Fuzzy Soft LA-(m,n)-Γ-ideals in LA-Γ-semigroups

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ABSTRACT

In this paper, we introduce some certain fuzzy soft algebraic notions of generalized concepts in LA-Γ-semigroups and study some properties of their families.

Keywords: (Fuzzy) Soft Sets; LA-Γ-semigroup; (Fuzzy) LA-(m,n)-Γ-ideals

1. Introduction

In[23], Zadeh introduced the notion of fuzzy subset. The notion of a fuzzy ideal in Γ -ring was first introduced by Jun and Lee[10]. The concept of an LA-semigroup (also known as AG-groupoids) was defined by Kazim and Naseeruddin[12]. The notion of Γ-semigroup was introduced by Sen[19,20].

Shah and Rehman[22], introduced the notion of LA-Γ-semigroup (Γ -AG-groupoid) and discussed some properties of Γ-ideals and Γ-bi-ideals in Γ-AG-groupoids. Moreover in[23], they defined fuzzy Γ -ideals in a Γ -AG-groupoid and studied some of its properties. Abbasi and Basar[1], introduced the notion of (m,n) - Γ -ideal of an LA-Γ -semigroup. In[3], Akın investigated generalizations of some concepts in LA-Γ-semigroup.

Fuzzy soft sets which has drawn a steadily increasing attention of the researchers and has led to remarkable development in some research areas also has been a very dynamic area for the algebraists. In this paper, we study some certain concepts of the fuzzy soft sets in LA-(m,n)-Γ-semigroups.

2. Preliminaries

In this section we recall certain definitions and results in the notion of LA-Γ-semigroups from the references[1,3,13,14,22,23,25] and we also recall certain definitions in the concept of (fuzzy) soft sets from the references[2,4,9,11,15-18,24].

2.1 LA-Γ-semigroups

Let S and Γ be nonempty sets. We call S to be an LA-Γ-semigroup if there exists a mapping S×Γ×S→S, written as (a,γ,b) and denoted by ab such that S satisfies the identity (ab)c=a(bc) for all a,b,c∈S and γ,α∈Γ. If S is an LA-Γ-semigroup and A,B⊆S, then we denote AB={a+b|a∈A,b∈B and γ ∈ Γ}. For a positif integer m, the power of B is defined as follows: B^m=...((BΓB)...ΓB.

Example 2.1

(i) (22), Example 2) Let Γ = {1,2,3} . Define a mapping Z×Γ×Z→Z by ab = b−γ−a for all a,b∈Z and γ ∈ Γ , where “−” is a usual subtraction of integers. Then Z is a LA-Γ-semigroup.

(ii) Let S = (0, + ∞) = Γ . Define a mapping S×Γ×S→S by ab = bγ for all a,b∈S and γ ∈ Γ. Then S is a LA-Γ-semigroup.

(iii) Let S = M_n(ℝ) = Γ . Define a mapping S×Γ×S→S by AB = AT + γ + B for all A,B∈S and γ ∈ Γ. Then S is a LA-Γ-semigroup.

Definition 2.2 Let S be an LA-Γ-semigroup.

(i) An element e of S is called left (right) identity if ea = a(ay = a) for all a ∈ S and γ ∈ Γ.

(ii) S is called a band if its all elements are idempotent, i.e., aγa = a for all a ∈ S and γ ∈ Γ.

(iii) S is called a locally associative LA-Γ-semigroup if (aγa)α = aγ(αa) for all a ∈ S and γ,α ∈ Γ.
Example 2.3 (\cite{21}, Example 2.8.) Let $S$ be a locally associative AG-groupoid (LA-semigroup) defined by the following Cayley table.

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Let for all $a,b \in S$ and $\alpha \in \Gamma$, define a mapping $S \times \Gamma \times S \rightarrow S$ by $\alpha \cdot b = a \cdot b$. Then $S$ is a locally associative LA-$\Gamma$-semigroup.

Lemma 2.4 (\cite{14}, Proposition 60) Let $S$ be a locally associative LA-$\Gamma$-semigroup and $m$ be a positif integer. Then $a^{m}\Gamma b^{m} = (a^{m} \Gamma b)^{m}$ for all $a,b \in S$.

Proposition 2.5 Let $S$ be an LA-$\Gamma$-semigroup.

(i) The left (right) identity in an LA-$\Gamma$-semigroup is unique if it exists.

(ii) If $S$ contains a right identity, then it becomes a commutative $\Gamma$-semigroup.

(iii) Every LA-$\Gamma$-semigroup with left identity satisfy the equalities $\gamma (a \cdot b) = b \cdot (a \cdot c)$ and $(\gamma a)(c \cdot b) = (dyc) (a \cdot (b\beta))$ for all $a,b,c,d \in S$ and $\gamma,\alpha,\beta \in \Gamma$.

