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ORIGINAL RESEARCH ARTICLE

Spectral Properties of Transmission Amplitude Gratings

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ABSTRACT

The core components of the precision instruments such as spectroscope and spectrograph is grating. And one-dimensional multi-slot transmission amplitude grating is the most simple, and its basic theory is also an important basis used as reference when perform grating design, this shows the important status of transmission amplitude grating in spectroscopy study. In this paper, the theory of Fraunhofer Diffraction in optics was used in introducing the basic conclusion and spectral pattern characteristics, with the concrete experiment the spectral pattern of transmission amplitude grating in the monochromatic light and complex light is showed, then the theoretical analysis was done by mathematical reasoning and numerical simulation, the specific expression of spectral characteristic parameters in transmission amplitude was study to confirm the effects of these parameters on the produced spectral, results show that these parameters are independently. Through these studies, we understand that in the design of gratings should pay attention to the full consideration of these parameters and how to specifically improve the performance of the grating.

KEYWORDS: transmission amplitude grating, spectral properties, Fraunhofer diffraction

1. Introduction

1.1. The research background and significance of this thesis

The spectrometer and spectrograph is not only used as discovering tools in extending human body function, it is also widely used in the detection of air pollution, water pollution, food hygiene and metal industry, and also in order to find out the elements in items. These instruments recently being actively used in high-tech aerospace activities in order to analyze the chemical composition of unknown celestial bodies.

As we know that the emission of the bright line spectrum is also different when the atoms are different, each element of the atoms have a certain line of the spectrum, each atom can only emit light in certain wavelength with its own characteristics, so the line of bright line spectrum is also called the characteristic lines of atom. The use of atomic characteristic lines can identify matter and study the structure of atoms. But it can not be seen by naked eye. The spectrometer and spectrograph can be a very good spectral analysis, people able to know the spectral analysis through these instruments. Historically these instruments played an important role and help human discover many new elements. For example, rubidium and cesium were found via spectrum by discovering its unknown line characteristics. Spectral analysis is also useful for studying the chemical composition of celestial bodies. At the beginning of the nineteenth century, when studying the solar spectrum, it was found that there were many dark lines in its continuous spectrum, the formation of these dark lines is unknown, then when people understood the cause of the absorption spectrum, it was absorption spectrum of the strong light which emitted from sun through the relatively low temperature of the solar atmosphere. These darklines were analyzed carefully and compared to the characteristics line of various atom, people know the sun's atmosphere contains hydrogen, helium, nitrogen, carbon, oxygen, iron, magnesium, silicon, calcium, sodium and other dozens of elements.

Another great achievement is that from the middle of the 19th century, hydrogen atomic spectroscopy has been one of the important topics in spectroscopy research. In the process of exploring the spectrum of hydrogen atoms, the achievements played important role in the law of quantum mechanics which is one of the most important theory in physic. These rules are not only be applied to hydrogen atoms, satisfactorily explain the cause of the line, but also can be applied to other atoms, molecules and condensed matter.

The core part of the components of these precision instruments is the grating. The grating design must be guided by theory. The transmission grating is the simplest gratings. It is a optical component of flat mirror or metal sheet with a large number of evenly spaced parallel slits with different width (engraved line) and disperse the light based on the principle of slit diffraction (which breakdown as spectrum), thus the theory of transmission amplitude grating is the theoretical basis for people to study. Therefore, it is important to analyze the spectral characteristics of the transmission amplitude grating in detail.

1.2. The research progress and current situation of this thesis

As an undergraduate education, it is not important to introduce a transmission amplitude grating, but rather to introduce typical Fraunhofer single-slit diffraction and rectangular aperture diffraction in optical class. Let the students lay the foundation, familiar with the important theory of optical, cultivate people's theoretical analysis. In contrast, the practical transmission amplitude grating and its theory is less introduced. Although the price of the grating is not particularly expensive, but the manufacturing process is still demanding, thus people continue to study the theory of grating, and continuously improve the manufacturing process, therefore, high requirement towards the companies which is specialized in producing grating design become high.

As time passing, the development of science and technology is fast. Transmission amplitude gratings work as a simplest in the grating have its advantages and disadvantages. The transmission amplitude grating is just an introduction in grating, only being used in universities for experiment purpose. The gratings on the project are varied, and the following is a brief review of the grating research process: the earliest diffraction gratings are winding gratings. In 1786, American scientist Ritenhouse made a width of 12.7 mm of 50 to 60 lines in parallel wires in Philadelphia Diffraction grating. In 1821, Fraunhofer in order to observe the solar spectrum, made of wire diffraction grating, two years later, he was coated with gold on the plane glass, and then made in the gold foil groove with a larger dispersion of the diffraction grating. In 1870, Rutherford used a diamond knife to engrave 3500 lines in the 50 mm wide reflector mirror, which is the world's first grating that it's resolution is comparable to the prism, with great significance.

