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Fuzzy Soft LA-(m,n)-Γ-ideals in LA-Γ-semigroups

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ABSTRACT

In this paper, we introduce some certain fuzzy soft algebraic notions of generalized concepts in LA-Γ-semigroups and study some properties of their families.

***Keywords*:** (Fuzzy) Soft Sets; LA-Γ-semigroup; (Fuzzy) LA-(m,n)-Γ-ideals

1. Introduction

In[25], Zadeh introduced the notion of fuzzy subset. The notion of a fuzzy ideal in Γ-ring was first introduced by Jun and Lee[10]. The concept of an LA-semigroup (also known as AG-groupoids) was defined by Kazim and Naseeruddin[12]. The notion of Γ-semigroup was introduced by Sen[19,20].

Shah and Rehman[22], introduced the notion of LA-Γ-semigroup (Γ-AG-groupoid) and discussed some properties of Γ-ideals and Γ-bi-ideals in Γ-AG-groupoids. Moreover in[23], they defined fuzzy Γ-ideals in a Γ-AG-groupoid and studied some of its properties. Abbasi and Basar[1], introduced the notion of (m,n)-Γ-ideal of an LA-Γ-semigroup. In[3], Akın investigated generalizations of some concepts in LA-Γ-semigroup.

Fuzzy soft sets which has drawn a steadily increasing attention of the researchers and has led to remarkable development in some research areas also has been a very dynamic area for the algebraists. In this paper, we study some certain concepts of the fuzzy soft sets in LA-(m,n)-Γ-semigroups.

2. Preliminaries

In this section we recall certain definitions and results in the notion of LA-Γ-semigroups from the references[1,3,13,14,22,23,25] and we also recall certain definitions in the concept of (fuzzy) soft sets from the references[2,4-9,11,15-18,24].

2.1 LA-Γ-semigroups

Let S and Γ be nonempty sets. We call S to be an LA-Γ-semigroup if there exists a mapping S×Γ×S→S, written as (a,γ,b) and denoted by aγb such that S satisfies the identity (aγb)αc=(cγb)αa for all a,b,c∈S and γ,α∈Γ. If S is an LA-Γ-semigroup and A,B⊆S, then we denote AΓB:={aγb|a∈A,b∈B and γ∈Γ}. For a positif integer m, the power of B is defined as follows:B^m=(...((BΓB)ΓB)...)ΓB.

Example 2.1

(i) ([22], Example 2) Let $\Gamma=\{1,2,3\}$. Define a mapping $\mathbb{Z}\times\Gamma\times\mathbb{Z}\to\mathbb{Z}$ by $a\gamma b=b-\gamma-a$ for all $a,b\in\mathbb{Z}$ and $\gamma\in\Gamma$, where "$-$" is a usual subtraction of integers. Then $\mathbb{Z}$ is a LA-Γ-semigroup.

(ii) Let $S=(0,+\infty)=\Gamma$. Define a mapping $S\times\Gamma\times S\to S$ by $a\gamma b=\frac{b}{\gamma a}$ for all $a,b\in S$ and $\gamma\in\Gamma$. Then S is a LA-Γ-semigroup.

(iii) Let $S=M_n(\Re)=\Gamma$. Define a mapping $S\times\Gamma\times S\to S$ by $A\gamma B=A^T+\gamma+B$ for all $A,B\in S$ and $\gamma\in\Gamma$. Then S is a LA-Γ-semigroup.

Definition 2.2 Let S be an LA-Γ-semigroup.

(i) An element e of S is called left (right) identity if $e\gamma a=a$ ($a\gamma e=a$) for all $a\in S$ and $\gamma\in\Gamma$.

(ii) S is called a band if its all elements are idempotent, i.e., $a\gamma a=a$ for all $a\in S$ and $\gamma\in\Gamma$

(iii) S is called a locally associative LA-Γ-semigroup if $(a\gamma a)\alpha a=a\gamma(a\alpha a)$ for all $a\in S$ and $\gamma,\alpha\in\Gamma$.

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Example 2.3 ([21], Example 2.8.) Let S be a locally associative AG-groupoid (LA-semigroup) defined by the following Cayley table.

$\cdot$	a	b	c
a	c	c	b
b	b	b	b
c	b	b	b

Let for all $a,b \in S$ and $\alpha \in \Gamma$, define a mapping $S \times \Gamma \times S \to S$ by $a\alpha b = a \cdot b$. Then S is a locally associative LA-Γ-semigroup.

Lemma 2.4 ([14], Proposition 60) Let S be a locally associative LA-Γ-semigroup and m be a positif integer. Than $a^m\Gamma b^m = (a\Gamma b)^m$ for all $a,b \in S$.

Proposition 2.5 Let S be an LA-Γ-semigroup.

(i) The left (right) identity in an LA-Γ-semigroup is unique if it exists.

(ii) If S contains a right identity, then it becomes a commutative Γ-semigroup.

(iii) Every LA-Γ-semigroup with left identity satisfy the equalities $a\gamma(b\alpha c) = b\gamma(a\alpha c)$ and $(a\gamma b)\alpha(c\beta d) = (d\gamma c)\alpha(b\beta a)$for all $a,b,c,d \in S$ and $\gamma,\alpha,\beta \in \Gamma$.

(iv) S is Γ-medial, i.e., $(a\alpha b)\beta(c\gamma d) = (a\alpha c)\beta(b\gamma d)$ for all $a,b,c \in S$ and $\gamma,\alpha,\beta \in \Gamma$ in S.

Definition 2.6 Let S be an LA-Γ-semigroup.

(i) A nonempty subset A of S is called a LA-Γ-subsemigroup of S if $a\gamma b \in A$ for all $a,b \in A$ and $\gamma \in \Gamma$,

(ii) A nonempty subset A of S is called a left (right) Γ-ideal of S if $S\Gamma A \subseteq A$ ($A\Gamma S \subseteq A$). A nonempty subset A of S is called a Γ-ideal of S if it is both a left and a right Γ-ideal of S.

(iii)A nonempty subset A of S is called a generalized Γ-bi-ideal of S if $(A\Gamma S)\Gamma A \subseteq A$.

(iv) An LA-Γ-subsemigroup A of S is called a Γ-bi-ideal of S if $(A\Gamma S)\Gamma A \subseteq A$.

(v) A nonempty subset A of S is called a Γ-interior ideal of S if $(S\Gamma A)\Gamma S \subseteq A$.

Lemma 2.7[22,23] Let S be an LA-Γ-semigroup. If S is an LA-Γ-semigroup with left identity e, then every right Γ-ideal of S is a left Γ-ideal of S .

2.2 Fuzzy subsets of LA-Γ-semigroups

A function f from a nonempty set S to the unit interval [0,1] is called a fuzzy subset of S. Let f,g be fuzzy subsets of S, then $f \subseteq g$ means that $f(a) \leq g(b)$ for all $a,b \in S$. For $t \in [0,1]$, the set $f_t = \{a \in S | f(a) \geq t\}$ is called a level set of f. Let A be a subset of S, then χ_A is denoted the characteristic function of A defined as, for all $x \in S$,

$$\chi_A(x) = \begin{cases} 1 & , \ x \in A \\ 0 & , \ x \notin A. \end{cases}$$

For all $(f_i)_{i\in\Lambda},(g_i)_{i\in\Lambda} \in \mathcal{F}(S)$ and $x \in S$, $(\wedge_{i\in\Lambda} f_i)(x) = \wedge_{i\in\Lambda} f_i(x)$ and $(\vee_{i\in\Lambda} f_i)(x) = \vee_{i\in\Lambda} f_i(x)$. Let f,g be any fuzzy subsets of a LA-Γ-semigroup S and $x \in S$, then the Γ-product $f\circ_\Gamma g$ is defined by

$$(f\circ_\Gamma g)(x) = \begin{cases} \bigvee_{x=a\gamma b} f(a) \wedge g(b) & , \ \exists a,b \in S \text{ and } \gamma \in \Gamma \text{ such that } x = a\gamma b \\ 0 & , \ \text{otherwise.} \end{cases}$$

Definition 2.8 Let S be an LA-Γ-semigroup and f be a fuzzy subset of S.