(iv) $S$ is $\Gamma$-medial, i.e., $(\alpha \cdot b) \cdot (\gamma \cdot c) = (\alpha \cdot c) \cdot (\beta \cdot d)$ for all $a,b,c \in S$ and $\gamma,\alpha,\beta \in \Gamma$ in $S$.

Definition 2.6 Let $S$ be an LA-$\Gamma$-semigroup.

(i) A nonempty subset $A$ of $S$ is called a LA-$\Gamma$-subsemigroup of $S$ if $a \cdot b \in A$ for all $a,b \in A$ and $\gamma \in \Gamma$.

(ii) A nonempty subset $A$ of $S$ is called a left (right) $\Gamma$-ideal of $S$ if $\gamma A \subseteq A$ (\Gamma A \subseteq A). A nonempty subset $A$ of $S$ is called a $\Gamma$-ideal of $S$ if it is both a left and a right $\Gamma$-ideal of $S$.

(iii) A nonempty subset $A$ of $S$ is called a generalized $\Gamma$-bi-ideal of $S$ if $(\Gamma A) \Gamma A \subseteq A$.

(iv) An LA-$\Gamma$-subsemigroup $A$ of $S$ is called a $\Gamma$-bi-ideal of $S$ if $(\Gamma A) \Gamma A \subseteq A$.

(v) A nonempty subset $A$ of $S$ is called a $\Gamma$-interior ideal of $S$ if $(\Gamma A) \Gamma S \subseteq A$.

Lemma 2.7 (\cite{22,23}) Let $S$ be an LA-$\Gamma$-semigroup. If $S$ is an LA-$\Gamma$-semigroup with left identity $e$, then every right $\Gamma$-ideal of $S$ is a left $\Gamma$-ideal of $S$.

2.2 Fuzzy subsets of LA-$\Gamma$-semigroups

A function $f$ from a nonempty set $S$ to the unit interval $[0,1]$ is called a fuzzy subset of $S$. Let $f,g$ be fuzzy subsets of $S$, then $f \subseteq g$ means that $f(a) \leq g(b)$ for all $a,b \in S$. For $t \in [0,1]$, the set $f_{t} = \{a \in S | f(a) \geq t \}$ is called a level set of $f$. Let $A$ be a subset of $S$, then $\chi_{A}$ is the characteristic function of $A$ defined as, for all $x \in S$,

$$\chi_{A}(x) = \begin{cases} 1 & , \ x \in A \\ 0 & , \ x \notin A \end{cases}$$

For all $(f_{t})_{t \in (\alpha \subseteq A) \in \mathbb{P}(S)}$ and $x \in S$ ,

$$(\Lambda_{\alpha \subseteq A})_{t \in (\alpha \subseteq A)}(x) = \Lambda_{\alpha \subseteq A} f_{t}(x) \text{ and } (\vee_{\alpha \subseteq A})_{t \in (\alpha \subseteq A)}(x) = \vee_{\alpha \subseteq A} f_{t}(x).$$

Let $f,g$ be any fuzzy subsets of a LA-$\Gamma$-semigroup $S$ and $x \in S$, then the $\Gamma$-product $f \cdot g$ is defined by

$$(f \cdot g)(x) = \begin{cases} \vee_{x \in y \gamma} f(y) \wedge g(b) & , \exists \gamma \in \Gamma \text{ such that } x = ayb \\ 0 & , \text{otherwise} \end{cases}$$

Definition 2.8 Let $S$ be an LA-$\Gamma$-semigroup and $f$ be a fuzzy subset of $S$.

(i) $f$ is called a fuzzy LA-$\Gamma$-subsemigroup of $S$ if $f(ab) \geq f(a) \wedge f(b)$ for all $a,b \in S$ and $\gamma \in \Gamma$.

(ii) $f$ is called a fuzzy left (right) $\Gamma$-ideal of $S$ if $f(ab) \geq f(b)$ ($f(ab) \geq f(a)$) for all $a,b \in S$ and $\gamma \in \Gamma$. $f$ is called a fuzzy $\Gamma$-two-sided ideal of $S$ if it is both a fuzzy left and fuzzy right ideal of $S$.

(iii) $f$ is called a fuzzy generalized $\Gamma$-bi-ideal of $S$ if $f((ab)c) \geq f(a) \wedge f(c)$ for all $a,b,c \in S$ and $\gamma,\alpha \in \Gamma$.

(iv) A fuzzy LA-$\Gamma$-subsemigroup $f$ is called a fuzzy $\Gamma$-bi-ideal of $S$ if $f((ab)c) \geq f(a) \wedge f(c)$ for all $a,b,c \in S$ and $\gamma,\alpha \in \Gamma$.

(v) $f$ is called a fuzzy $\Gamma$-interior ideal of $S$ if $f((ab)x) \geq f(x)$ for all $a,b,x \in S$ and $\gamma,\alpha \in \Gamma$.

Lemma 2.9 (\cite{23}) Let $S$ be an LA-$\Gamma$-semigroup.