In the 1880s, Rowland invented diffraction gratings engrave machine and concave gratings spectroscopy device, then gratings spectrometer became the leading role in the field of spectral analysis. Later, Anderson and Wood studied the influence of the grating groove on the light intensity distribution, and then proposed the 'shine' theory of the grating. The 'shine' greatly improved the diffraction efficiency of the grating, most of the light energy can be concentrated at a predetermined diffraction. In 1948, Gabor proposed the holographic optics principle, after the laser was invented. The emergence of a technique specifically for recording laser interference stripe has led to the emergence of 'holographic gratings', which are mainly used as dispersion elements to select and tune the laser output spectrum. With the rapid development of silicon microfabrication technology, people began to use silicon substrate in making grating due to the characteristic of silicon and its compatibility in processing. In 1975, Tang and S. Wang first reported the use of silicon processing technology to produce grating, since then the silicon grating is used in many different areas. With the development of microfabrication technology and the development of binary optical applications, the characteristic size of periodic binary optical element grating is narrowing and its structure becomes more and more complicated. From single-cycle grating to double-cycle cross grating, Medium grating to metal grating, from single-layer grating to multi-layer grating, optical components more and more miniaturization, high efficiency, array. In recent years, the emergence of a series of new gratings on the development of science and technology and industrial production technology innovation also plays an important role in making the grating inside the fiber, resulting in fiber grating, and promote the development of optical fiber communications industry. The combination of grating and waveguide produces an array of waveguide gratings that are very important for optical fiber communication in wavelength division multiplexing devices; grating's femtosecond pulse chirped amplification techniques facilitate the generation of intense lasers; large size pulse compression gratings are the beam splitter for laser fusion device. Darnmann grating is used in photoelectron array lighting technology. The volume holographic grating has been used in the practical stage of optical storage and wavelength division multiplexing.

1.3. The main content of this paper

The main content of this paper is the spectral characteristics of the transmission amplitude grating, and the concrete situation is obtained. In particular, use of optical knowledge to analysis the optical theory of the single-seam Fraunhofer diffraction field theory and simple review how to use light diffraction principle to conduct reasoning analysis, and introduced its basic conclusions and characteristics of the pattern. The specific formula of the light intensity distribution of the single-slot Fraunhofer diffraction is recognized as the diffraction factor for the diffraction intensity distribution of the later transmission amplitude grating.

Then, based on the experiment of transmission amplitude grating, the spectral pattern of the transmission amplitude grating under monochromatic light and complex light irradiation is given. Through the experimental device and diffraction pattern, we also know the specific work of the grating, and the optical path is different when the light go through different slits. And then to conduct the theoretical analysis by mathematical reasoning and numerical simulation. In obtaining the diffraction intensity distribution formula of the transmission amplitude grating, we clearly see that there are two factors, one is the structural factor and the other is the diffraction factor. Combined with the influence of these factors, we also know the specific expression of the spectral characteristic parameters of the transmission amplitude grating and determine the influence of these spectral characteristic parameters on the spectrum it produces. This is also the main purpose of this paper, on the other hand, through these studies, we have deepened the understanding of the working principle of the spectroscope and spectrograph where the grating is located, and know that the core theory of these instruments is the spectral characteristic of the grating. The content of this part of the paper is very large, but also the core of the article.

In the final part, this paper mentioned the concern in the design of gratings, and how to improve the performance of gratings. Combined with the theory of transmission amplitude grating, several spectral characteristic parameters of the transmission amplitude grating are independent of each other, and the design should be considered, so that the grating performance is the best. References were listed in this paper, consists of Chinese and also foreign language reference, which the main help for the paper. The article finally attached a tribute to the mentor and appendix.

2. Diffraction principle of light and Fraunhofer diffraction

2.1. Basic theory of light diffraction

Huygens principle: light waves spread in space, is the spread of vibration, waves induces vibration in the space, any point of wave is consider as new point of vibration center, and the wave following produced from the center is called sub-wave. Sub-wave can produce a new vibration center, continue to issue sub-wave, which makes the light waves continue to spread forward. The new wavefront or the envelope surface is the each sub-wave produced from these vibrating centers. The sub-wave model can easily explain the diffraction phenomenon of light.

Huygens - Fresnel principle: Fresnel developed Huygens' theory of light propagation, he proposed that those sub-wave interference is the co-superposition of the second wave. In particular, the wave of all sub-waves emitted by the center of the wavefront is superimposed on the vibration at point P, that is, the vibration of the wave before the wave is transmitted to point P. The general formula is:

$$U(P) = K \oint_{(\Sigma)} f(\theta_0, \theta) \tilde{U}_0(Q) \frac{e^{ikr}}{r} ds$$

Where the $\frac{e^{ikr}}{r}$ is the spherical wave arriving at the field point; $\tilde{U}_0(Q)$ of the sub-wave source itself; ds is the sub-wave wavefront differential element; f is the tilt factor; K is the proportional coefficient.

Fresnel-Kirchhoff diffraction integration principle: Kirchhoff further clarify the expression of each factor based on the principle of Fresnel, specifically into $U(P) = \frac{-i}{\lambda} \oint_{\Sigma} \frac{1}{2}(\cos\theta_0 + \cos\theta) \tilde{U}_0(Q) \frac{e^{ikr}}{r} ds$

In the case of paraxial diffraction, the boundary conditions combined with Erhf diffraction can be further

$$U(P) = \frac{-i}{\lambda r_0} \oint_{\Sigma} \tilde{U}_0(Q) e^{ikr} ds$$

2.2. Experimental apparatus and pattern of Fraunhofer diffraction

Figure for the diffraction device. Parallel light ray in, with convex lens is imaged in the image side of the square focal plane. This equivalent of each point issued by the sub-wave convergence at infinity. That is, the coherent superposition of parallel light. Where the width of the single slit is a

