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Dynamics of Bianchi Type-V Anisotropic Dark Energy Cosmological Model in A Scale Covariant Theory of Gravitation

Dandala Radhakrishna Reddy

Andhra University, Visakhapatnam, India

ABSTRACT

This paper is devoted to the discussion of dynamical properties of anisotropic dark energy cosmological model of the universe in a Bianchi type-V space time in the framework of scale covariant theory of gravitation formulated by Canuto *et al*. (phys.Rev.Lett.39:429,1977). A dark energy cosmological model is presented by solving the field equations of this theory by using some physically viable conditions. The dynamics of the model is studied by computing the cosmological parameters, dark energy density, equation of state (EoS) parameter, skewness parameters, deceleration parameter and the jerk parameter. This being a scalar field model gives us the quintessence model of the universe which describes a significant dark energy candidate of our accelerating universe. All the physical quantities discussed are in agreement with the recent cosmological observations.

***Keywords*:** Scale Covariant Theory; BIANCHI Type-V Model; DARK Energy Model; Cosmological Model

1. Introduction

Scalar-tensor theories of gravitation in which ,in addition to the metric tensor field, a scalar field has been introduced have a significant role to play in modern cosmology. In spite of the fact that Einstein's theory of gravitation , even today, is considered to be the precise and successful theory of gravitation, it does not fully explain certain concepts like Mach's Principle, cosmological problems and dark energy. Hence alternative theories of gravitation like scalar-tensor theories and other modified theories of gravitationhave been proposed from time to time. The most significant among the scalar-tensor theories of gravitation are Brans-Dicke[1] and Saez-Ballester[2] theories of gravitation.Brans-Dicke theory advocates variation of gravitational constantG[3,4] while Saez-Ballester theory suggests a possible way to solve the 'missing mass'problem in non-flat FRW cosmologies. These two theories are attracting several researchers in modern cosmology.Canuto*et al*.[5,6] formulated a scale covariant theory of gravitation by associating the mathematical operation of scale transformation with the physics of different dynamicalsystems to measure space time distances. They obtained the generalized Einstein's field equations invariant under scale transformations and studied several astrophysical tests. This theory is aviable alternative to general relativity which allowsa natural interpretation of the possible variation ofthe gravitational constant. In this theory, Einstein field equations are valid in gravitational units whereas physical quantities are measured in atomic units. In the two systems of units the metric tensor components are related by a conformal transformation

$$\overline{g_{ij}} = \varphi^2(x^k)g_{ij} \quad , \qquad i, j=1,2,3,4 \tag{1}$$

Here the barred quantities denote gravitational units and unbarred quantities refer to atomic units and φ ($0<\varphi<\infty$) is the scalar gauge function which in its more general expression is function of all space-time coordinates. Thus using the conformal transformation of the type given by Eq.(1), Canuto *et al*.[5,6] transformed the usual Einstein equations into

$$R_{ij} - \frac{1}{2}g_{ij}R + f_{ij}(\varphi) - 8\pi G(\varphi)T_{ij} + \Lambda(\varphi)g_{ij} \tag{2}$$

where

$$\varphi^2 f_{ij} = 2\varphi\varphi_{i;j} - 4\varphi_{,i}\varphi_{,j} - g_{ij}(\varphi\varphi^{k}{}_{;k} - \varphi^{,k}\varphi_{,k}) \tag{3}$$

Here semicolon denotes covariant differentiation and comma indicates ordinary differentiation with respect to the coordinates x^k .The other symbols have their usual meaning. In our discussion we take the cosmological constant $\Lambda = 0$. The above equations have been written in a form such that their form remains invariant under arbitrary coordinate as well as scale transformation. A particular feature of this theory is that no independent equation for φ exists as in the other scalar-tensor theories. One has to go outside the theory in order to specify the form of φ and its relationship with G. The speculations that have been considered for this gauge function are (Cannuto *et al*.[6])

$$\varphi(t) = \left(\frac{t}{t_0}\right)^{\varepsilon}, \ \varepsilon = \pm 1, \pm\frac{1}{2} \quad (4)$$

Where t_0 is a constant. But the form

$$\varphi \sim t^{\frac{1}{2}} \quad (5)$$

is the one most favored to fit observation (Cannuto and Goldman[7]). Inspired by the importance of modified theories of gravitation in cosmology, several researchers are attracted to scale covariant theory of gravitation and have investigated some interesting Bianchi type cosmological models in the presence of different physical sources[8–19].