(i) Let $\emptyset \neq A \subseteq S$. Then $A$ is a LA-$\Gamma$-subsemigroup of $S$ if and only if the characteristic function $\chi_{A}$ of $A$ is a fuzzy LA-$\Gamma$-subsemigroup of $S$.

(ii) Let $\emptyset \neq A \subseteq S$. Then $A$ is a left (right) LA-$\Gamma$-ideal of $S$ if and only if the characteristic function $\chi_{A}$ of $A$ is a fuzzy left (right) LA-$\Gamma$-ideal of $S$.

(iii) A fuzzy subset $f$ of $S$ is fuzzy LA-$\Gamma$-subsemigroup of $S$ if and only if the level set of $f$ is LA-$\Gamma$-subsemigroup of $S$.

(iv) A fuzzy subset $f$ of $S$ is fuzzy left (right) LA-$\Gamma$-ideal of $S$ if and only if the level set of $f$ is left (right) LA-$\Gamma$-ideal of $S$.
2.3 (Fuzzy) LA-(m,n)-\(\Gamma\)-ideals

The following definition introduces some certain concepts of LA-(m,n)-\(\Gamma\)-semigroups.

**Definition 2.10**[3] Let \(S\) be LA-\(\Gamma\)-semigroup.

(i) For an element \(a \in S\) and positive integer \(m\), the power of \(a\) is defined as the set \(a^m = ((a(a\Gamma a))_n)\Gamma a\), where \(a^1 = (a)\) and \(a^2 = a^1a\) (See [14]).

(ii) A nonempty subset \(A\) of \(S\) is called an LA-(m,n)-\(\Gamma\)-subsemigroup of \(S\) if \(a^m\Gamma b^n \subseteq A\) for all \(a,b \in A\).

(iii) A nonempty subset \(A\) of \(S\) is called an LA-(m,n)-\(\Gamma\)-bi-ideal of \(S\) if \((a^m\Gamma s^n)\Gamma b^n \subseteq A\) for all \(s \in S\) and \(a \in A\). A nonempty subset \(A\) of an LA-\(\Gamma\)-semigroup \(S\) is called a LA-(m,n)-\(\Gamma\)-two-sided ideal of \(S\) if it is both an LA-(m,n)-\(\Gamma\)-left and an LA-(m,n)-\(\Gamma\)-right ideal of \(S\).

(iv) A nonempty subset \(A\) of \(S\) is called a generalized LA-(m,n)-\(\Gamma\)-bi-ideal of \(S\) if \((a^m\Gamma s^n)\Gamma b^n \subseteq A\) for all \(s \in S\) and \(a,b \in A\).

(v) An LA-(m,n)-\(\Gamma\)-subsemigroup \(A\) of \(S\) is called an LA-(m,n)-\(\Gamma\)-bi-ideal of \(S\) if \((a^m\Gamma s^n)\Gamma b^n \subseteq A\) for all \(s \in S\) and \(a,b \in A\).

(vi) A nonempty subset \(A\) of \(S\) is called an LA-(m,n)-\(\Gamma\)-interior ideal of \(S\) if \(((s_1)^m\Gamma a)^c(s_2)^n \subseteq A\) for all \(s_1,s_2 \in S\) and \(a \in A\).

The following definition introduces some certain fuzzy concepts of LA-(m,n)-\(\Gamma\)-semigroups.

**Definition 2.11**[3] Let \(f\) be a fuzzy subset of \(S\).

(i) \(f\) is called a fuzzy LA-(m,n)-\(\Gamma\)-subsemigroup of \(S\) if \(f(a) \wedge f(b)\) for all \(a,b \in S\) and \(x \in a^m\Gamma b^n\).

(ii) \(f\) is called a fuzzy left (right) LA-(m,n)-\(\Gamma\)-ideal of \(S\) if \(f(a) \subseteq f(b)\) \((f(a) \supseteq f(b))\) for all \(a,b \in S\) and \(x \in a^m\Gamma b^n\).

(iii) \(f\) is called a fuzzy generalized LA-(m,n)-\(\Gamma\)-bi-ideal of \(S\) if \(f(x) \geq f(a) \wedge f(c)\) for all \(a,b,c \in S\) and \(x \in (a^m\Gamma c)\Gamma b^n\).

(iv) A fuzzy LA-(m,n)-\(\Gamma\)-subsemigroup \(f\) is called a fuzzy LA-(m,n)-\(\Gamma\)-bi-ideal of \(S\) if it is a fuzzy generalized LA-(m,n)-\(\Gamma\)-bi-ideal of \(S\).

(v) \(f\) is called a fuzzy LA-(m,n)-\(\Gamma\)-interior ideal of \(S\) if \(f(x) \geq f(c)\) for all \(a,b,c \in S\) and \(x \in (a^m\Gamma c)\Gamma b^n\).