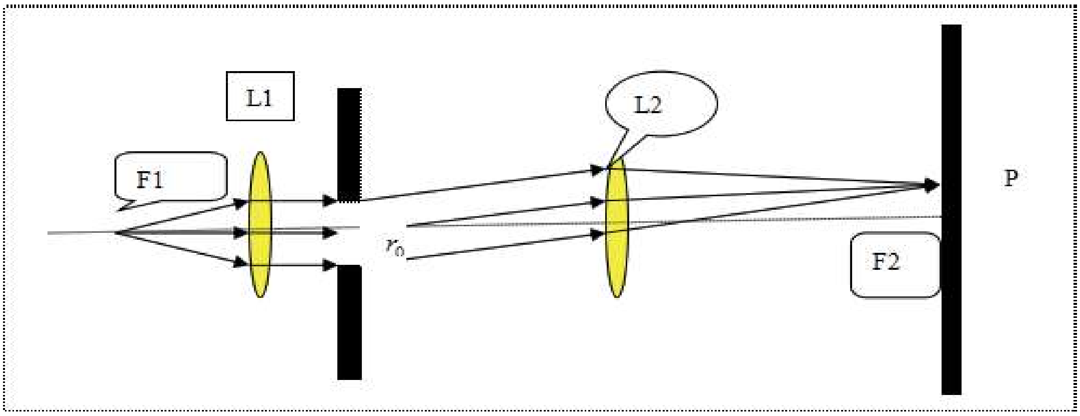

Figure 2.2 (Experimental arrangement of Fraunhofer diffraction) (Section 2.2, figure 1)

The diagram of resulting diffraction experiment is

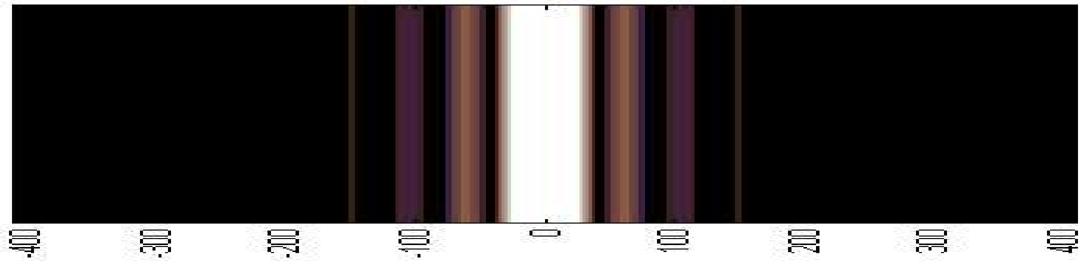

Figure 2.2 (Experimental picture of Fraunhofer diffraction) (Section 2.2, section 2)

2.3. Basic theory analysis of Fraunhofer diffraction

2.3.1 Fraunhofer diffraction intensity distribution

Integral method:

P light is from the same direction, same tilt factor. Different directions of light, to meet the paraxial conditions

The tilt factor is constant. Then on the type into

$$U(P)=\frac{-i}{\lambda r_0}\oint_{\Sigma}\tilde{U}_0(Q)e^{ikr}d\sigma=K\tilde{U}_0\int_{-a/2}^{a/2}\frac{e^{ikr}}{r}dx$$

$\Delta r=-x\sin\theta$ $\qquad$ $r=r_0+\Delta r$ $\qquad$ $K_0=K\tilde{U}_0(Q)$

$$U(P) = K\tilde{U}_0 \int_{-a/2}^{a/2} \frac{e^{ikr}}{r} dx = K_0 \frac{1}{r_0} e^{ikr_0} \frac{i}{-k\sin\theta}[e^{-k\frac{a}{2}\sin\theta} - e^{k\frac{a}{2}\sin\theta}]$$

$$= K_0 \frac{1}{r_0} e^{ikr_0} \frac{-2i(\frac{ka}{2}\sin\theta)}{-k\sin\theta} = K\tilde{U}_0(Q)\frac{1}{r_0}a^{ikr_0} \frac{(\frac{ka}{2}\sin\theta)}{\frac{1}{2}ka\sin\theta}$$

$$= \tilde{U}_0 \frac{\sin u}{u}$$

Where $\tilde{U}_0(Q)\frac{1}{r_0}e^{ikr_0}$ is the complex amplitude where the sub-wave emitted from Point Q along the directions of the optical axis of Point F. $\tilde{U}_0 = K\tilde{U}_0(Q)\frac{1}{r_0}a^{ikr_0}$ is the complex amplitude from the point F on the optical axis when the light passing through the narrow slit. $I_0 = \tilde{U}_0 \tilde{U}_0^*$ is the light intensity at point F on the optical axis. $u = \frac{1}{2}ka\sin\theta = \frac{\pi a\sin\theta}{\lambda}$ is the single slit (unit) diffraction factor.

The intensity distribution is derived from the above $I(P) = I_0 \frac{\sin^2 u}{u^2}$

If the inclination of the incident light is θ_0 ,

Then, $\Delta r = -x\sin\theta_0 - x\sin\theta = -x(\sin\theta_0 + \sin\theta)$ in the same side of the normal line, take +; different side,

take-. $u = \frac{1}{2}ka(\sin\theta_0 \pm \sin\theta) = \frac{\pi a}{\lambda}(\sin\theta_0 \pm \sin\theta)$

2.3.2 Characteristics of Fraunhofer Diffraction Patterns

(1) Extreme point: by the intensity distribution formula

$\frac{d}{d}(\frac{\sin^2 u}{u^2})' = 0$,which is $\frac{2\sin u(u\cos u - \sin u)}{u^3} = 0,$

Thus was $\sin u = 0$, $\tan u = u$

to solve the extreme points where two equations have

The position of the central maximum position of the single seam Fraunhofer diffraction

By the $\sin u = 0$ solution that satisfies $u_0 = \frac{1}{2}k$ $\sin\theta = \frac{\pi a \sin\theta_0}{\lambda}$ that direction $\sin\theta_0 = 0$

(Central maximum position)

That is, in the focus, the strongest light. Here, the sub-wave phase difference is zero, so the amplitude superposition is enhanced with each other.