The recent scenario of modern cosmology is that our universe is in a state of accelerated expansion. This fact has been confirmed by the cosmological observations of type1a Supernova[20–22]. It has been suggested that the existence of a large negative pressure dubbed as dark energy (DE) is responsible for this cosmic acceleration. Two approaches have been mostly used to clarify this situation. One of these approaches is dark energy models and the other approach is the modified theories of gravitation. Scalar-tensor theories of gravitation, scale covariant theory of gravitation[5,6] provide the most natural generalization of general relativity by incorporating additional fields like scalar fields. Field equations, in these theories, become even more complex than in Einstein's theory of gravitation. However, these scalar fields play a vital role in the discussion of dark energy cosmology.

It is a well established fact that the present day universe is described by the homogeneous and isotropic Friedman-Robertson-Walker (FRW) model. However, the recent experimental data and the theoretical arguments support the existence of considerable amount of anisotropy at the early stages of evolution of the universe which evolves into an isotropic one. Hence it becomes very much necessary to study the universe and DE models with anisotropic back ground. It is well known that Bianchi type models are the models with homogeneous and anisotropic background. Hence many authors have studied Bianchi models in alternative theories of gravitation. Here, we confine ourselves to the Bianchi models studied in scale covariant theory of gravitation. Zeyauddin and Rao[23]obtained Bianchi type cosmological model in scale covariant theory of gravitation using a variable deceleration parameter while ShriRam *et al*.[24] investigated Bianchi type cosmological model with perfect fluid source. Reddy[25] presented a higher dimensional Kaluza-Klein perfect fluid model while Raju *et al*[26] obtained a spherically symmetric five dimensional cosmological model in this theory. Pradhan *et al*.[27] and Prasad *et al*.[28] discussed bulk viscous string cosmological models in this theory. Reddy and Kumar[29] obtained Kaluza-Klein DE model whileKatore *et al*.[30] and Singh and Sharma[31] and Dasu Naidu *et al*.[32] have found Bianchi DE cosmological models in this particular theory of gravitation. However, anisotropic DE model in Bianchi space-time has not, so far, been investigated in this theory of gravitation.

Keeping in view the above discussion, the objective of this work is to investigate spatially homogeneous and anisotropic DE cosmological model in this theory of gravity. This work is organized in the following format. The next section deals with the metric and derivation of explicit field equations of the theory. In sec.3, we present the DE model by solving the field equations using some physically valid conditions. In sec.4 we determine the dynamical parameters of the model and discuss their physical significance in the universe. Finally, Sec.5 contains concluding remarks.

2. Basic Equations of Scale covariant theory

Here we obtain the explicit field equations of scale covariant theory of gravitation in the background of Bianchi type-V space-time. We consider spatially homogeneous and anisotropic Bianchi type-V metric in the form

$$ds^2 = dt^2 - a^2dx^2 + e^{2mx}(b^2dy^2 + c^2dz^2) \quad (6)$$

Where a,b,c are functions of cosmic time t and m is a positive constant which can be set equal to unity for simplicity.

The energy momentum tensor for anisotropic DE fluid is given by

$$T_{ij} = (\rho_\lambda + p_\lambda)u_i u_j - g_{ij}p_\lambda \quad (7)$$

where ρ_λ is the DE density and p_λ is the pressure of DE fluid. This can be parameterized as

$$T^i_j = diag(1,\omega_x,\omega_y,\omega_z)\rho_\lambda = diag[1,\omega_\lambda,(\omega_\lambda+\gamma),(\omega_\lambda+\delta)]\rho_\lambda \quad (8)$$

Here

$$\omega_\lambda = \frac{p_\lambda}{\rho_\lambda} \quad (9)$$

is theEoS parameter of DE , γ and δ are the skewness parameters which are the deviations from ω_λ along y and *z* directions respectively.