(i) f is called a fuzzy LA-Γ-subsemigroup of S if $f(a\gamma b) \geq f(a) \wedge f(b)$ for all $a,b \in S$ and $\gamma \in \Gamma$.

(ii) f is called a fuzzy left (right) Γ-ideal of S if $f(a\gamma b) \geq f(b)$ ($f(a\gamma b) \geq f(a)$) for all $a,b \in S$ and $\gamma \in \Gamma$. f is called a fuzzy Γ-two-sided ideal S if it is both a fuzzy left and fuzzy right ideal of S.

(iii) f is called a fuzzy generalized Γ-bi-ideal of S if $f((a\gamma b)\alpha c) \geq f(a) \wedge f(c)$ for all $a,b,c \in S$ and $\gamma,\alpha \in \Gamma$.

(iv) A fuzzy LA-Γ-subsemigroup f is called a fuzzy Γ-bi-ideal of S if $f((a\gamma b)\alpha c) \geq f(a) \wedge f(c)$ for all $a,b,c \in S$ and $\gamma,\alpha \in \Gamma$.

(v) f is called a fuzzy Γ-interior ideal of S if $f((a\gamma x)\alpha b) \geq f(x)$ for all $a,b,x \in S$ and $\gamma,\alpha \in \Gamma$.

Lemma 2.9[23] Let S be an LA-Γ-semigroup.

(i) Let $\emptyset \neq A \subseteq S$. Then A is a LA-Γ-subsemigroup of S if and only if the characteristic function χ_A of A is a fuzzy LA-Γ-subsemigroup of S.

(ii Let $\emptyset \neq A \subseteq S$. Then A is a left (right) LA-Γ-ideal of S if and only if the characteristic function χ_A of A is a fuzzy left (right) LA-Γ-ideal of S.

(iii) A fuzzy subset f of S is fuzzy LA-Γ-subsemigroup of S if and only if the level set of f is LA-Γ-subsemigroup of S.

(iv) A fuzzy subset f of S is fuzzy left (right) LA-Γ-ideal of S if and only if the level set of f is left (right) LA-Γ-ideal of S.

2.3 (Fuzzy) LA-(m,n)-Γ-ideals

The following definition introduces some certain concepts of LA-(m,n)-Γ-semigroups.

Definition 2.10[3] Let S be LA-Γ-semigroup.

(i) For an element $a \in S$ and positive integer m, the power of a is defined as the set $a^m = (...((a\Gamma a)\Gamma a)...)\Gamma a$, where $a^1 = \{a\}$ and $a^2 = a\Gamma a$ (See[14]).

(ii) A nonempty subset A of S is called an LA-(m,n)-Γ-subsemigroup of S if $a^m\Gamma b^n \subseteq A$ for all $a,b \in A$.

(iii) A nonempty subset A of S is called an LA-(m,n)-Γ-left (LA-(m,n)-Γ-right) ideal of S if $s^m\Gamma a^n \subseteq A$ ($a^m\Gamma s^n \subseteq A$) for all $s \in S, a \in A$. A nonempty subset A of an LA-Γ-semigroup S is called a LA-(m,n)-Γ-two sided ideal of S if it is both an LA-(m,n)-Γ-left and an LA-(m,n)-Γ-right ideal of S.

(iv) A nonempty subset A of S is called a generalized LA-(m,n)-Γ-bi-ideal of S if $(a^m\Gamma s)\Gamma b^n \subseteq A$ for all $s \in S, a,b \in A$.

(v) An LA-(m,n)-Γ-subsemigroup A of S is called an LA-(m,n)-Γ-bi-ideal of S if $(a^m\Gamma s)\Gamma b^n \subseteq A$ for all $s \in S, a,b \in A$.

(vi) A nonempty subset A of S is called an LA-(m,n)-Γ-interior ideal of S if $((s_1)^m\Gamma a)\Gamma(s_2)^n \subseteq A$ for all $s_1,s_2 \in S, a \in A$.

The following definition introduces some certain fuzzy concepts of LA-(m,n)-Γ-semigroups.

Definition 2.11[3] Let f be a fuzzy subset of S.

(i) f is called a fuzzy LA-(m,n)-Γ-subsemigroup of S if $f(x) \geq f(a) \wedge f(b)$ for all $a,b \in S$ and $x \in a^m\Gamma b^n$.

(ii) f is called a fuzzy left (right) LA-(m,n)-Γ-ideal of S if $f(x) \geq f(b)$ ($f(x) \geq f(a)$) for all $a,b \in S$ and $x \in a^m\Gamma b^n$. f is called a fuzzy LA-(m,n)-Γ-two-sided ideal S if it is both a fuzzy left and fuzzy right ideal of S.

(iii) f is called a fuzzy generalized LA-(m,n)-Γ-bi-ideal of S if $f(x) \geq f(a) \wedge f(c)$ for all $a,b,c \in S$ and $x \in (a^m\Gamma b)\Gamma c^n$.

(iv) A fuzzy LA-Γ-subsemigroup f is called a fuzzy LA-(m,n)-Γ-bi-ideal of S if it is a fuzzy generalized LA-(m,n)-Γ-bi-ideal of S.

(v) f is called a fuzzy LA-(m,n)-Γ-interior ideal of S if $f(x) \geq f(c)$ for all $a,b,c \in S$ and $x \in (a^m\Gamma c)\Gamma b^n$.

Proposition 2.12[3] Let S be an LA-Γ-semigroup.

(i) A is an LA-(m,n)-Γ-subsemigroup of S if and only if χ_A is a fuzzy LA-(m,n)-Γ-subsemigroup of S.

(ii) A is an left (right) LA-(m,n)-Γ-ideal of S if and only if χ_A is a fuzzy left (right) LA-(m,n)-Γ-ideal of S.

(iii) A is a generalized LA-(m,n)-Γ-bi-ideal of S if and only if χ_A is a fuzzy generalized LA-(m,n)-Γ-bi-ideal of S.

(iv) A is an LA-(m,n)-Γ-bi-ideal of S if and only if χ_A is a fuzzy LA-(m,n)-Γ-bi-ideal of S.

(v) A is an LA-(m,n)-Γ-interior ideal of S if and only if χ_A is a fuzzy LA-(m,n)-Γ-interior ideal of S.

Proposition 2.13[3] Let S be an LA-Γ-semigroup.

(i) f is a fuzzy LA-(m,n)-Γ-subsemigroup of S if and only if all of the nonempty level sets of f is an LA-(m,n)-Γ-subsemigroup of S.

(ii) f is a fuzzy left (right) LA-(m,n)-Γ-ideal of S if and only if all of the nonempty level sets of f is an left (right) LA-(m,n)-Γ-ideal of S.

(iii) f is a fuzzy generalized LA-(m,n)-Γ-bi-ideal of S if and only if all of the nonempty level sets of f is a generalized LA-(m,n)-Γ-bi-ideal of S.

(iv) f is a fuzzy LA-(m,n)-Γ-bi-ideal of S if and only if all of the nonempty level sets of f is an LA-(m,n)-Γ-bi-ideal of S.

(v) f is a fuzzy LA-(m,n)-Γ-interior ideal of S if and only if all of the nonempty level sets of f is an LA-(m,n)-Γ-interior ideal of S.

2.4 Soft sets

Let U be an initial universe set and P be a set of parameters. The power set of U is denoted by P(U) and A is a subset of P. A pair (F,A) is called a soft set over U where F is a mapping given by $F:A \rightarrow P(U)$[18]. The pair (U,P) denotes the collection of all soft sets on U with the attributes from P and is called a soft class[15].

Definition 2.14[18] Let (F,A) and (G,B) be two soft sets over U, (F,A) is called a soft subset of (G,B), denoted by $(F,A) \subseteq (G,B)$, if (i) $B \subseteq A$, (ii) $F(x) \subseteq G(x)$ for each $x \in B$.