Single seam Fraunhofer diffraction minimum position

By $\sin u = 0$ solution to satisfy $u_k = \frac{1}{2}k$ $\sin\theta_k = \frac{\pi a \sin\theta_k}{\lambda} = k\pi$

Some of the diffraction direction, ie $\sin\theta_k = k\frac{\lambda}{a}$ $(\pm 1, \pm 2, \pm 3, ...)$

(Minimum position)

These locations are dark spots. This type is called zero point condition of single slit diffraction.

Single seam Fraunhofer diffraction sub - maximum position

There is a maximum value between each of the two adjacent minimum values, and the specific positions of these maxima can be solved by the transcendental equation. By plotting, we can obtain the following values

$$\sin\theta = \pm 1.43\frac{\lambda}{a}, \pm 2.46\frac{\lambda}{a}, \pm 3.47\frac{\lambda}{a}, ...$$

(2) bright stripes angular width (angular distance between adjacent dark stripes)

From the zero point condition of the single-slit diffraction $\sin\theta_k = k\frac{\lambda}{a}$ $(\pm1, \pm2, \pm3, ...)$, θ it can be seen that at very small time, $\theta_k = k\frac{\lambda}{a}$ the angular distance between adjacent dark fringes is $\Delta\theta_k = (k+1)\frac{\lambda}{a} - k\frac{\lambda}{a} = \frac{\lambda}{a}$

And the zero order is very large $\Delta\theta_0 = \frac{\lambda}{a} - (-1)\frac{\lambda}{a} = 2\frac{\lambda}{a}$, and its half width is the $\Delta\theta_0 = \frac{\lambda}{a}$ inverse relationship of diffraction.

3. Transmission of diffraction gratings

3.1. Overview of transmission amplitude gratings

There are two types of grating: reflection grating and transmission grating. We have only discussed the diffraction of the transmission grating, the commonly used transmission grating is made of optical glass, engraved on the glass plate with a large number of even spacing of the parallel notation, in the notch, due to the glass become hairy, the incident light scattering, almost opaque, and in between the two djacent marks the smooth part equivalent to the slit, allow the light ray in. Refined grating has up to 10,000 lines with 1 cm with. If the width of the engrave is a, $a+b$ it is called the grating constant, when the lines is 10000, then the grating constant is $a+b = 1/10000cm = 10^{-4}cm$.

When a bundle of parallel monochromatic light is incident on the grating vertically, the diffraction effect can be generated for each slit in the grating and the respective diffraction patterns are formed on the screen. The distribution of the grating diffraction fringes is different in the case of single-slit diffraction, the width of the central stripes is large, and the width of the other lines is smaller and the intensity decreases with the order. The more number of slits is in the grating diffraction, the stripes become more bright and narrow, and each other is separate clearly, that is, the bright stripes between the dark areas is widening. In the grating the width of each slit is same, they produce the same intensity and position coincidence on the screen. But since the gratings contain a series of parallel slits of equal area and the beams emitted from the slits interfere with each other, the final formation of the grating diffraction stripes is not just a single slit and the more important is the interference between the beams emitted by many slits, that is, the interference effect of the multi-beam. Therefore, the grating diffraction stripes are based on the single-slit diffraction, is the total effect of the interference.

3.2. Experimental model of monochromatic light irradiation transmission amplitude grating

Diagram showed the experimental device according to the spatial orientation. It can be seen from the figure that it is similar to the experimental device of single-seamed of Fraunhofer diffraction, but only change from single slit to transmission amplitude grating.

experimental picture obtained

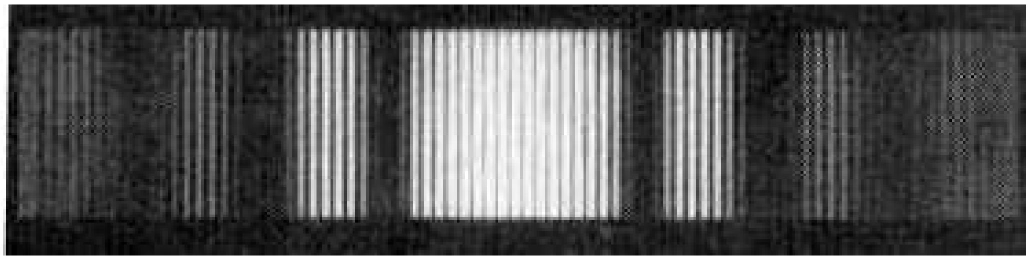

Figure 3.2 (monochromatic light transmission amplitude grating experimental picture) (Section 3.2 of the first two)

3.3. Theoretical analysis of monochromatic light irradiation transmission amplitude grating

3.3.1 Diffraction intensity distribution of monochromatic light irradiation transmission amplitude gratings

Monochromatic light is incident parallel to meet the paraxial conditions. The formula in the paraxial state can be used and the diffractive part of the diffractive screen is only integrated, that is, the pupil function of the opaque part is zero.

$$\tilde{U}(P) = \frac{-i}{\lambda r_0} f \oint_{\Sigma} \tilde{U}_0(Q) e^{ikr} d\Sigma = \frac{-i\tilde{U}_0(0)}{\lambda r_0} f \int_{\Sigma} e^{ikr} d\Sigma$$

$$= \frac{-i\tilde{U}_0(0)}{\lambda r_0} f \sum_{j=1}^{N} [\int_{\Sigma_j} e^{ikr} d\Sigma_j]$$

Where N is the total number of slits, and the latter part is the integral of each slit, and the vibration caused by the diffraction of the incident light through the slit is obtained at the point P, i.e., the complex amplitude,

The sum of all the slits is to superimpose the vibrations caused by the light emitted by each slit at the point P, which is superimposed on each other, for the interference of light. The physical process is that the light of each single slit is diffracted at the point P and then the complex amplitude is diffracted.

