearly interpreted.

Now we will give a new definition of the isometric latitude q on the ellipsoid using the Mercator projection of the ellipsoid.

Definition 1. *The isometric latitude of any point on the rotational ellipsoid is proportional to the y-ordinate of the image of that point in the normal aspect Mercator projection,* $q = \frac{y}{a}$. *The proportionality factor is* $\frac{1}{a} = \frac{1}{N(\varphi_0) \cos \varphi_0}$, *where* φ_0 *is the geodetic latitude of the standard parallel. If* $a = 1$, *then the isometric latitude of a point on the ellipsoid is equal to the y-ordinate of the image of that point in the normal aspect Mercator projection.*

5. A new approach to the Meractor projection

A common approach to deriving the equations of the normal aspect Mercator projection is to look for a cylindrical projection that satisfies the conformality condition (section 4 in this article). When we have the equations of the normal aspect Mercator projection, then we derive from them the equation of the rhumb line in that projection and show that it is always a straight line.

The new approach to the derivation of the equations of that projection does not start with setting the conformity condition. Instead, we set the condition that each rhumb line in the normal aspect cylindrical projection is mapped as a straight line. The equations of the normal aspect Mercator projection will emerge from this condition.

When Mercator made his map, he had in mind the rectilinearity of the loxodrome, not its conformality. Thus, the following derivation, in a way, connects Mercator's original idea with today's usual approach to his projection as a conformal cylindrical projection.

Let us start from the equations of a normal aspect cylindrical projection (24), where φ and λ are the geodetic latitude and longitude, respectively, a is a constant, and $y(\varphi)$ is a function to be determined assuming that each rhumb line on the ellipsoid is mapped in the normal aspect cylindrical projection to the straight line, which forms the same angle α with the positive direction of the y axis in the projection plane that the rhumb line encloses with all meridians on the ellipsoid. The equation of the rhumb line on the ellipsoid is (21), where α and β are constants, q is the isometric latitude, and λ is the generalized longitude. If we substitute Equation (21) in Equation (24), we will get the equation of the image of the rhumb line in the plane of the cylindrical projection

$$x = a(q \tan \alpha + \beta), y = y(\varphi). \quad (35)$$

In order for Equation (35) to be the equation of a straight line in parametric form with the parameter q, which closes the angle α with the positive direction of the y axis, the equation for y must be of the form

$$y = aq + b, \quad (36)$$

where b is a constant. According to Equation (17)

$$y = a \ln \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)\left(\frac{1 - e \sin \varphi}{1 + e \sin \varphi}\right)^{\frac{e}{2}} + b. \quad (37)$$

Therefore, the equations of the normal aspect cylindrical projection of the ellipsoid, which has the property that every rhumb line on the ellipsoid that encloses an angle α with the meridians is mapped to a straight line in the projection plane that encloses the same angle α with the images of the meridians are

$$x = a\lambda,\ y = a \ln \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)\left(\frac{1 - e \sin\varphi}{1 + e \sin\varphi}\right)^{\frac{e}{2}} + b. \tag{38}$$

With the usual condition in map projections that $y = 0$ for $\varphi = 0$, it follows $b = 0$, so we have

$$x = a\lambda,\ y = a \ln \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)\left(\frac{1 - e \sin\varphi}{1 + e \sin\varphi}\right)^{\frac{e}{2}}, \tag{39}$$

where we recognize the equations of the normal aspect Mercator projection of the rotational ellipsoid.

6. Conclusion

It is known that instead of the geodetic latitude, it is convenient to introduce the isometric latitude as a parameter when it comes to the issue of preserving angles. We have shown that in this way we arrive at a very simple equation of the rhumb line on the ellipsoid. This is a linear relationship between the isometric latitude and geodetic longitude, with the fact that geodetic longitude should be taken in a generalized sense, i.e., from the interval $(-\infty, \infty)$. This made it possible to define the isometric latitude using the rhumb line and geodetic longitude.

The traditional definition of isometric latitude is found in the article, Equation (17), and has nothing to do with the rhumb line. In the article, it was shown that isometric latitude is in a simple linear relationship with longitude, Equation (21). On the other hand, the ordinate of each point in the Mercator projection is also linearly related to the isometric latitude. From there follows a new definition of isometric latitude based on the Mercator projection.

The traditional definition of the Mercator projection can be found in the article, Equation (30). Since the image of each rhumb line in the Mercator projection is a straight line, thus the image of a linear function, this is a possible new definition of the Mercator projection as a projection in which each rhumb line is mapped as a straight line. So, one property of the Mercator projection gave its definition, and its previous definition as a conformal cylindrical projection became a property of that projection.

The new definitions shed new light on the issue and new insight, thus expanding the field of the theory of map projections. For isometric latitude, it was not known that it could be clearly interpreted with the help of longitude or with the help of the ordinate in the Mercator projection. The Mercator projection is always defined as a cylindrical conformal projection, although in Mercator's time such a classification did not exist. Mercator constructed his projection in a way that is called a new definition in the article, i.e., a projection in which all loxodromes from a sphere or ellipsoid are mapped into straight lines.

Future research could explore the impact of new definitions on solving practical tasks in maritime affairs.

Conflict of interest: The author declares no conflict of interest.

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