Definition 2.15[6,8,9,11] Let $\{(F_i,A_i)|i \in \Lambda\}$ be a family of soft sets in a soft class (U,P). Then

(i) The restricted intersection of the family $\{(F_i,A_i)|i \in \Lambda\}$, denoted by $(\cap_r \ )_{i\in\Lambda}(F_i,A_i)$, is the soft set (F,A) defined as: $A = \cap_{i\in\Lambda} \ A_i$, $F(x) = \cap_{i\in\Lambda} \ F_i(x)$ $(\forall x \in A)$,

(ii) The extended intersection of the family $\{(F_i,A_i)|i \in \Lambda\}$, denoted by $(\cap_e \ )_{i\in\Lambda}(F_i,A_i)$, is the soft set (F,A) defined as: $A = \cup_{i\in\Lambda} \ A_i$, $F(x) =$

$\bigcap_{i\in\Lambda(x)} F_i(x)$ ($\forall x \in A$) where $\Lambda(x) = \{i | x \in A_i\}$,

(iii) The restricted union of the family $\{(F_i,A_i)|i \in \Lambda\}$, denoted by $(\bigcap_r)_{i\in\Lambda}(F_i,A_i)$, is the soft set (F,A) defined as: $A = \bigcap_{i\in\Lambda} A_i$, $F(x) = \bigcup_{i\in\Lambda} F_i(x)$ ($\forall x \in A$).

(iv) The extended union of the family $\{(F_i,A_i)|i \in \Lambda\}$, denoted by $(\bigcap_e)_{i\in\Lambda}(F_i,A_i)$, is the soft set (F,A) defined as: $A = \bigcup_{i\in\Lambda} A_i$, $F(x) = \bigcup_{i\in\Lambda(x)} F_i(x)$ ($\forall x \in A$) where $\Lambda(x) = \{i | x \in A_i\}$,

Definition 2.16[6,8,11,16] Let $\{(F_i,A_i)|i \in \Lambda\}$ be a family of soft sets in a soft class (U,P). Then

(i) The $\wedge$-intersection of the family $\{(F_i,A_i)|i \in \Lambda\}$, denoted by $\wedge_{i\in\Lambda}(F_i,A_i)$, is the soft set (F,A) defined as: $A = \prod_{i\in\Lambda} A_i$, $H((x_i)_{i\in\Lambda}) = \bigcap_{i\in\Lambda} F_i(x_i)$ ($\forall (x_i)_{i\in\Lambda} \in A$),

(ii) The $\vee$-union of the family $\{(F_i,A_i)|i \in \Lambda\}$, denoted by $\vee_{i\in\Lambda}(F_i,A_i)$, is the soft set (F,A) defined as: $A = \prod_{i\in\Lambda} A_i$, $H((x_i)_{i\in\Lambda}) = \bigcup_{i\in\Lambda} F_i(x_i)$ ($\forall (x_i)_{i\in\Lambda} \in A$),

(iii) The product of the family $\{(F_i,A_i)|i \in \Lambda\}$, denoted by $\prod_{i\in\Lambda} (F_i,A_i)$, is the soft set (F,A) defined as: $A = \prod_{i\in\Lambda} A_i$, $H((x_i)_{i\in\Lambda}) = \prod_{i\in\Lambda} F_i(x_i)$ ($\forall (x_i)_{i\in\Lambda} \in A$).

2.5 Fuzzy soft sets

Let U be an initial universe set and P be a set of parameters. $F(U)$ denotes the set of all fuzzy sets of U. A pair (f,E) is called a fuzzy soft set over U, where $f{:}E \to F(U)$ is a mapping[17]. The pair $\widetilde{(U,P)}$ denotes the collection of all fuzzy soft sets on U as initial set with the attributes from P and is called a fuzzy soft class[2].

Definition 2.17[5] Let (f,E) be a fuzzy soft set over U. For each $\alpha \in [0,1]$, the set $(f,E)_\alpha = (f_\alpha,E)$ is called an α-level set of (f,E), where $f_\alpha(a) = f(a)_\alpha$ for each $a \in E$. Obviously, $(f,E)_\alpha$ is a soft set over U.

Definition 2.18[17] Let (f,E) and (g,H) be two fuzzy soft sets over U, (f,E) is called a fuzzy soft subset of (g,H), denoted by $(f,E)\tilde{\subseteq}(g,H)$, if

(i) $E \subseteq H$,

(ii) for each $a \in E$, $f(a) \subseteq g(a)$.

Definition 2.19[2,16,17] Let $\{(f_i,E_i)|i \in \Lambda\}$ be a family of fuzzy soft sets in a fuzzy soft class $\widetilde{(U,P)}$. Then

(i) The restricted intersection of the family $\{(f_i,E_i)|i \in \Lambda\}$, denoted by $\bigcap^r_{i\in\Lambda} (f_i,E_i)$, is a fuzzy soft set (f,E), $E = \bigcap_{i\in\Lambda} E_i$ and for all $x \in E$, $f(x) = \wedge_{i\in\Lambda} f_i(x)$.

(ii) The extended intersection of the family $\{(f_i,E_i)|i \in \Lambda\}$, denoted by $\bigcap^e_{i\in\Lambda} (f_i,E_i)$, is a fuzzy soft set (f,E), $E = \bigcup_{i\in\Lambda} E_i$ and for all $x \in E$, $f(x) = \wedge_{i\in\Lambda(x)} f_i(x)$ where $\Lambda(x) = \{i | x \in E_i\}$,

(iii) The restricted union of the family $\{(f_i,E_i)|i \in \Lambda\}$, denoted by $\bigcup^r_{i\in\Lambda}(f_i,E_i)$, is a fuzzy soft set (f,E), $E = \bigcap_{i\in\Lambda} E_i$ and for all $x \in E$, $f(x) = \vee_{i\in\Lambda} f_i(x)$.

(iv) The extended union of the family $\{(f_i,E_i)|i \in \Lambda\}$, denoted by $\bigcup^e_{i\in\Lambda}(f_i,E_i)$, is a fuzzy soft set (f,E), $E = \bigcup_{i\in\Lambda} E_i$ and for all $x \in E$, $f(x) = \vee_{i\in\Lambda(x)} f_i(x)$ where $\Lambda(x) = \{i | x \in E_i\}$.

Definition 2.20[7,17] Let $\{(f_i,E_i)|i \in \Lambda\}$ be a family of fuzzy soft sets in a fuzzy soft class $\widetilde{(U,P)}$. Then

(i) The fuzzy $\wedge$-intersection of the family $\{(f_i,E_i)|i \in \Lambda\}$, denoted by $\wedge_{i\in\Lambda}(f_i,E_i)$, is the soft set (f,E) defined as: $E = \prod_{i\in\Lambda} E_i$, $f((x_i)_{i\in\Lambda}) = \wedge_{i\in\Lambda} f_i(x_i)$ ($\forall (x_i)_{i\in\Lambda} \in E$),

(ii) The fuzzy $\vee$-union of the family $\{(f_i,E_i)|i \in \Lambda\}$, denoted by $\vee_{i\in\Lambda}(f_i,E_i)$, is the soft set (f,E) defined as: $E = \prod_{i\in\Lambda} E_i$, $f((x_i)_{i\in\Lambda}) = \vee_{i\in\Lambda} f_i(x_i)$ ($\forall (x_i)_{i\in\Lambda} \in E$).

(iii) The product of the family $\{(f_i,E_i)|i \in \Lambda\}$, denoted by $\prod_{i\in\Lambda}(f_i,E_i)$, is a fuzzy soft set (f,E), $E = \prod_{i\in\Lambda} E_i$ and, $f((x_i)_{i\in\Lambda}) = \vee_{J\subseteq\Lambda} (\wedge_{j\in J} f_j(x_j))$.

2.6 Fuzzy Soft Sets in Γ-Semigroups

Akram et. al introduce some definitions of algebraic structure of fuzzy soft sets in Γ-semigroups.

Definition 2.21[4] Let S be an Γ-semigroup and (f,E) be a fuzzy soft set in the fuzzy soft class $\widetilde{(S,P)}$.

(i) (f,E) is called a fuzzy soft Γ-subsemigroup of S if $f(x)$ is a fuzzy Γ-subsemigroup of S for all $x \in E$.

(ii) (f,E) is called a fuzzy soft left (right) Γ-ideal of S if $f(x)$ is a fuzzy left (right) Γ-ideal of S for all $x \in E$. (f,E) is called a fuzzy soft Γ-ideal of S if $f(x)$ is both a fuzzy left and fuzzy right Γ-ideal of S for all $x \in E$.

(iii) (f,E) is called a fuzzy soft Γ-bi-ideal of S if $f(x)$ is a fuzzy Γ-bi-ideal of S for all $x \in E$.