**Proposition 2.12**[3] Let \(S\) be an LA-\(\Gamma\)-semigroup.

(i) \(A\) is an LA-(m,n)-\(\Gamma\)-subsemigroup of \(S\) if and only if \(\chi_A\) is a fuzzy LA-(m,n)-\(\Gamma\)-subsemigroup of \(S\).

(ii) \(A\) is an left (right) LA-(m,n)-\(\Gamma\)-ideal of \(S\) if and only if \(\chi_A\) is a fuzzy left (right) LA-(m,n)-\(\Gamma\)-ideal of \(S\).

(iii) \(A\) is a generalized LA-(m,n)-\(\Gamma\)-bi-ideal of \(S\) if and only if \(\chi_A\) is a fuzzy generalized LA-(m,n)-\(\Gamma\)-bi-ideal of \(S\).

(iv) \(A\) is an LA-(m,n)-\(\Gamma\)-bi-ideal of \(S\) if and only if \(\chi_A\) is a fuzzy LA-(m,n)-\(\Gamma\)-bi-ideal of \(S\).

(v) \(A\) is an LA-(m,n)-\(\Gamma\)-interior ideal of \(S\) if and only if \(\chi_A\) is a fuzzy LA-(m,n)-\(\Gamma\)-interior ideal of \(S\).

**Proposition 2.13**[3] Let \(S\) be an LA-\(\Gamma\)-semigroup.

(i) \(f\) is a fuzzy LA-(m,n)-\(\Gamma\)-subsemigroup of \(S\) if and only if all of the nonempty level sets of \(f\) is an LA-(m,n)-\(\Gamma\)-subsemigroup of \(S\).

(ii) \(f\) is a fuzzy left (right) LA-(m,n)-\(\Gamma\)-ideal of \(S\) if and only if all of the nonempty level sets of \(f\) is an left (right) LA-(m,n)-\(\Gamma\)-ideal of \(S\).

(iii) \(f\) is a fuzzy generalized LA-(m,n)-\(\Gamma\)-bi-ideal of \(S\) if and only if all of the nonempty level sets of \(f\) is a generalized LA-(m,n)-\(\Gamma\)-bi-ideal of \(S\).

(iv) \(f\) is a fuzzy LA-(m,n)-\(\Gamma\)-bi-ideal of \(S\) if and only if all of the nonempty level sets of \(f\) is an LA-(m,n)-\(\Gamma\)-bi-ideal of \(S\).

(v) \(f\) is a fuzzy LA-(m,n)-\(\Gamma\)-interior ideal of \(S\) if and only if all of the nonempty level sets of \(f\) is an LA-(m,n)-\(\Gamma\)-interior ideal of \(S\).

2.4 Soft sets

Let \(U\) be an initial universe set and \(P\) be a set of parameters. The power set of \(U\) is denoted by \(P(U)\) and \(A\) is a subset of \(P\). A pair \((f,A)\) is called a soft set over \(U\) where \(F\) is a mapping given by \(F:A \rightarrow P(U)\)[18]. The pair \((U,P)\) denotes the collection of all soft sets on \(U\) with the attributes from \(P\) and is called a soft class[15].

**Definition 2.14**[18] Let \((f,A)\) and \((G,B)\) be two soft sets over \(U\), \((f,A)\) is called a soft subset of \((G,B)\), denoted by \((f,A) \subseteq (G,B)\), if \((i)\ B \subseteq A\), \((ii)\ F(x) \subseteq G(x)\) for each \(x \in B\).

**Definition 2.15**[6,8,9,11] Let \(\{(F_i,A_i)\mid i \in \Lambda\}\) be a family of soft sets in a soft class \((U,P)\). Then

(i) The restricted intersection of the family \(\{(F_i,A_i)\mid i \in \Lambda\}\), denoted by \((\cap_{i \in \Lambda})F_i\), is the soft set \((F,A)\) defined as: \(A = \bigcap_{i \in \Lambda} A_i\), \(F(x) = \bigcap_{i \in \Lambda} F_i(x)\) \((\forall x \in A)\).

(ii) The extended intersection of the family \(\{(F_i,A_i)\mid i \in \Lambda\}\), denoted by \((\cap_{i \in \Lambda})F_i\), is the soft set \((F,A)\) defined as: \(A = \bigcup_{i \in \Lambda} A_i\), \(F(x) = \bigcup_{i \in \Lambda} F_i(x)\) \((\forall x \in A)\).
\[ \bigcap_{i \in E(x)} F_i(x) \quad (\forall x \in A) \] where \( A(x) = \{ i | x \in A_i \} \).

(iii) The restricted union of the family \( \{ (f_i, E) \mid i \in A \} \), denoted by \( \bigcap_{f \in E} (f_i, E) \), is the soft set \( (F, E) \) defined as: \( A = \bigcap_{i \in E} A_i \), \( F(x) = \bigcup_{i \in E(x)} F_i(x) \) (\( \forall x \in A \)).