Using co moving coordinates the field equations (2) and (3) for the metric (6) with the help of Eqs.(7)-(9) can be , explicitly, written as

$$\frac{\dot a}{a}\frac{\dot b}{b}+\frac{\dot a}{a}\frac{\dot c}{c}+\frac{\dot b}{b}\frac{\dot c}{c}-\frac{3}{a^2}+\frac{\dot\varphi}{\varphi}\left(\frac{\dot a}{a}+\frac{\dot b}{b}+\frac{\dot c}{c}\right)-\frac{\ddot\varphi}{\varphi}+3\frac{\dot\varphi^2}{\varphi^2}=8\pi G\rho_\lambda \quad (10)$$

$$\frac{\ddot b}{b}+\frac{\ddot c}{c}+\frac{\dot b}{b}\frac{\dot c}{c}-\frac{1}{a^2}-2\frac{\dot a}{a}\frac{\dot\varphi}{\varphi}+\frac{\dot\varphi}{\varphi}\left(\frac{\dot a}{a}+\frac{\dot b}{b}+\frac{\dot c}{c}\right)+\frac{\ddot\varphi}{\varphi}-\frac{\dot\varphi^2}{\varphi^2}=-8\pi G\omega_\lambda\rho_\lambda \quad (11)$$

$$\frac{\ddot a}{a}+\frac{\ddot c}{c}+\frac{\dot a}{a}\frac{\dot c}{c}-\frac{1}{a^2}-2\frac{\dot b}{b}\frac{\dot\varphi}{\varphi}+\frac{\dot\varphi}{\varphi}\left(\frac{\dot a}{a}+\frac{\dot b}{b}+\frac{\dot c}{c}\right)+\frac{\ddot\varphi}{\varphi}-\frac{\dot\varphi^2}{\varphi^2}=-8\pi G\rho_\lambda(\omega_\lambda+\gamma) \quad (12)$$

$$\frac{\ddot a}{a}+\frac{\ddot b}{b}+\frac{\dot a}{a}\frac{\dot b}{b}-\frac{1}{a^2}-2\frac{\dot c}{c}\frac{\dot\varphi}{\varphi}+\frac{\dot\varphi}{\varphi}\left(\frac{\dot a}{a}+\frac{\dot b}{b}+\frac{\dot c}{c}\right)+\frac{\ddot\varphi}{\varphi}-\frac{\dot\varphi^2}{\varphi^2}=-8\pi G\rho_\lambda(\omega_\lambda+\delta) \quad (13)$$

$$\frac{2\dot a}{a}-\frac{\dot b}{b}-\frac{\dot c}{c}=0 \quad (14)$$

Here an overhead dot denotes differentiation with respect to cosmic time *t*.

In this theory of gravitation the energy conservation equation which is a consequence of the field equations is given by (Canuto *et al.*[5])

$$\dot\rho+(\rho+p)u^i_{;i}+\rho\left(\frac{\dot G}{G}+\frac{\dot\varphi}{\varphi}\right)+3p\frac{\dot\varphi}{\varphi}=0 \quad (15)$$

For the case ofanisotropic DE fluid Eq.(15), with the help of Eqs.(6)-(9), takes the form

$$\dot\rho_\lambda+(1+\omega_\lambda)\rho_\lambda\left(\frac{\dot a}{a}+\frac{\dot b}{b}+\frac{\dot c}{c}\right)+\rho_\lambda\left(\frac{\dot G}{G}+\frac{\dot\varphi}{\varphi}\right)+3\rho_\lambda\omega_\lambda\frac{\dot\varphi}{\varphi}=0 \quad (16)$$

Now we shall define some dynamical parameters which are significant in the discussion of cosmology and in solving the above field equations.

The spatial volume V and the average scale factor A(t) for the space-time(6) are given by

$$V = A^3(t) = (abc) \quad (17)$$

The expansion scalar θ, the mean anisotropy parameter A_h and shear scalar σ^2 are defined as

$$\theta = 3H = 3(H_1+H_2+H_3) = 3\left(\frac{\dot a}{a}+\frac{\dot b}{b}+\frac{\dot c}{c}\right) \quad (18)$$

$$A_h = \frac{1}{3}\left(\sum_1^3\frac{H_i-H}{H}\right)^2 \quad (19)$$

$$\sigma^2 = \frac{1}{2}\sigma^{ik}\sigma_{ik} = 3A_hH^2 \quad (20)$$

where H is the mean Hubble parameter and $H_i, i=1,2,3$ are the directional Hubble parameters

3. Anisotropic Dark energy model

Here we solve the field equations of this theory using some physically valid conditions and present the anisotropic DE model .