(iv) (f,E) is called a fuzzy soft Γ-interior ideal of S if $f(x)$ is a fuzzy Γ-interior ideal of S for all $x \in E$.

3. Main results

In this paper, we consider an LA-Γ-semigroup S as the initial universe and we introduce the fuzzy soft concepts in LA-(m,n)-Γ-semigroups.

Definition 3.1 Let S be an LA-Γ-semigroup and (f,E) be a fuzzy soft set in the fuzzy soft class $\widetilde{(S,P)}$.

(i) (f,E) is called a fuzzy soft LA-(m,n)-Γ-subsemigroup of S if $f(x)$ is a fuzzy

LA-(m,n)-Γ-subsemigroup of S for all $x \in E$.

(ii) (f,E) is called a fuzzy soft left (right) LA-(m,n)-Γ-ideal of S if f(x) is a fuzzy left (right) LA-(m,n)-Γ-ideal of S for all $x \in E$. (f,E) is called a fuzzy soft LA-(m,n)-Γ-ideal of S if f(x) is both a fuzzy left and fuzzy right LA-(m,n)-Γ-ideal of S for all $x \in E$.

(iii) (f,E) is called a fuzzy soft generalized LA-(m,n)-Γ-bi-ideal of S if f(x) is a fuzzy generalized LA-(m,n)-Γ-bi-ideal of S for all $x \in E$.

(iv) (f,E) is called a fuzzy soft LA-(m,n)-Γ-bi-ideal of S if f(x) is a fuzzy LA-(m,n)-Γ-bi-ideal of S for all $x \in E$.

(v) (f,E) is called a fuzzy soft LA-(m,n)-Γ-interior ideal of S if f(x) is a fuzzy LA-(m,n)-Γ-interior ideal of S for all $x \in E$.

Example 3.2 Let μ be a fuzzy LA-(m,n)-Γ-subsemigroup of S. Then (f,[0,1]), defined by $f(\alpha) = \chi_{\mu_\alpha}$ for all $\alpha \in [0,1]$, is a fuzzy soft LA-(m,n)-Γ-subsemigroup of S.

Lemma 3.3 Let S be an LA-Γ-semigroup. Then

(i) (f,E) is a fuzzy soft LA-(m,n)-Γ-subsemigroup if and only if all of the nonempty level sets of f is an LA-(m,n)-Γ-subsemigroup,

(ii) (f,E) is a fuzzy soft left (right) LA-(m,n)-Γ-ideal if and only if all of the nonempty level sets of f is a left (right) LA-(m,n)-Γ-ideal,

(iii) (f,E) is a fuzzy soft generalized LA-(m,n)-Γ-bi-ideal if and only if all of the nonempty level sets of f is a generalized LA-(m,n)-Γ-bi-ideal,

(iv) (f,E) is a fuzzy soft LA-(m,n)-Γ-bi-ideal if and only if all of the nonempty level sets of f is an LA-(m,n)-Γ-bi-ideal,

(v) (f,E) is a fuzzy soft LA-(m,n)-Γ-interior ideal if and only if all of the nonempty level sets of f is an LA-(m,n)-Γ-interior ideal.

Proof. Straightforward from Definition 3.1 and Proposition 2.13.

Lemma 3.4 Let S be an LA-Γ-semigroup and $A \subseteq E$. Then

(i) If (f,E) is a fuzzy soft LA-(m,n)-Γ-subsemigroup, then (f,A) is a fuzzy soft LA-(m,n)-Γ-subsemigroup,

(ii) If (f,E) is a fuzzy soft left (right) LA-(m,n)-Γ-ideal, then (f,A) is a fuzzy soft left (right) LA-(m,n)-Γ-ideal,

(iii) If (f,E) is a fuzzy soft generalized LA-(m,n)-Γ-bi-ideal, then (f,A) is a fuzzy soft generalized LA-(m,n)-Γ-bi-ideal,

(iv) If (f,E) is a fuzzy soft LA-(m,n)-Γ-bi-ideal, then (f,A) is a fuzzy soft LA-(m,n)-Γ-bi-ideal,

(v) If (f,E) is a fuzzy soft LA-(m,n)-Γ-interior ideal, then (f,A) is a fuzzy soft LA-(m,n)-Γ-interior ideal.

Proof. Straightforward.

Theorem 3.5 Let $\{(f_i,E_i) | i \in \Lambda\}$ be a family of fuzzy soft LA-(m,n)-Γ-subsemigroups of S. Then

(i) $\cap^r_{i\in\Lambda}(f_i,E_i)$, if $\cap_{i\in\Lambda} E_i \neq \emptyset$,

(ii) $\cap^e_{i\in\Lambda}(f_i,E_i)$,

(iv) $\wedge_{i\in\Lambda}(f_i,E_i)$,

(v) $\prod_{i\in\Lambda}(f_i,E_i)$ are fuzzy soft LA-(m,n)-Γ-subsemigroups of S.

Proof. Let $a,b \in S$ and $u \in a^m\Gamma b^n$.

(i) Let $\cap^r_{i\in\Lambda}(f_i,E_i) = (f,E)$. Then $E = \cap_{i\in\Lambda} E_i$ and $f(x) = \wedge_{i\in\Lambda} f_i(x)$ for all $x \in E$. Since $f(x)(u) = (\wedge_{i\in\Lambda} f_i(x))(u) = \wedge_{i\in\Lambda}(f_i(x)(u)) \geq \wedge_{i\in\Lambda}(f_i(x)(a) \wedge f_i(x)(b)) = \wedge_{i\in\Lambda}(f_i(x)(a)) \wedge \wedge_{i\in\Lambda}(f_i(x)(b)) = (\wedge_{i\in\Lambda} f_i(x))(a) \wedge (\wedge_{i\in\Lambda} f_i(x))(b) = f(x)(a) \wedge f(x)(b)$, therefore $\cap^r_{i\in\Lambda}(f_i,E_i)$ is a fuzzy soft LA-(m,n)-Γ-subsemigroup of S if $\cap_{i\in\Lambda} E_i \neq \emptyset$.

(ii) Let $\cap^e_{i\in\Lambda}(f_i,E_i) = (f,E)$. Then $E = \cup_{i\in\Lambda} E_i$ and $f(x) = \wedge_{i\in\Lambda(x)} f_i(x)$ for all $x \in E$, where $\Lambda(x) = \{i | x \in E_i\}$. Since $f(x)(u) = (\wedge_{i\in\Lambda(x)} f_i(x))(u) = \wedge_{i\in\Lambda(x)}(f_i(x)(u)) \geq \wedge_{i\in\Lambda(x)}(f_i(x)(a) \wedge f_i(x)(b)) = \wedge_{i\in\Lambda(x)}(f_i(x)(a)) \wedge \wedge_{i\in\Lambda(x)}(f_i(x)(b)) = (\wedge_{i\in\Lambda(x)} f_i(x))(a) \wedge (\wedge_{i\in\Lambda(x)} f_i(x))(b) = f(x)(a) \wedge f(x)(b)$, therefore $\cap^e_{i\in\Lambda}(f_i,E_i)$ is a fuzzy soft LA-(m,n)-Γ-subsemigroup of S.

(iii) Let $\wedge_{i\in\Lambda}(f_i,E_i) = (f,E)$. Then $E = \prod_{i\in\Lambda} E_i$ and $f((x_i)_{i\in\Lambda}) = \wedge_{i\in\Lambda} f_i(x_i)$ for all $x \in E$. Since $f((x_i)_{i\in\Lambda})(u) = (\wedge_{i\in\Lambda} f_i(x_i))(u) = \wedge_{i\in\Lambda}(f_i(x_i)(u)) \geq \wedge_{i\in\Lambda}(f_i(x_i)(a) \wedge f_i(x_i)(b)) = (\wedge_{i\in\Lambda} f_i(x_i))(a) \wedge (\wedge_{i\in\Lambda} f_i(x_i))(b) = f((x_i)_{i\in\Lambda})(a) \wedge f((x_i)_{i\in\Lambda})(b)$, therefore $\wedge_{i\in\Lambda}(f_i,E_i)$ is a fuzzy soft LA-(m,n)-Γ-subsemigroup of S.