(iv) The extended union of the family \( \{ (f_i, E) \mid i \in A \} \), denoted by \( \bigcap_{i \in E} (f_i, E) \), is the soft set \( (F, E) \) defined as: \( A = \bigcup_{i \in E} A_i \), \( F(x) = \bigcup_{i \in E(x)} F_i(x) \) (\( \forall x \in A \)).

**Definition 2.16** \[6,8,11,16\] Let \( \{(f_i, E) \mid i \in A \} \) be a family of soft sets in a soft class \((U, P)\). Then

(i) The \( \Lambda \)-intersection of the family \( \{ (f_i, E) \mid i \in A \} \), denoted by \( \Lambda_{i \in E} (F_i, A_i) \), is the soft set \((F, E)\) defined as: \( A = \prod_{i \in E} A_i \), \( H((x_i)_{i \in E}) = \bigcap_{i \in E} H_i(x_i) \) (\( \forall (x_i)_{i \in E} \in A \)).

(ii) The \( \vee \)-union of the family \( \{ (f_i, E) \mid i \in A \} \), denoted by \( \vee_{i \in E} (F_i, A_i) \), is the soft set \((F, E)\) defined as: \( A = \prod_{i \in E} A_i \), \( H((x_i)_{i \in E}) = \bigcup_{i \in E} H_i(x_i) \) (\( \forall (x_i)_{i \in E} \in A \)).

(iii) The product of the family \( \{ (F_i, A_i) \mid i \in A \} \), denoted by \( \prod_{i \in E} (F_i, A_i) \), is the soft set \((F, E)\) defined as: \( A = \prod_{i \in E} A_i \), \( H((x_i)_{i \in E}) = \prod_{i \in E} H_i(x_i) \) (\( \forall (x_i)_{i \in E} \in A \)).

2.5 Fuzzy soft sets

Let \( U \) be an initial universe set and \( P \) be a set of parameters. \( F(U) \) denotes the set of all fuzzy sets of \( U \). A pair \((f, E)\) is called a fuzzy soft set over \( U \), where \( f : E \rightarrow F(U) \) is a mapping\[17\]. The pair \((U, P)\) denotes the collection of all fuzzy soft sets on \( U \) as initial set with the attributes from \( P \) and is called a fuzzy soft class\[2\].

**Definition 2.17** \[5\] Let \((f, E)\) be a fuzzy soft set over \( U \). For each \( \alpha \in [0, 1] \), the set \( (f(E))_{\alpha} = (f_\alpha)_{E} \) is called an \( \alpha \)-level set of \((f, E)\), where \( f_\alpha(a) = f(a)_{\alpha} \) for each \( a \in E \). Obviously, \((f, E))_{\alpha} \) is a soft set over \( U \).

**Definition 2.18** \[17\] Let \((f, E)\) and \((g, H)\) be two fuzzy soft sets over \( U \), \((f, E)\) is called a fuzzy soft subset of \((g, H)\), denoted by \((f, E) \subseteq (g, H)\), if

(i) \( E \subseteq H \),

(ii) for each \( a \in E \), \( f(a) \subseteq g(a) \).

**Definition 2.19** \[2,16,17\] Let \( \{(f_i, E) \mid i \in A \} \) be a family of fuzzy soft sets in a fuzzy soft class \((U, P)\). Then

(i) The restricted intersection of the family \( \{(f_i, E) \mid i \in A \} \), denoted by \( \bigcap_{i \in E} (f_i, E) \), is a fuzzy soft set \((f, E)\) defined as: \( E = \bigcap_{i \in E} E_i \) and for all \( x \in E \), \( f(x) = \Lambda_{i \in E} f_i(x) \).

(ii) The extended intersection of the family \( \{(f_i, E) \mid i \in A \} \), denoted by \( \bigcap_{i \in E} (f_i, E) \), is a fuzzy soft set \((f, E)\) defined as: \( E = \bigcup_{i \in E} E_i \) and for all \( x \in E \), \( f(x) = \Lambda_{i \in E} f_i(x) \).

2.6 Fuzzy Soft Sets in \( \Gamma \)-Semigroups

Akram et. al introduce some definitions of algebraic structure of fuzzy soft sets in \( \Gamma \)-semigroups.

**Definition 2.21** \[4\] Let \( S \) be a \( \Gamma \)-semigroup and \((f, E)\) be a fuzzy soft set in the fuzzy soft class \((S, P)\). Then

(i) \((f, E)\) is called a fuzzy soft \( \Gamma \)-subsemigroup of \( S \) if \( f(x) \) is a fuzzy \( \Gamma \)-subsemigroup of \( S \) for all \( x \in E \).

(ii) \((f, E)\) is called a fuzzy soft left (right) \( \Gamma \)-ideal of \( S \) if \( f(x) \) is a fuzzy left (right) \( \Gamma \)-ideal of \( S \) for all \( x \in E \).

(iii) \((f, E)\) is called a fuzzy soft \( \Gamma \)-ideal of \( S \) if \( f(x) \) is both a fuzzy left and fuzzy right \( \Gamma \)-ideal of \( S \) for all \( x \in E \).