Each slit width of the transmission amplitude grating is b, the width of the opaque portion is a, a + b = d, d is the distance from the center of the adjacent slit, that is, the period of the grating. The constant a set here is different from the meaning of the single slit width a in the single seam of Fraunhofer, and the difference is calculated.

It can be seen from the figure when the light emitted by the center of the slit reaches the optical path of the screen L_j, there are: $L_2 = L_1 + \delta, L_3 = L_1 + 2\delta, . \quad . \quad . \quad L_n = L_1 + (n-1)\delta$

$\delta = d \sin\theta$ The light emitted by the adjacent two slit centers reaches the optical path difference of point P.

In the j-th slit, the light emitted from the point light source and the light emitted from the center of the slit reach the optical path difference $\Delta r_j = -x_j \sin\theta$ of point P, that is $r_j = L_j - x_j \sin\theta$, the above-

$$\tilde{U}(P) = \frac{-i\tilde{U}_0(0)}{\lambda r_0} f \sum_{j=1}^{N} [\int_{\Sigma_j} e^{ikr_j} d\Sigma_j] = \frac{-i\tilde{U}_0(0)}{\lambda r_0} f \sum_{j=1}^{N} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{ikL_j - ikx_j \sin\theta} dx_j$$

$$= \frac{-i\tilde{U}_0(0)}{\lambda r_0} f \sum_{j=1}^{N} e^{ikL_j} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{-ikx_j \sin\theta} dx_j = [\frac{-i\tilde{U}_0(0)}{\lambda r_0} fb \frac{\sin u}{u}] \sum_{j=1}^{N} e^{ikL_j}]$$

It can be seen that the results of the single-slit diffraction in the preceding [] are the same for each slit; the result of the interference between the multiple joints in the following [].

The vibration at point P in the end is the product of both. The above formula is further

$$\tilde{U}(P) = [\frac{-i\tilde{U}_0(0)}{\lambda r_0} fb \frac{\sin u}{u}] \sum_{j=1}^{N} e^{ikL_j}] = [\frac{-i\tilde{U}_0(0)e^{ikL_1}}{\lambda r_0} fb \frac{\sin u}{u}] \sum_{j=1}^{N} e^{k(j-1)d\sin\theta}]$$

$$= \tilde{U}_0[\frac{\sin u}{u}] \sum_{j=1}^{N} e^{k(j-1)d\sin\theta}]$$

Where in a single set

$$\tilde{U}_0 = \frac{-i\tilde{U}_0(0)e^{ikL_1}}{\lambda r_0} fb \qquad U(\theta) = \frac{\sin u}{u} = \frac{\sin(\frac{\pi b}{\lambda}\sin\theta)}{\frac{\pi b}{\lambda}\sin\theta}$$

Element (single seam) diffraction factor, determined by the pupil function.

$$\tilde{N}(\theta) = \sum_{j=1}^{N} e^{k(j-1)d\sin\theta} = \frac{1 - e^{2N\frac{\pi d}{\lambda}\sin\theta}}{1 - e^{2i\frac{\pi d}{\lambda}\sin\theta}}$$

Separate

$$\beta = kd \sin\theta = \frac{\pi}{\lambda} d \sin\theta$$

So that on the type into $$\tilde{N}(\theta) = \frac{e^{iN\beta}}{e^{i\beta}} \frac{e^{-N\beta} - e^{N\beta}}{e^{-i\beta} - e^{i\beta}} = e^{i(N-1)\beta} \frac{\sin(N\beta)}{\beta} = e^{i(N-1)\beta} N(\theta)$$

Where $N(\theta)$ is the interference factor.

Through the above derivation, now we can see $\tilde{U}(P)=\tilde{U}_0 U(\theta)e^{i(N-1)\beta}N(\theta)$

The light intensity at the point P, that is, the light intensity distribution $I(P)=I_0(\frac{\sin u}{u})^2(\frac{\sin N\beta}{\sin\beta})^2$

3.3.2 Numerical simulation of the diffraction intensity of a monochromatic light irradiation transmission amplitude grating

By diffraction, the intensity distribution function of the transmission amplitude grating by monochromatic light, we know that there are two factors, but the intuitive image of how it is distributed is not clear. In this paper, the diffraction intensity distribution function of monaural light transmission amplitude grating is numerically simulated with MATLAB, and its function curve is drawn to establish the intuitionistic image for the further study of the spectral characteristics of the transmission amplitude grating. This figure is a special case of the diffraction factor, the structural factor and the total diffraction intensity distribution, and its specific preparation is given later in the appendix.