Integration of Eq.(14) gives us

$$a^2 = k_1bc \quad (21)$$

where k_1 is a constant of integration which can be set equal to unity with out loss of any generality so that we have

$$a^2 = bc \tag{22}$$

Now the field equations (10)-(14) and (16) are a system of five independentequations[since conservation equation(16) is a consequence of the field equations] in seven unknowns a(t), b(t), c(t), $\rho_\lambda, \omega_\lambda, \gamma, \delta$ [in view of the fact G=G(φ) and Eq.(5)].Hence to find an exact solution we need two more conditions. We use the following physically significant conditions:

(i) The shear scalar σ is proportional to scalar expansion θ which leads to a relationship between the metric potentials(Collins *et al.*[33]) so that we can take

$$b = c^k \tag{23}$$

where $k \neq 1$ is a positive constant and preserves anisotropy character of the space-time (6).The physical reason for this assumption is warranted from the observation of the velocity redshift relation for extragalactic sources which suggest that the Hubble expansion of the universe may achieve isotropy when $\frac{\sigma}{\theta}$ is constant(Kantowski and Sachs[34]).

(ii) We consider the average scale factor, following Akarsu *et al.*[35], in the form

$$A(t) = (abc)^{\frac{1}{3}} = A_0 t^\alpha e^{\beta t} \tag{24}$$

where α and β are non negative constants and A_0 is the present value of the scale factor. The physical interpretation for the above proposed law is that it gives the time varying deceleration parameter which describes the transitioning universe Also, Eq.(24) is called the hybrid expansion law because when $\alpha = 0$, we have power law and when β =0 we get exponential law cosmology. This has been extensively used in literature[36–40].

Now using Eqs.(17) and (22)-(24) we obtain the metric potentials as

$$a(t) = A_0 t^\alpha e^{\beta t},$$

$$b(t) = (A_0 t^\alpha e^{\beta t})^{\frac{2k}{k+1}}, \quad c(t) = (A_0 t^\alpha e^{\beta t})^{\frac{2}{k+1}} \tag{25}$$

and the scalar field in the model is given by Eq.(5). Using Eq.(25) , we can write Bianchi typemetric in the form

$$ds^2 = dt^2 - (A_0 t^\alpha e^{\beta t})^2 dx^2 + e^{2x}[(A_0 t^\alpha e^{\beta t})^{\frac{4k}{k+1}} dy^2 + ((A_0 t^\alpha e^{\beta t})^{\frac{4}{k+1}} dz^2] \tag{26}$$

Thus Eq.(26) along with Eq.(5) describes the anisotropic DE model in Bianchi type-V space-time.

4. Dynamical properties of cosmological parameters

This section deals with the determination and discussion of the physical parameters of the cosmological model represented by the metric given by Eq.(26). For this purpose we use Eqs.(17)-(20) and Eqs.(25)

Spatial volume of the universe is

$$V = (A_0 t^\alpha e^{\beta t})^3 \tag{27}$$

This shows that spatial volume of the universe increases with the increase in cosmic time which tells us that our model is an expanding model. Also at the initial epoch, i.e., at t=0, it vanishes. Hence the universe evolves from zero volume.

The scalar expansionof the model is

$$\theta = 3\left(\frac{\alpha + \beta t}{t}\right) \tag{28}$$

This shows that θ of the model diverges at t=0 and it becomes constant for infinitely large values of t.

The shear scalarof the model becomes

$$\sigma^2 = \left(\frac{\alpha + \beta t}{t}\right)^2 \left(\frac{k-1}{k+1}\right)^2 \tag{29}$$

This shows that the shear of the model diverges at the initial epoch and will be constant for large values of cosmic time *t*. It can be seen that shear vanishes for k=1 and in this case the model becomes shear free.

The average anisotropy parameter of our model is determined as

$$A_h = \frac{2}{3}\left(\frac{k-1}{k+1}\right)^2 \tag{30}$$

It can be seen that for our model average anisotropy is constant It can, also, be observed that when k=1, $A_h = 0$ .Hence, in this case, we observe , in view of Eqs.(29) and (30), the universe is isotropic and shear free.

The average Hubble parameter is found to be

$$H = \left(\frac{\alpha + \beta t}{t}\right) \tag{31}$$

This diverges for $t=0$ and attains constant value as $t \to \infty$.