(iv) Let $\prod_{i\in\Lambda}(f_i,E_i) = (f,E)$. Then $E = \prod_{i\in\Lambda} E_i$ and $f((x_i)_{i\in\Lambda}) = \vee_{\substack{J\subseteq\Lambda \\ J \text{is finite}}} (\wedge_{j\in J} f_j(x_j))$.

Since

$$\begin{aligned} f((x_i)_{i\in\Lambda})(u) &= (\bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} (\wedge_{j\in J} f_j(x_j)))(u) \\ &= \bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} ((\wedge_{j\in J} f_j(x_j))(u)) \\ &= \bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} (\wedge_{j\in J}(f_j(x_j)(u))) \\ &\geq \bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} (\wedge_{j\in J}(f_i(x)(a) \wedge f_i(x)(b))) \\ &= (\bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} (\wedge_{j\in J}(f_i(x)(a)))) \wedge (\bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} (\wedge_{j\in J}(f_i(x)(b)))) \\ &= (\bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} (\wedge_{j\in J}(f_i(x))))(a) \wedge (\bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} (\wedge_{j\in J}(f_i(x))))(b) \\ &= f((x_i)_{i\in\Lambda})(a) \wedge f((x_i)_{i\in\Lambda})(b) \end{aligned}$$

, therefore $\prod_{i\in\Lambda}(f_i,E_i)$ is a fuzzy soft LA-(m,n)-Γ-subsemigroup of S.

Example 3.6 Let S be the LA-Γ-semigroup in Example 2.3 (i). Since $a\cdot b \neq b\cdot a$ and $(b\Gamma S)\Gamma(b\Gamma S) \subseteq (b\Gamma b)\Gamma S = b\Gamma S$, $(S\Gamma b)\Gamma(S\Gamma b) \subseteq S\Gamma(b\Gamma b) = S\Gamma b$, then $b\Gamma S$ and $S\Gamma$ are two different LA-Γ-subsemigroups of S. By Lemma 2.9, $\chi_{b\Gamma S}$ and $\chi_{S\Gamma b}$ are fuzzy LA-Γ-subsemigroups of S. Let $(f,A),(g,A) \in \tilde{(}S,P)$ be defined by $f(a) = \chi_{b\Gamma S}$ and $g(a) = \chi_{S\Gamma b}$ for all $a \in A$. Thus $(f,A)\cup^r(g,A)$ is not a fuzzy soft LA-(1,1)-Γ-subsemigroups of S although (f,A),(g,A) are fuzzy soft LA-(1,1)-Γ-subsemigroups of S. Indeed, $(f(a)\cup^r g(a))(x\alpha y) = (\chi_{b\Gamma S}\cup^r\chi_{S\Gamma b})(x\alpha y) = 0$ for any $a \in A$, where $x = y = a\alpha b$ for any $\alpha \in \Gamma$. However $(f(a)\cup^r g(a))(x) \wedge (f(a)\cup^r g(a))(y) = 1$ since $(f(a)\cup^r g(a))(x) = (\chi_{b\Gamma S}\cup^r\chi_{S\Gamma b})(x) = 1$.

Theorem 3.7 Let $\{(f_i,E_i)|i \in \Lambda\}$ be a family of fuzzy soft LA-(m,n)-Γ-subsemigroups of S.

(i) Let $\cup^r_{i\in\Lambda}(f_i,E_i) = (f,E)$ and let $x \in E$. For all $i,j \in \Lambda$, if $f_i(x) \subseteq f_j(x)$ or $f_j(x) \subseteq f_i(x)$, then $\cup^r_{i\in\Lambda}(f_i,E_i)$ is fuzzy soft LA-(m,n)-Γ-subsemigroups of S.

(ii) Let $\cup^e_{i\in\Lambda}(f_i,E_i) = (f,E)$ and let $x \in E$. For all $i,j \in \Lambda$, if $f_i(x) \subseteq f_j(x)$ or $f_j(x) \subseteq f_i(x)$, then $\cup^e_{i\in\Lambda}(f_i,E_i)$ is fuzzy soft LA-(m,n)-Γ-subsemigroups of S.

(iii) Let $\vee_{i\in\Lambda}(f_i,E_i) = (f,E)$ and let $(x_i)_{i\in\Lambda} \in E$. For all $i,j \in \Lambda$, if $f_i(x_i) \subseteq f_j(x_j)$ or $f_j(x_j) \subseteq f_i(x_i)$, then $\vee_{i\in\Lambda}(f_i,E_i)$ is fuzzy soft LA-(m,n)-Γ-subsemigroups of S.

Proof. Straightforward.

Theorem 3.8 Let $\{(f_i,E_i)|i \in \Lambda\}$ be a family of fuzzy soft left (right) LA-(m,n)-Γ-ideal of S. Then

(i) $\cap^r_{i\in\Lambda}(f_i,E_i)$, if $\cap_{i\in\Lambda} E_i \neq \emptyset$,

(ii) $\cap^e_{i\in\Lambda}(f_i,E_i)$,

(iii) $\cup^r_{i\in\Lambda}(f_i,E_i)$,

(iv) $\cup^e_{i\in\Lambda}(f_i,E_i)$,

(v) $\wedge_{i\in\Lambda}(f_i,E_i)$,

(vi) $\vee_{i\in\Lambda}(f_i,E_i)$,

(vii) $\prod_{i\in\Lambda}(f_i,E_i)$ are fuzzy soft left (right) LA-(m,n)-Γ-ideals of S.

Proof. Let $a,b \in S$ and $u \in a^m\Gamma b^n$.

(i) Let $\cap^r_{i\in\Lambda}(f_i,E_i) = (f,E)$. Then $E = \cap_{i\in\Lambda} E_i$ and $f(x) = \wedge_{i\in\Lambda} f_i(x)$ for all $x \in E$. Since $f(x)(u) = (\wedge_{i\in\Lambda} f_i(x))(u) = \wedge_{i\in\Lambda}(f_i(x)(u)) \geq \wedge_{i\in\Lambda}(f_i(x)(b)) = (\wedge_{i\in\Lambda} f_i(x))(b) = f(x)(b)$, therefore $\cap^r_{i\in\Lambda}(f_i,E_i)$ is a fuzzy soft left LA-(m,n)-Γ-ideal of S if $\cap_{i\in\Lambda} E_i \neq \emptyset$.

(ii) Let $\cap^e_{i\in\Lambda}(f_i,E_i) = (f,E)$. Then $E = \cup_{i\in\Lambda} E_i$ and $f(x) = \wedge_{i\in\Lambda(x)} f_i(x)$ for all $x \in E$, where $\Lambda(x) = \{i|x \in E_i\}$. Since $f(x)(u) = (\wedge_{i\in\Lambda(x)} f_i(x))(u) = \wedge_{i\in\Lambda(x)}(f_i(x)(u)) \geq \wedge_{i\in\Lambda(x)}(f_i(x)(b)) = (\wedge_{i\in\Lambda(x)} f_i(x))(b) = f(x)(b)$, therefore $\cap^e_{i\in\Lambda}(f_i,E_i)$ is a fuzzy soft left LA-(m,n)-Γ-ideal of S.

(iii) Let $\cup^r_{i\in\Lambda}(f_i,E_i) = (f,E)$. Then $E = \cap_{i\in\Lambda} E_i$ and $f(x) = \vee_{i\in\Lambda} f_i(x)$ for all $x \in E$. Since $f(x)(u) = (\vee_{i\in\Lambda} f_i(x))(u) = \vee_{i\in\Lambda}(f_i(x)(u)) \geq \vee_{i\in\Lambda}(f_i(x)(b)) = (\vee_{i\in\Lambda} f_i(x))(b) = f(x)(b)$, therefore $\cup^r_{i\in\Lambda}(f_i,E_i)$ is a fuzzy soft left LA-(m,n)-Γ-ideal of S.