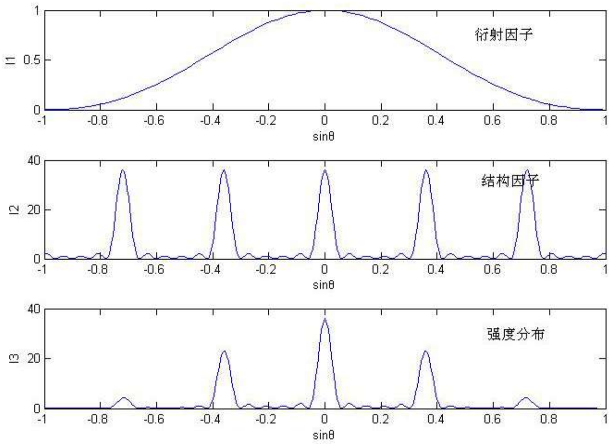

Figure 3.3.2 (Monochromatic light irradiation transmission amplitude grating diffraction intensity numerical simulation) (Section 3.3.2, first figure)

3.3.3 Spectral characteristics of monochromatic light irradiation transmission amplitude gratings

Diffraction maximum position and missing position:

The position of this series of bright stripes can be obtained by the light intensity distribution function, ie $\beta=k\pi$,

that is $d\sin\theta_k=k\lambda,\qquad k=0,\pm1,\pm2,\ldots$

Have a maximum value $(\frac{\sin N\beta}{\sin\beta})^2=N^2$ at this time $I(\theta_k)=N^2\times i(\theta_k)$. The visible spectral position is independent of N and is determined by d, j, λ. But the strength is N^2, this is a very large number because the number of grating slits is very large. Considering the influence of another factor, it is clear that the factor is the same as the intensity distribution function of the diffraction of the single-spin, and therefore it can be called the diffraction factor.

When the maximum value of the interference and the minimum value of the diffraction coincidence occurs, there will be a missing phenomenon, the following discussion on the occurrence of the phenomenon of missing level

By the multi-seam k-level main peak position $d\sin\theta_k = k\lambda, \qquad k = 0, \pm 1, \pm 2, \ldots$

Single seam grade main peak position $b\sin\theta_{k'} = k'\lambda, \qquad k' = 0, \pm 1, \pm 2, \ldots$

When $\theta_k = \theta_{k'}$ satisfied, it means that the k-level main peak position happens to fall in the single point of the first k' zero point position, then $I(\theta_k) = 0$ the K-class main peak disappears. Which was so $\frac{d}{b} = \frac{k}{k'}$

Diffraction min. Position:

Diffraction factor $U(\theta) = 0, \qquad \sin u = 0, \qquad u \neq 0$

$$b\sin\theta = n\lambda, \qquad n = \pm 1, \pm 2, \pm 3, \ldots\ldots$$

Interference factor $N(\theta) = 0, \qquad \sin N\beta = 0, \qquad \sin\beta \neq 0$

$$d\sin\theta = k\frac{\lambda}{N}, \qquad d\sin\theta \neq j\lambda$$

as k ≠ jN?

(Λ / N), N (λ / N), (N / 1) (λ / N), N (1) (Λ / N), ... 2N (λ / N), ... j (λ / N), ... between the two main values are N-1 is the minimum, N-2 secondary is big.

The maximum width of the main peak of the diffraction peak:

K-level main peak, the left and right of the first zero point that is $(\theta_k \pm \Delta\theta)$ the location of dark spots should be satisfied

$$d\sin(\theta_k \pm \Delta\theta) = (k \pm \frac{1}{N})\lambda$$

At this point, the structural factor is zero and the denominator is nonzero. Which is not difficult to export

$$d\cos\theta_k * \Delta\theta = \frac{\lambda}{N}$$

Thus, the formula of the half-width of the k-level main peak is obtained

$$\Delta\theta_k = \frac{\lambda}{Nd\cos\theta_k},$$

This indicates that the larger the grating size, the smaller the width of the main peak half-width, the two are inversely proportional relationship

In addition to introduce oblique incident when the situation, then the optical path difference $\delta = -d(\sin\theta_0 \pm \sin\theta)$

3.4. Theoretical analysis of transmission grating with complex light irradiation

3.4.1 Multi-color light irradiation transmission amplitude grating introduction

Complex light exposure is also a real concern in the practical, and the direct theory which required from the instrument when we conducting spectral analysis. Therefore, it is important to discuss the spectrum obtained by studying the complex color light transmission amplitude grating. To analyze the characteristics of the spectrum in actually is to analysis the different wavelengths of monochromatic light irradiation pattern. The intensity distribution of each monochromatic light has been given by the above analysis.

3.4.2 Spectral characteristics of transmission light gratings with complex light irradiation

The grating spectrometer is a dispersive spectrometer. Where the dispersion type spectrometer has three basic performance indicators - the role of scattered ability, line dispersion skills and color resolution skills. The role of scattered ability is the ability to separate the different wavelengths of light. Now analyze the three performance indicators of the transmission amplitude grating expression:

First set the wavelength of the k-th line, the diffraction angle θ_k , the location of the l wavelength $\lambda + \delta\lambda$ of the k-th line, the diffraction angle $\theta_k + \delta\theta_k$, the position is $l + \delta l$.

(1) the transmission amplitude grating of the ability to disperse

The role factor is defined as $D_\theta \equiv \frac{\delta\theta}{\delta\lambda}$ the diffraction angle interval produced by the difference in the unit wavelength of the k-th spectral line wavelength.

For the transmission amplitude grating, according to the grating equation $d\sin\theta_k = k\lambda, \quad k = 0, \pm 1, \pm 2, \ldots$,Both sides of the differential

$$d\cos\theta_k \delta\theta = k\delta\lambda\,, \quad k = 0, \pm 1, \pm 2, \ldots$$

So there are $D_\theta \equiv \frac{\delta\theta}{\delta\lambda} = \frac{k}{d\cos\theta_k}$

Obviously reduce d , but increase k the ability of dispersion, for higher levels of the spectrum, the dispersion ability can be further increased. And the zero-order spectrum is free of dispersion, that is, all zero-order spectral lines of different wavelengths are concentrated at the same position because the optical path difference of the zero-order spectrum is equal to zero.