Other physically significant parameters which are useful to study the behavior of the universe are the following

The deceleration parameter q which is defined and determined as

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) = -1 + \frac{\alpha}{(\alpha + \beta t)^2} \tag{32}$$

It is well known that for the positive and negative values of , the model decelerates and accelerates respectively. It can be observed that as $t \to \infty$, $q = -1, \frac{dH}{dt} = 0$ which gives us the greatest value of Hubble parameter and fastest rate of(accelerated) expansion of the universe. This is in agreement with the observation of modern cosmologywhich confirm the accelerated expansion of the universe.

The jerk parameterj(t) is defined and is given as[41]

$$j(t) = \frac{\dddot{A}}{AH^3} = 1 + \frac{2\alpha}{(\alpha + \beta t)^3} - \frac{3\alpha}{(\alpha + \beta t)^2} \tag{33}$$

Jerk parameter is useful in cosmology to describe models close to ΛCDM. In cosmology it is believed that the transition of the universe occurs for models with positive value of jerk parameter and negative value of deceleration parameter. It may be observed that as $t \to \infty$, that is at late times we have $q<0$ and $j>0$. Hence for our universe there occurs a smooth transition from decelerated phase to the present accelerated phase. This confirms the observational data.

From Eq.(5) we observe that the scalar gauge function vanishes at the initial epoch and increases with cosmic time.

From Eqs.(5), (10) and (25) the anisotropic DE ρ_λ can be computed as

$$8\pi G\rho_\lambda = \frac{4(\alpha + \beta t)^2 (k^2 + 4k + 1) + (k+1)^2 [3(\alpha + \beta t) + 2 - 6t^2 (A_0 t^\alpha e^{\beta t})^{-2}]}{2(k+1)^2 t^2} \tag{34}$$

From Eqs.(5),(11), (25) and (34) we obtain The EoS parameter for our model as

$$\omega_\lambda = -\left(\frac{8(\alpha + \beta t)^2 (k^2 + k + 1) + (k+1)^2 [\beta t - 3\alpha - 1 - 2t^2 (A_0 t^\alpha e^{\beta t})^{-2}]}{4(\alpha + \beta t)^2 (k^2 + 4k + 1) + (k+1)^2 [3(\alpha + \beta t) + 2 - 6t^2 (A_0 t^\alpha e^{\beta t})^{-2}]}\right) \tag{35}$$

From Eqs.(5),(11), (12), (13), (25) and (34) we find the skewness parameters γ and δ as

$$\gamma = \left(\frac{(k-1)[6(k+1)(\alpha + \beta t)^2 - 2\alpha(k+1) + 2(\alpha + \beta t)]}{4(\alpha + \beta t)^2 (k^2 + 4k + 1) + (k+1)^2 [3(\alpha + \beta t) + 2 - 6t^2 (A_0 t^\alpha e^{\beta t})^{-2}]}\right) \tag{36}$$

$$\delta = \left(\frac{2(k-1)[(\alpha + \beta t)^2 - (k+1)(\alpha + \beta t - 1)]}{4(\alpha + \beta t)^2 (k^2 + 4k + 1) + (k+1)^2 [3(\alpha + \beta t) + 2 - 6t^2 (A_0 t^\alpha e^{\beta t})^{-2}]}\right) \tag{37}$$

From Eq.(34) we observe that the anisotropic DE is always positive, decrease with the increase in cosmic time and attains a constant value at late times. It can be observed from Eq.(35) the EoS parameter is always negative and varies in the quintessence region which should be the case since we are dealing with a scalar field model. It may be observed from Eqs.(36) and (37) that the skewness parameters decrease with time and will vanish when k=1so that the universe exhibits isotropy at late times. Our results confirm the cosmological observations.

5. Concluding remarks

Anisotropic DE models help to explain the recent scenario of accelerated expansion of the universe. In particular DE models in modified theories of gravitation are being investigated in literature to throw light on this mysterious concept. Here, we have investigated anisotropic DE model in Bianchi type-V space-time in the frame work of scale covariant theory of gravitation[5,6] which is a modified Einstein's theory of gravitation. The basic field equations of this theory have been solved using physical conditions and presented an exact cosmological model in the presence of anisotropic DE. The significant cosmological parameters of the model are determined and discussed in detail. We have observed that our model exhibits a smooth transition from early deceleration to late time acceleration. We also observe that we have a quintessence model. Our model agrees with the cosmological observations.

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