(iv) Let $\cup^e_{i\in\Lambda}(f_i,E_i) = (f,E)$. Then $E = \cup_{i\in\Lambda} E_i$ and $f(x) = \vee_{i\in\Lambda(x)} f_i(x)$ for all $x \in E$, where $\Lambda(x) = \{i|x \in E_i\}$. Since $f(x)(u) = (\vee_{i\in\Lambda(x)} f_i(x))(u) = \vee_{i\in\Lambda(x)}(f_i(x)(u)) \geq \vee_{i\in\Lambda(x)}(f_i(x)(b)) = (\vee_{i\in\Lambda(x)} f_i(x))(b) = f(x)(b)$, therefore $\cup^e_{i\in\Lambda}(f_i,E_i)$ is a fuzzy soft left LA-(m,n)-Γ-ideal of S.

(v) Let $\wedge_{i\in\Lambda}(f_i,E_i) = (f,E)$. Then $E = \prod_{i\in\Lambda} E_i$ and $f((x_i)_{i\in\Lambda}) = \wedge_{i\in\Lambda} f_i(x_i)$ for all $x \in E$. Since $f((x_i)_{i\in\Lambda})(u) = (\wedge_{i\in\Lambda} f_i(x_i))(u) = \wedge_{i\in\Lambda}(f_i(x_i)(u)) \geq \wedge_{i\in\Lambda}(f_i(x_i)(b)) = (\wedge_{i\in\Lambda} f_i(x_i))(b) = f((x_i)_{i\in\Lambda})(b)$, therefore $\wedge_{i\in\Lambda}(f_i,E_i)$ is a fuzzy soft left LA-(m,n)-Γ-ideal of S.

(vi) Let $\vee_{i\in\Lambda}(f_i,E_i) = (f,E)$. Then $E = \prod_{i\in\Lambda} E_i$ and $f((x_i)_{i\in\Lambda}) = \vee_{i\in\Lambda} f_i(x_i)$ for all $x \in E$. Since $f((x_i)_{i\in\Lambda})(u) = (\vee_{i\in\Lambda} f_i(x_i))(u) = \vee_{i\in\Lambda}(f_i(x_i)(u)) \geq \vee_{i\in\Lambda}(f_i(x_i)(b)) = (\vee_{i\in\Lambda} f_i(x_i))(b) = f((x_i)_{i\in\Lambda})(b)$, therefore $\vee_{i\in\Lambda}(f_i,E_i)$ is a fuzzy soft left LA-(m,n)-Γ-ideal of S.

(vii) Let $\prod_{i\in\Lambda}(f_i,E_i) = (f,E)$. Then $E = \prod_{i\in\Lambda} E_i$ and $f((x_i)_{i\in\Lambda}) = \bigvee_{J\subseteq\Lambda} (\wedge_{j\in J} f_j(x_j))$. Since

$$f((x_i)_{i\in\Lambda})(u) = (\vee_{J\subseteq\Lambda}^{J \text{ is finite}} (\wedge_{j\in J} f_j(x_j)))(u)$$
$$= \vee_{J\subseteq\Lambda}^{J \text{ is finite}} ((\wedge_{j\in J} f_j(x_j))(u))$$
$$= \vee_{J\subseteq\Lambda}^{J \text{ is finite}} (\wedge_{j\in J}(f_j(x_j)(u)))$$
$$\geq \vee_{J\subseteq\Lambda}^{J \text{ is finite}} (\wedge_{j\in J}(f_i(x)(b)))$$
$$= (\vee_{J\subseteq\Lambda}^{J \text{ is finite}} (\wedge_{j\in J}(f_i(x)(b))))$$
$$= (\vee_{J\subseteq\Lambda}^{J \text{ is finite}} (\wedge_{j\in J}(f_i(x))))(b)$$
$$= f((x_i)_{i\in\Lambda})(b)$$

, therefore $\prod_{i\in\Lambda} (f_i,E_i)$ is a fuzzy soft left LA-(m,n)-Γ-ideal of S.

Proofs are similar for fuzzy soft right LA-(m,n)-Γ-ideals of S.

Theorem 3.9 Let $\{(f_i,E_i)|i\in\Lambda\}$ be a family of fuzzy soft generalized LA-(m,n)-Γ-bi-ideal S. Then

(i) $\cap_{i\in\Lambda}^{r}(f_i,E_i)$, if $\cap_{i\in\Lambda} E_i \neq \emptyset$,

(ii) $\cap_{i\in\Lambda}^{e}(f_i,E_i)$,

(iii) $\wedge_{i\in\Lambda}(f_i,E_i)$,

(iv) $\prod_{i\in\Lambda} (f_i,E_i)$ are fuzzy soft generalized LA-(m,n)-Γ-bi-ideals of S.

Proof. Let a,b,c ∈ S and $u \in (a^m\Gamma b)\Gamma c^n$.

(i) Let $\cap_{i\in\Lambda}^{r}(f_i,E_i) = (f,E)$. Then $E = \cap_{i\in\Lambda} E_i$ and $f(x) = \wedge_{i\in\Lambda} f_i(x)$ for all $x \in E$. Since $f(x)(u) = (\wedge_{i\in\Lambda} f_i(x))(u) = \wedge_{i\in\Lambda}(f_i(x)(u)) \geq \wedge_{i\in\Lambda}(f_i(x)(a) \wedge f_i(x)(c)) = \wedge_{i\in\Lambda}(f_i(x)(a)) \wedge \wedge_{i\in\Lambda}(f_i(x)(c)) = (\wedge_{i\in\Lambda} f_i(x))(a) \wedge (\wedge_{i\in\Lambda} f_i(x))(c) = f(x)(a) \wedge f(x)(c)$, therefore $\cap_{i\in\Lambda}^{r}(f_i,E_i)$ is a fuzzy soft generalized LA-(m,n)-Γ-bi-ideal of S if $\cap_{i\in\Lambda} E_i \neq \emptyset$.

(ii) Let $\cap_{i\in\Lambda}^{e}(f_i,E_i) = (f,E)$. Then $E = \cup_{i\in\Lambda} E_i$ and $f(x) = \wedge_{i\in\Lambda(x)} f_i(x)$ for all $x \in E$, where $\Lambda(x) = \{i|x \in E_i\}$. Since $f(x)(u) = (\wedge_{i\in\Lambda(x)} f_i(x))(u) = \wedge_{i\in\Lambda(x)}(f_i(x)(u)) \geq \wedge_{i\in\Lambda(x)}(f_i(x)(a) \wedge f_i(x)(c)) = \wedge_{i\in\Lambda(x)}(f_i(x)(a)) \wedge \wedge_{i\in\Lambda(x)}(f_i(x)(c)) = (\wedge_{i\in\Lambda(x)} f_i(x))(a) \wedge (\wedge_{i\in\Lambda(x)} f_i(x))(c) = f(x)(a) \wedge f(x)(c)$, therefore $\cap_{i\in\Lambda}^{e}(f_i,E_i)$ is a fuzzy soft generalized LA-(m,n)-Γ-bi-ideal of S.

(iii) Let $\wedge_{i\in\Lambda}(f_i,E_i) = (f,E)$. Then $E = \prod_{i\in\Lambda} E_i$ and $f((x_i)_{i\in\Lambda}) = \wedge_{i\in\Lambda} f_i(x_i)$ for all $x \in E$. Since $f((x_i)_{i\in\Lambda})(u) = (\wedge_{i\in\Lambda} f_i(x_i))(u) = \wedge_{i\in\Lambda}(f_i(x_i)(u)) \geq \wedge_{i\in\Lambda}(f_i(x_i)(a) \wedge f_i(x_i)(c)) = (\wedge_{i\in\Lambda} f_i(x_i))(a) \wedge (\wedge_{i\in\Lambda} f_i(x_i))(c) = f((x_i)_{i\in\Lambda})(a) \wedge f((x_i)_{i\in\Lambda})(c)$, therefore $\wedge_{i\in\Lambda}(f_i,E_i)$ is a fuzzy soft generalized LA-(m,n)-Γ-bi-ideal of S.