(2) transmission line grating line dispersion ability

The line dispersion capability is defined as $D_l \equiv \frac{\delta l}{\delta\lambda}$ the spatial line distance interval produced by the adjacent unit wavelength difference of the k-th spectral line wavelength.

For the transmission amplitude grating, the relationship between the line distance interval δl and the angular interval $\delta\theta$ is approximately

$$\delta l \approx f\delta\theta$$ f - the focal length of the lens after the grating,

So there are
$$D_l = f \cdot D_\theta = f\frac{k}{d\cos\theta_k}$$
in order to increase the line dispersion ability, but also increase the lens focal length (often up to several meters).

(3) transmission amplitude grating color resolution ability

Dispersion skills only reflect the separation of the main line, but can not explain whether the lines overlap, because the line itself is a width, therefore the color resolution skills was introduced.

The color resolution capability is defined as $R \equiv \frac{\lambda}{\delta\lambda}$ the number of wavelengths that are resolved by the adjacent unit wavelength difference at the k-th spectral line wavelength.

For transmission amplitude gratings, taking into account that each principal itself has a half-width $\Delta\theta_k$, the monochromatic light has been given its expression. So the angular interval $\delta\theta$ between the two main strong between the intensity distribution of the overlap may occur. According to Rayleigh, when $\delta\theta > \Delta\theta_k$, you can distinguish between two lines; $\delta\theta < \Delta\theta_k$, cannot tell the boundaries of the two lines; to $\delta\theta = \Delta\theta_k$, as can distinguish the boundaries of the two lines. Distinguish the minimum wavelength interval: according to

$$\delta\theta = D_\theta\delta\lambda = \frac{k}{d\cos\theta_k}\delta\lambda \quad , \quad \Delta\theta_k = \frac{\lambda}{Nd\cos\theta_k}$$

Make $\delta\theta = \Delta\theta_k$ it $\delta\lambda_m = \frac{\lambda}{kN}$

So available
$$R \equiv \frac{\lambda}{\delta\lambda} = kN$$

At this point, and the k-th spectral line has just been able to distinguish, the k-th main maximum, should be with the k-level line of the inner edge of coincidence.

In addition to the above three performance indicators, but also introduced another important parameter -

(4) Free spectral range, ie dispersion range

As the dispersion is proportional to the level k, for a certain light band, with the level increase, expand the wider, there will be spectral overlap phenomenon. For example, in the visible spectrum band (400nm-760nm), there is a spectral overlap from the second stage, the long band of the second order spectrum and the short band of the third stage overlap; the third stage of the long band and the fourth stage of the short band overlap ....

When the k + 1th order of the shorter wavelength of the spectrum coincides with the k-th order of the longer wavelength of the spectrum, $(k+1)\lambda_1 = k(\lambda_1 + \Delta k)$

which is $(\Delta\lambda)_f = \frac{\lambda_1}{k}$

Where, $(\Delta\lambda)_f$ for the free spectral range.

The wavelength range is $\lambda_m \sim \lambda_M = \lambda_m + \Delta\lambda$ from the above $\Delta\lambda < \frac{\lambda_m}{k}$ formula $\lambda_M - \lambda_m < \frac{\lambda_m}{k}$. Grating equation requirements $\lambda_M < d$.

Thus, in the first order spectrum $\lambda_m > \frac{1}{2}\lambda_M$, the free spectral range of the first order spectrum is $(\frac{1}{2}\lambda_M, \lambda_M)$, ie $(\frac{1}{2}d, d)$.

4. Combined with the theory of transmission amplitude gratings in the design and use of grating should pay attention to the problem

For the transmission amplitude grating of the three performance indicators, Dispersion ability, line dispersion skills and color resolution skills, each with independent functions, each other can not be replaced. In the design and use, to consider the coherence of the three. As mentioned above, in order to improve the performance of the ability to reduce the grating constant, if you can also observe the higher level of the spectrum, the dispersion ability can be further increased; in order to improve the line dispersion ability, can increase the lens focal length; and to achieve the performance of the realization, the requirements must met.

Of course, we can not blindly pursue a small grating constant, but should be based on the spectral range to made the appropriate choice. Because if choose $d \leq \lambda$, according to the grating equation $d \sin\theta_k = k\lambda, \quad k = 0, \pm 1, \pm 2, \ldots$, it is very difficult to solve the solution, and then it is unable to see Fraunhofer far field spectrum. Therefore it cannot blindly pursue the smaller grating constant.

Through the above analysis we can also find that θ if the grating spectrum is not observed at all, $\cos\theta$ almost do not change θ , so D_θ and D_l almost are constant. At this time the spectrum is called the uniform spectrum. A convenient spectroscopic method can be used when measuring unknown wavelengths based on a convenient method of spectral spectrum.

5. Conclusion

Based on the theory of diffraction in optics and the theory of diffraction field, and the experimental results are given, the spectral pattern of the transmission amplitude grating under monochromatic light irradiation is obtained, and then by using detailed mathematical calculus and the numerical simulation to conduct the analysis theoretically, and the concrete expression of the spectral characteristic parameters of the transmission amplitude grating is obtained.

From the specific expression of the spectral characteristic parameters of the transmission amplitude grating, we know the three basic performance indicators of the dispersion type spectrometer - the factor of the dispersion, the line dispersion ability and the grating constant of the chromatic abstraction grating. The diffraction angle, the number of wavelengths of the measured light even related to the number of slits and the lens focal length used in the experiment. The results also show that these parameters are independent of each other. With these finding, we can consider the problem of these parameter when design gratings and how to specifically improve the performance of the grating.