3. Main results

In this paper, we consider an LA-\( \Gamma \)-semigroup \( S \) as the initial universe and we introduce the fuzzy soft concepts in LA-(\( m,n \))-\( \Gamma \)-semigroups.

**Definition 3.1** Let \( S \) be an LA-\( \Gamma \)-semigroup and \((f, E)\) be a fuzzy soft set in the fuzzy soft class \((S, P)\). Then

(i) \((f, E)\) is called a fuzzy soft \( \Lambda \)-subsemigroup of \( S \) if \( f(x) \) is a fuzzy...
LA-(m,n)-Γ-subsemigroup of S for all x ∈ E.

(ii) (f,E) is called a fuzzy soft left (right) LA-(m,n)-Γ-ideal of S if f(x) is a fuzzy left (right) LA-(m,n)-Γ-ideal of S for all x ∈ E. (f,E) is called a fuzzy soft LA-(m,n)-Γ-ideal of S if f(x) is both a fuzzy left and fuzzy right LA-(m,n)-Γ-ideal of S for all x ∈ E.

(iii) (f,E) is called a fuzzy soft generalized LA-(m,n)-Γ-bi-ideal of S if f(x) is a fuzzy generalized LA-(m,n)-Γ-bi-ideal of S for all x ∈ E.

(iv) (f,E) is called a fuzzy soft LA-(m,n)-Γ-bi-ideal of S if f(x) is a fuzzy LA-(m,n)-Γ-bi-ideal of S for all x ∈ E.

(v) (f,E) is called a fuzzy soft LA-(m,n)-Γ-interior ideal of S if f(x) is a fuzzy LA-(m,n)-Γ-interior ideal of S for all x ∈ E.

Example 3.2 Let μ be a fuzzy LA-(m,n)-Γ-subsemigroup of S. Then (f,[0,1]), defined by f(x) = μxα for all α ∈ [0,1], is a fuzzy soft LA-(m,n)-Γ-subsemigroup of S.

Lemma 3.3 Let S be an LA-Γ-semigroup. Then

(i) (f,E) is a fuzzy soft LA-(m,n)-Γ-subsemigroup if and only if all of the nonempty level sets of f is an LA-(m,n)-Γ-subsemigroup,

(ii) (f,E) is a fuzzy soft left (right) LA-(m,n)-Γ-ideal if and only if all of the nonempty level sets of f is a left (right) LA-(m,n)-Γ-ideal,

(iii) (f,E) is a fuzzy soft generalized LA-(m,n)-Γ-bi-ideal if and only if all of the nonempty level sets of f is a generalized LA-(m,n)-Γ-bi-ideal,

(iv) (f,E) is a fuzzy soft LA-(m,n)-Γ-bi-ideal if and only if all of the nonempty level sets of f is an LA-(m,n)-Γ-bi-ideal,

(v) (f,E) is a fuzzy soft LA-(m,n)-Γ-interior ideal if and only if all of the nonempty level sets of f is an LA-(m,n)-Γ-interior ideal.

Proof. Straightforward from Definition 3.1 and Proposition 2.13.

Lemma 3.4 Let S be an LA-Γ-semigroup and A ⊆ E. Then

(i) If (f,E) is a fuzzy soft LA-(m,n)-Γ-subsemigroup, then (f,A) is a fuzzy soft LA-(m,n)-Γ-subsemigroup,

(ii) If (f,E) is a fuzzy soft left (right) LA-(m,n)-Γ-ideal, then (f,A) is a fuzzy soft left (right) LA-(m,n)-Γ-ideal,

(iii) If (f,E) is a fuzzy soft generalized LA-(m,n) - Γ -bi-ideal, then (f,A) is a fuzzy soft generalized LA-(m,n) - Γ -bi-ideal,

(iv) If (f,E) is a fuzzy soft LA-(m,n) - Γ -bi-ideal, then (f,A) is a fuzzy soft LA-(m,n) - Γ -bi-ideal,

(v) If (f,E) is a fuzzy soft LA-(m,n) - Γ -bi-ideal, then (f,A) is a fuzzy soft LA-(m,n) - Γ -bi-ideal.

Proof. Straightforward.

Theorem 3.5 Let \( (f_i,E_i) \{i ∈ Λ \} \) be a family of fuzzy soft LA-(m,n)-Γ-subsemigroups of S. Then

(i) \( \bigcap_{i ∈ Λ} f_i(E_i) \), if \( \bigcap_{i ∈ Λ} E_i ≠ \emptyset \),

(ii) \( \bigcup_{i ∈ Λ} f_i(E_i) \),

(iv) \( Λ_{i ∈ Λ}(f_i,E_i) \),

(v) \( \prod_{i ∈ Λ}(f_i,E_i) \) are fuzzy soft LA-(m,n)-Γ-subsemigroups of S.