(iv) Let $\prod_{i\in\Lambda} (f_i,E_i) = (f,E)$. Then $E = \prod_{i\in\Lambda} E_i$ and $f((x_i)_{i\in\Lambda}) = \vee_{J\subseteq\Lambda} (\wedge_{j\in J} f_j(x_j))$. Since

$$f((x_i)_{i\in\Lambda})(u) = (\vee_{J\subseteq\Lambda}^{J \text{ is finite}} (\wedge_{j\in J} f_j(x_j)))(u)$$
$$= \vee_{J\subseteq\Lambda}^{J \text{ is finite}} ((\wedge_{j\in J} f_j(x_j))(u))$$
$$= \vee_{J\subseteq\Lambda}^{J \text{ is finite}} (\wedge_{j\in J}(f_j(x_j)(u)))$$
$$\geq \vee_{J\subseteq\Lambda}^{J \text{ is finite}} (\wedge_{j\in J}(f_i(x)(a) \wedge f_i(x)(c)))$$
$$= (\vee_{J\subseteq\Lambda}^{J \text{ is finite}} (\wedge_{j\in J}(f_i(x)(a))))$$
$$\wedge (\vee_{J\subseteq\Lambda}^{J \text{ is finite}} (\wedge_{j\in J}(f_i(x)(c))))$$
$$= (\vee_{J\subseteq\Lambda}^{J \text{ is finite}} (\wedge_{j\in J}(f_i(x))))(a)$$
$$\wedge (\vee_{J\subseteq\Lambda}^{J \text{ is finite}} (\wedge_{j\in J}(f_i(x))))(c)$$
$$= f((x_i)_{i\in\Lambda})(a) \wedge f((x_i)_{i\in\Lambda})(c)$$

, therefore $\prod_{i\in\Lambda} (f_i,E_i)$ is a fuzzy soft generalized LA-(m,n)-Γ-bi-ideal of S.

Theorem 3.10 Let $\{(f_i,E_i)|i\in\Lambda\}$ be a family of fuzzy soft generalized LA-(m,n)-Γ-bi-ideal S. Then

(i) Let $\cup_{i\in\Lambda}^{r}(f_i,E_i) = (f,E)$ and let $x \in E$. For all $i,j \in \Lambda$, if $f_i(x) \subseteq f_j(x)$ or $f_j(x) \subseteq f_i(x)$, then $\cup_{i\in\Lambda}^{r}(f_i,E_i)$ is fuzzy soft generalized LA-(m,n)-Γ-bi-ideals of S.

(ii) Let $\cup_{i\in\Lambda}^{e}(f_i,E_i) = (f,E)$ and let $x \in E$. For all $i,j \in \Lambda$, if $f_i(x) \subseteq f_j(x)$ or $f_j(x) \subseteq f_i(x)$, then $\cup_{i\in\Lambda}^{e}(f_i,E_i)$ is fuzzy soft generalized LA-(m,n)-Γ-bi-ideals of S.

(iii) Let $\vee_{i\in\Lambda}(f_i,E_i) = (f,E)$ and let $(x_i)_{i\in\Lambda} \in E$. For all $i,j \in \Lambda$, if $f_i(x_i) \subseteq f_j(x_j)$ or $f_j(x_j) \subseteq f_i(x_i)$, then $\vee_{i\in\Lambda}(f_i,E_i)$ is fuzzy soft generalized LA-(m,n)-Γ-bi-ideals of S.

Proof. Straightforward.

Theorem 3.11 Let $\{(f_i,E_i)|i\in\Lambda\}$ be a family of fuzzy soft LA-(m,n)-Γ-bi-ideal of S. Then

(i) $\cap_{i\in\Lambda}^{r}(f_i,E_i)$, if $\cap_{i\in\Lambda} E_i \neq \emptyset$,

(ii) $\cap_{i\in\Lambda}^{e}(f_i,E_i)$,

(iii) $\wedge_{i\in\Lambda}(f_i,E_i)$,

(iv) $\prod_{i\in\Lambda} (f_i,E_i)$ are fuzzy soft LA-(m,n)-Γ-bi-ideals of S.

Proof. Straightforward from Theorem 3.5 and Theorem 3.9.

Theorem 3.12 Let $\{(f_i,E_i)|i\in\Lambda\}$ be a family of fuzzy soft LA-(m,n)-Γ-bi-ideal of S. Then

(i) Let $\cup_{i\in\Lambda}^{r}(f_i,E_i) = (f,E)$ and let $x \in E$. For all $i,j \in \Lambda$, if $f_i(x) \subseteq f_j(x)$ or $f_j(x) \subseteq f_i(x)$, then $\cup_{i\in\Lambda}^{r}(f_i,E_i)$ is fuzzy soft LA-(m,n)-Γ-bi-ideals of S.

(ii) Let $\cup_{i\in\Lambda}^{e}(f_i,E_i) = (f,E)$ and let $x \in E$. For all $i,j \in \Lambda$, if $f_i(x) \subseteq f_j(x)$ or $f_j(x) \subseteq f_i(x)$, then $\cup_{i\in\Lambda}^{e}(f_i,E_i)$ is fuzzy soft LA-(m,n)-Γ-bi-ideals of S.

(iii) Let $\vee_{i\in\Lambda}(f_i,E_i) = (f,E)$ and let $(x_i)_{i\in\Lambda} \in E$. For all $i,j \in \Lambda$, if $f_i(x_i) \subseteq f_j(x_j)$ or $f_j(x_j) \subseteq f_i(x_i)$, then $\vee_{i\in\Lambda}(f_i,E_i)$ is fuzzy soft LA-(m,n)-Γ-bi-ideals of S.

Proof. Straightforward.

Theorem 3.13 Let $\{(f_i,E_i)|i \in \Lambda\}$ be a family of fuzzy soft LA-(m,n)-Γ-interior ideal of S. Then

(i) $\cap^{r}_{i\in\Lambda}(f_i,E_i)$, if $\cap_{i\in\Lambda} E_i \neq \emptyset$,

(ii) $\cap^{e}_{i\in\Lambda}(f_i,E_i)$,

(iii) $\cup^{r}_{i\in\Lambda}(f_i,E_i)$,

(iv) $\cup^{e}_{i\in\Lambda}(f_i,E_i)$,

(v) $\wedge_{i\in\Lambda}(f_i,E_i)$,

(vi) $\vee_{i\in\Lambda}(f_i,E_i)$,

(vii) $\prod_{i\in\Lambda}(f_i,E_i)$ are fuzzy soft LA-(m,n)-Γ-interior ideals of S.

Proof. Let $a,b,c \in S$ and $u \in (a^m\Gamma b)\Gamma c^n$.

(i) Let $\cap^{r}_{i\in\Lambda}(f_i,E_i) = (f,E)$. Then $E = \cap_{i\in\Lambda} E_i$ and $f(x) = \wedge_{i\in\Lambda} f_i(x)$ for all $x \in E$. Since $f(x)(u) = (\wedge_{i\in\Lambda} f_i(x))(u) = \wedge_{i\in\Lambda}(f_i(x)(u)) \geq \wedge_{i\in\Lambda}(f_i(x)(b)) = (\wedge_{i\in\Lambda} f_i(x))(b) = f(x)(b)$, therefore $\cap^{r}_{i\in\Lambda}(f_i,E_i)$ is a fuzzy soft LA-(m,n)-Γ-interior ideal of S if $\cap_{i\in\Lambda} E_i \neq \emptyset$.

(ii) Let $\cap^{e}_{i\in\Lambda}(f_i,E_i) = (f,E)$. Then $E = \cup_{i\in\Lambda} E_i$ and $f(x) = \wedge_{i\in\Lambda(x)} f_i(x)$ for all $x \in E$, where $\Lambda(x) = \{i|x \in E_i\}$. Since $f(x)(u) = (\wedge_{i\in\Lambda(x)} f_i(x))(u) = \wedge_{i\in\Lambda(x)}(f_i(x)(u)) \geq \wedge_{i\in\Lambda(x)}(f_i(x)(b)) = (\wedge_{i\in\Lambda(x)} f_i(x))(b) = f(x)(b)$, therefore $\cap^{e}_{i\in\Lambda}(f_i,E_i)$ is a fuzzy soft LA-(m,n)-Γ-interior ideal of S.

(iii) Let $\cup^{r}_{i\in\Lambda}(f_i,E_i) = (f,E)$. Then $E = \cap_{i\in\Lambda} E_i$ and $f(x) = \vee_{i\in\Lambda} f_i(x)$ for all $x \in E$. Since $f(x)(u) = (\vee_{i\in\Lambda} f_i(x))(u) = \vee_{i\in\Lambda}(f_i(x)(u)) \geq \vee_{i\in\Lambda}(f_i(x)(b)) = (\vee_{i\in\Lambda} f_i(x))(b) = f(x)(b)$, therefore $\cup^{r}_{i\in\Lambda}(f_i,E_i)$ is a fuzzy soft LA-(m,n)-Γ-interior ideal of S.