Appendix A M file preparation for MATLAB simulation of diffractive intensity numerical simulation of monochromatic light irradiation

After entering the MATLAB program, first write the M command file, save the file name yanshe.m

Numerical Simulation of the Diffraction Intensity of the Transmission Amplitude Grating with Monochromatic Light

Disp ('This M command file is a numerical simulation of the diffraction intensity of the transmission amplitude grating for monochromatic light. The following are some of the input constants');

N = input ('\\ nPlease enter the number of slits in the transmission amplitude grating. Note here for the demo, just enter 1-6 on the line \\ nN = \\ n');

D = input ('\\ nPlease enter the grating constant of the transmission amplitude grating. The laboratory is generally 1 / 600mm, ie 1666nm \\ nd = \\ n');

B = input ('\\ nPlease enter the light transmission part of the grating constant of the transmission amplitude grating \\ nb = \\ n');

L = input ('\\ nPlease enter the wavelength measured with the transmission amplitude grating \\ nl = \\ n');

Subplot (3,1,1), fplot ('danfeng', [- 1,1], [], [], [], b, l);

Xlabel ('sinθ'); ylabel ('I1'); gtext ('diffraction factor');

Subplot (3,1,2), fplot ('jiegou', [- 1,1], [], [], [], N, d, l);

Xlabel ('sinθ'); ylabel ('I2'); gtext ('structural factor');

Subplot (3,1,3), fplot ('duofeng', [- 1,1], [], [], [], N, d, b, l);

Xlabel ('sinθ'); ylabel ('I3'); gtext ('intensity distribution');

Also write three M function files, as follows

The first saved as danfeng.m file, the specific content is

Function I = danfeng (sint, b, l)

Numerical simulation of the diffraction factor in the diffraction intensity of the monochromatic light irradiation transmission grating

I = (sin (pi * b / l * sint) ./ (pi * b / l * sint)). ^ 2;

The second saved as jiegou.m file, the specific content for

Function I = jiegou (sint, N, d, l)

% The numerical simulation of the structural factors in the diffraction intensity of the monaural light irradiated transmission amplitude grating

I = (sin (N * pi * d / l * sint) ./ sin (pi * d / l * sint)). ^ 2;

The third saved as duofeng.m file, the specific content is

Function I = duofeng (sint, N, d, b, l)

Numerical Simulation of Diffraction Intensity Distribution of Transmission Amplitude Gratings with Monochromatic Light

(Sin * (pi * b / l * sint). / (Pi * b / l * sint)). ^ 2. * (Sin (N * pi * d / l * sint). / Sin (pi * d /l*sint)).^2;

Appendix B Preparation of Command Window for MATLAB Numerical Simulation of Diffraction Intensity in Monochromatic Light Irradiation

This appendix gives the specific parameters of the graph given in Section 3.3.2, 'Numerical Simulation of the Diffraction Intensity of a Monochromatic Light Irradiation Amplitude Grating'. This is entered by the matlab command window to ensure that the above file The versatility. The command window is written as follows

>> yanshe

The M command file is a numerical simulation of the diffraction intensity of the monaural light irradiation transmission amplitude grating. The following are some of the input constant

Please enter the number of slits in the transmission amplitude grating. Note here for the demo, just enter 1-6 on the line

N =6

Enter the grating constant of the transmission amplitude grating. Laboratory is generally 1 / 600mm, or 1666nm

D =1666

Enter the light transmission section of the grating constant of the transmission amplitude grating

B =600

Please enter the wavelength measured with the transmission amplitude grating

L =600>>

After entering the above values as required, a graph of section 3.3.2 is displayed in the graphics window.

References

1. Zhong Xihua. Modern optical foundation [M]. Beijing: Peking University Press, 2004: 31-267
2. Zhao Kaihua. New Concept Physics Course - Optics. Beijing: Higher Education Press, 2004: 1--200
3. the mother of light, war Yuanling. Optical [M]. Second Edition. Beijing: Higher Education Press, 2005: 237-245
4. Zheng Zhuren. Optical [M]. Beijing: Physics [M]. Harbin: Harbin Institute of Technology Press, 2006: 138-166
5. Zhu Zimin. Physical optics [M]. Wuhan: Huazhong University of Science and Technology Press, 2009: 146-152
6. Liang Zhu. Optical Principles of the tutorial [M]. Beijing: Beijing University of Aeronautics and Astronautics Press, 2005: 192-200
7. Wang Hongmin. Engineering Optics [M]. Beijing: Peking University Press, 2009: 216-219
8. Wu Jian, Yan Gang. Optical Principles of the tutorial [M]. Beijing: National Defense Industry Press, 2007: 141-177
9. Cai Lizhong. Optical [M]. Beijing: Science Press, 2007: 219-270
10. Yao Qijun. Optical tutorial [M]. Fourth edition. Beijing: Higher Education Press, 2008: 70-156
11. Xuan Guixin. Optical [M]. Shanghai: East China Normal University Press, 2006: 1-190
12. Ye Yutang, Rao Jianzhen, Xiao Jun. Optical tutorial [M]. Beijing: Tsinghua University Press, 2005: 270-273
13. Born M, wolf E. Principle Of Optics [M]. 7th ed. London: Cambridge University Press, 1999
14. Hecht E, Iajac A. Optics [M] .Massachusetts: Addision-wesley, 2003
15. Jenkins F A, White H E. Fundamentals Of Optics [M] .4th ed. New York: Mcgraw-hill, 1976