Proof. Let a,b ∈ S and u ∈ a\"Γ\"b". 

(i) Let \( \bigcap_{i ∈ Λ}(f_i,E_i) = (f,E) \). Then E = \( \bigcap_{i ∈ Λ} E_i \) and f(x) = \( Λ_{i ∈ Λ}(f_i(x)) \) for all x ∈ E. Since f(x) = \( Λ_{i ∈ Λ}(f_i(x)) \) (u) = \( Λ_{i ∈ Λ}(f_i(x))(u) \geq Λ_{i ∈ Λ}(f_i(x)(a) ∧ f_i(x)(b)) = \( Λ_{i ∈ Λ}(f_i(x))(a) ∧ Λ_{i ∈ Λ}(f_i(x)(b)) = \( Λ_{i ∈ Λ}(f_i(x))(a) ∧ Λ_{i ∈ Λ}(f_i(x)(b)) = \), therefore \( \bigcap_{i ∈ Λ}(f_i,E_i) \) is a fuzzy soft LA-(m,n)-Γ-subsemigroup of S.

(ii) Let \( \bigcup_{i ∈ Λ}(f_i,E_i) = (f,E) \). Then E = \( \bigcup_{i ∈ Λ} E_i \) and f(x) = \( Λ_{i ∈ Λ}(f_i(x)) \) for all x ∈ E, where \( Λ(x) = \{i | x ∈ E_i \} \). Since f(x) = \( Λ_{i ∈ Λ}(f_i(x))(x) \) = \( Λ_{i ∈ Λ}(f_i(x))(a) ∧ f_i(x)(b)) = \( Λ_{i ∈ Λ}(f_i(x))(a) ∧ Λ_{i ∈ Λ}(f_i(x)(b)) = \), therefore \( \bigcup_{i ∈ Λ}(f_i,E_i) \) is a fuzzy soft LA-(m,n)-Γ-subsemigroup of S.

(iii) Let \( Λ_{i ∈ Λ}(f_i,E_i) = (f,E) \). Then E = \( \prod_{i ∈ Λ} E_i \) and f(x) = \( Λ_{i ∈ Λ}(f_i,E_i) \) for all x ∈ E. Since f(x) = \( \bigvee_{i ∈ Λ}(f_i(x)) \) (u) = \( \bigvee_{i ∈ Λ}(f_i(x))(u) \geq \bigvee_{i ∈ Λ}(f_i(x)(a) ∧ f_i(x)(b)) = \), therefore \( \bigcup_{i ∈ Λ}(f_i,E_i) \) is a fuzzy soft LA-(m,n)-Γ-subsemigroup of S.

(iv) Let \( \prod_{i ∈ Λ}(f_i,E_i) = (f,E) \). Then E = \( \prod_{i ∈ Λ} E_i \) and f(x) = \( \bigvee_{i ∈ Λ}(f_i,E_i) \). Since
Indeed, let \( \Gamma \subseteq \text{any} \) and \( \bigvee_{E = f(x)(u)} = \text{finite} \). For all \( \alpha \in \Gamma \), the family \( (f,A),(g,A) \) of \( \text{fuzzy} \) \( \text{LA} \)-subsemigroups of \( S \) is finite. Let \( (f_A),(g_A) \in \langle S,P \rangle \) be defined by \( f(a) = x_{BFS} \) and \( g(a) = x_{SBF} \) for all \( a \in A \). Thus \( (f_A)U^g(g_A) \) is not a fuzzy soft \( \text{LA}-(1,1) \)-\( \Gamma \)-subsemigroups of \( S \) although \( (f_A),(g_A) \) are fuzzy soft \( \text{LA}-(1,1) \)-\( \Gamma \)-subsemigroups of \( S \). Indeed, \( (f(a)U^g(g)(x))(y) = (x_{BFS}U^x_{SBF})(x)(y) = 0 \) for any \( a \in A \). However, \( (f(a)U^g(g)(x))(y) = (x_{BFS}U^x_{SBF})(x)(y) = 1 \)

Theorem 3.7 Let \( \{f_iE\mid i \in A\} \) be a family of fuzzy soft \( \text{LA}-(m,n) \)-\( \Gamma \)-subsemigroups of \( S \).

(i) Let \( U^e_{\text{IA}}(f_iE) \) is fuzzy soft \( \text{LA}-(m,n) \)-\( \Gamma \)-subsemigroups of \( S \).

(ii) Let \( V_{\text{IA}}(f_iE) = \langle f,E \rangle \) and let \( x \in E \). For all \( i \in A \), if \( f_i(x) \not\subseteq f_i(x) \) or \( f_i(x) \not\subseteq f_i(x) \), then \( V_{\text{IA}}(f_iE) \) is fuzzy soft \( \text{LA}-(m,n) \)-\( \Gamma \)-subsemigroups of \( S \).

Proof. Straightforward.

Theorem 3.8 Let \( \{f_iE\mid i \in A\} \) be a family of fuzzy soft left (right) \( \text{LA}-(m,n) \)-\( \Gamma \)-ideal of \( S \).