(iv) Let $\cup^{e}_{i\in\Lambda}(f_i,E_i) = (f,E)$. Then $E = \cup_{i\in\Lambda} E_i$ and $f(x) = \vee_{i\in\Lambda(x)} f_i(x)$ for all $x \in E$, where $\Lambda(x) = \{i|x \in E_i\}$. Since $f(x)(u) = (\vee_{i\in\Lambda(x)} f_i(x))(u) = \vee_{i\in\Lambda(x)}(f_i(x)(u)) \geq \vee_{i\in\Lambda(x)}(f_i(x)(b)) = (\vee_{i\in\Lambda(x)} f_i(x))(b) = f(x)(b)$, therefore $\cup^{e}_{i\in\Lambda}(f_i,E_i)$ is a fuzzy soft LA-(m,n)-Γ-interior ideal of S.

(v) Let $\wedge_{i\in\Lambda}(f_i,E_i) = (f,E)$. Then $E = \prod_{i\in\Lambda} E_i$ and $f((x_i)_{i\in\Lambda}) = \wedge_{i\in\Lambda} f_i(x_i)$ for all $x \in E$. Since $f((x_i)_{i\in\Lambda})(u) = (\wedge_{i\in\Lambda} f_i(x_i))(u) = \wedge_{i\in\Lambda}(f_i(x_i)(u)) \geq \wedge_{i\in\Lambda}(f_i(x_i)(b)) = (\wedge_{i\in\Lambda} f_i(x_i))(b) = f((x_i)_{i\in\Lambda})(b)$, therefore $\wedge_{i\in\Lambda}(f_i,E_i)$ is a fuzzy soft LA-(m,n)-Γ-interior ideal of S.

(vi) Let $\vee_{i\in\Lambda}(f_i,E_i) = (f,E)$. Then $E = \prod_{i\in\Lambda} E_i$ and $f((x_i)_{i\in\Lambda}) = \vee_{i\in\Lambda} f_i(x_i)$ for all $x \in E$. Since $f((x_i)_{i\in\Lambda})(u) = (\vee_{i\in\Lambda} f_i(x_i))(u) = \vee_{i\in\Lambda}(f_i(x_i)(u)) \geq \vee_{i\in\Lambda}(f_i(x_i)(b)) = (\vee_{i\in\Lambda} f_i(x_i))(b) = f((x_i)_{i\in\Lambda})(b)$, therefore $\vee_{i\in\Lambda}(f_i,E_i)$ is a fuzzy soft LA-(m,n)-Γ-interior ideal of S.

(vii) Let $\prod_{i\in\Lambda}(f_i,E_i) = (f,E)$. Then $E = \prod_{i\in\Lambda} E_i$ and $f((x_i)_{i\in\Lambda}) = \vee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} (\wedge_{j\in J} f_j(x_j))$. Since,

$$\begin{aligned} f((x_i)_{i\in\Lambda})(u) &= (\vee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} (\wedge_{j\in J} f_j(x_j)))(u) \\ &= \vee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} ((\wedge_{j\in J} f_j(x_j))(u)) \\ &= \vee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} (\wedge_{j\in J}(f_j(x_j)(u))) \\ &\geq \vee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} (\wedge_{j\in J}(f_i(x)(b))) \\ &= (\vee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} (\wedge_{j\in J}(f_i(x)(b)))) \\ &= (\vee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} (\wedge_{j\in J}(f_i(x))))(b) \\ &= f((x_i)_{i\in\Lambda})(b) \end{aligned}$$

therefore $\prod_{i\in\Lambda}(f_i,E_i)$ is a fuzzy soft LA-(m,n)-Γ-interior ideal of S.

4. Conclusions

In this paper, we introduce the definitions of some certain fuzzy soft concepts in an LA-(m,n)-Γ-semigroup and investigate some algebraic properties of fuzzy soft sets in LA-(m,n)-Γ-semigroups. To extend this work, one could define fuzzy soft LA-(m,n)-Γ-quasi, -prime and -semiprime ideals of an LA-(m,n)-Γ-semigroup and examine algebraic properties of them.

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References

1. Abbasi MY and Basar A. On generalizations of ideals in LA-Γ-semigroups. Southeast Asian Bulletin of Mathematics 2015; 39 : 1-12.
2. Ahmad B and Kharal A. On Fuzzy Soft Sets. Advances in Fuzzy Systems 2009; Volume 2009, Article ID 586507, 6 pages, doi:10.1155/2009/586507
3. Akın C. Fuzzy LA-(m,n)-Γ-ideals in LA-Γ-semigroups. Advances in Fuzzy Sets and Systems 2017; 22 : 211-223.
4. Akram M, Kavikumar J and Khamis A. Fuzzy Soft Gamma Semigroups. Applied Mathematics and Information Sciences 2014; 8(2): 929-934.
5. Aygünoğlu Aand Aygün H. Introduction to fuzzy soft groups. Comput. Math. Appl. 2009; 58:1279-1286.
6. Çelik Y, Ekiz C and Yamak S. A new view on soft rings. Hacettepe Journal of Mathematics and Statistics 2011; 40 (2): 273-286.
7. Çelik Y, Ekiz C and Yamak S. Applications of fuzzy soft sets in ring theory. Annals Fuzzy Mathematics and Informatics 2013; 3 (5): 451-462.

8. Feng F, Jun -YB and Zhao -X. Soft semirings. Computers and Mathematics with Applications 2008; 56 : 2621-2628.
9. Irfan Ali M, Feng F, Liu X, Min WK and Shabir M. On some new operations in soft set theory. Computers and Mathematics with Applications 2009; 57: 1547-1553.
10. Jun YB and Lee CY. Fuzzy Γ-rings. Pusan Kyongnam Math. J. 1981; 84: 264-269.
11. Kazancı O, Yılmaz Ş and Yamak S. Soft sets and soft bch-algebrass. Hacettepe Journal of Mathematics and Statistics 2010; 39 (2) : 205-217.
12. Kazım MA and Naseeruddin M. On almost semigroups. The Alig. Bull. Math. 1972; 2 :1-7.
13. Khan M, Jun YB and Yousafzai F, Fuzzy ideals in right regular LA-semigroups. Hacettepe Journal of Mathematics and Statistics 2015; 44 (3): 569 – 586.
14. Khan M, Smarandache F and Anis S. Theory of Abel Grassmann's Groupoids. Educational Publisher, Columbus, 2015.
15. Kharal A and Ahmad B. Mappings on soft classes. New Mathematics and Natural Computation 2011; 7 (3): 471-481.
16. Maji PK, Biswas R and Roy R. Soft set theory. Computers and Mathematics with Applications 2003; 45 ; 555-562.
17. Maji PK, Biswas R and Roy R. Fuzzy soft sets, J. Fuzzy Math. 2001; 9 (3) ; 589-602.
18. Molodtsov D. Soft set theory first results. Computers and Mathematics with Applications1999; 37: 19–31.
19. Sen MK. On Γ-semigroups. Proceeding of International Symposium on Algebra and Its Applications. Decker Publication, New York, 30, 1981, 1-8.
20. Sen MK and Saha NK. On Γ-semigroups I. Bull. Cal. Math. Soc. 1986;78:180-186.
21. Shah T and Rehman I. Decomposition of Locally Associative Γ-AG-groupoids. Novi Sad J. Math. 2013; 4 ,Vol. 43, No. 1; 1-8.
22. Shah T and Rehman I. On Γ-ideals and Γ-bi-ideals in Γ-AG-groupoids. International Journal of Algebra 2010; 4 (6): 267-276.
23. Shah T and Rehman I and Khan A. Fuzzy Γ-ideals in Γ-AG-groupoids. Hacettepe Journal of Mathematics and Statistics 2014; 43(4): 625-634.
24. Yang CF. Fuzzy soft semigroups and fuzzy soft ideals. Computers and Mathematics with Applications 2011; 61 (2); 255-261.
25. Zadeh LA. Fuzzy Sets. Inform. and Control1965; 8 : 338-353.