(i) Let \( \cap_{\text{IA}}(f_iE_i) \) if \( \cap_{\text{IA}} E_i \neq \emptyset \),

(ii) Let \( U^r_{\text{IA}}(f_iE_i) \),

(iii) Let \( U^e_{\text{IA}}(f_iE_i) \),

(iv) Let \( V_{\text{IA}}(f_iE_i) \).

(v) \( \Lambda_{\text{IA}}(f_iE_i) \),

(vi) \( V_{\text{IA}}(f_iE_i) \),

(vii) \( \prod_{\text{IA}}(f_iE_i) \) fuzzy soft left (right) \( \text{LA}-(m,n) \)-\( \Gamma \)-ideals of \( S \).

Proof. Let \( a,b \in S \) and \( u \in a^*b^* \).

(i) Let \( \cap_{\text{IA}}(f_iE_i) = \langle f,E \rangle \) and \( f(x) = \Lambda_{\text{IA}A}(f_iE_i) \) for all \( x \in E \). Since \( f(x)(u) = (\Lambda_{\text{IA}A}(f_iE_i)(u))((\Lambda_{\text{IA}A}(f_iE_i)(u)) \geq (\Lambda_{\text{IA}A}(f_iE_i)(b)) = \text{fuzzy soft} \text{ left} \text {LA}-(m,n)-\Gamma\text{-ideal of } S \text{ if } \cap_{\text{IA}} E_i \neq \emptyset \).

(ii) Let \( \cap_{\text{IA}}(f_iE_i) = \langle f,E \rangle \). Then \( E = \prod_{\text{IA}}(f_iE_i) \) is fuzzy soft left \( \text{LA}-(m,n)-\Gamma\text{-ideal of } S \).

(iii) Let \( V_{\text{IA}}(f_iE_i) \). Then \( E \subseteq V_{\text{IA}}(f_iE_i) \) and \( f(x) = \Lambda_{\text{IA}A}(f_iE_i) \) for all \( x \in E \), where \( \Lambda(x) = \{i|x \in E_i\} \). Since \( f(x)(u) = (\Lambda_{\text{IA}A}(f_iE_i)(u)) = (\Lambda_{\text{IA}A}(f_iE_i)(u) \geq (\Lambda_{\text{IA}A}(f_iE_i)(b)) = \text{fuzzy soft} \text{ left} \text {LA}-(m,n)-\Gamma\text{-ideal of } S \).

(iv) Let \( U^r_{\text{IA}}(f_iE_i) = \langle f,E \rangle \). Then \( E = \prod_{\text{IA}}(f_iE_i) \) and \( f(x) = \Lambda_{\text{IA}A}(f_iE_i) \) for all \( x \in E \). Since \( f(x)(u) = (\Lambda_{\text{IA}A}(f_iE_i)(u)) \geq (\Lambda_{\text{IA}A}(f_iE_i)(b)) = \text{fuzzy soft} \text{ left} \text {LA}-(m,n)-\Gamma\text{-ideal of } S \).

(v) \( \Lambda_{\text{IA}}(f_iE_i) \),

(vi) \( V_{\text{IA}}(f_iE_i) \),

(vii) \( \prod_{\text{IA}}(f_iE_i) \) are fuzzy soft left (right) \( \text{LA}-(m,n)-\Gamma\text{-ideal of } S \).
\[ f((x_i)_{i \in \Lambda}) (u) = (\bigvee \text{finite } \lambda_{ij} f(x_i)(x_j))(u) = \bigvee \text{finite } \lambda_{ij} (\lambda_{ij} f(x_i)(x_j))(u) = \bigvee \text{finite } \lambda_{ij} (\lambda_{ij} f(x_j)(x_i))(u) \geq \bigvee \text{finite } \lambda_{ij} (\lambda_{ij} f(x_i)(x_j)) \geq (\bigvee \text{finite } \lambda_{ij} (\lambda_{ij} f(x_i)(x_j)))(b) = (\bigvee \text{finite } \lambda_{ij} (\lambda_{ij} f(x_i)(x_j)))(b) = f((x_i)_{i \in \Lambda}) (b) \]

Therefore \( \prod_{i \in \Lambda} (f_i, E_i) \) is a fuzzy soft left \( LA-(m,n)\)-\( \Gamma \)-ideal of \( S \).

Proofs are similar for fuzzy soft right \( LA-(m,n)\)-\( \Gamma \)-ideals of \( S \).

**Theorem 3.9** Let \( \{(f_i, E_i) \mid i \in \Lambda \} \) be a family of fuzzy soft generalized \( LA-(m,n)\)-\( \Gamma \)-bi-ideal \( S \). Then

(i) \( \cap_{i \in \Lambda} (f_i, E_i), \) if \( \cap_{i \in \Lambda} E_i \neq \emptyset \),

(ii) \( \cap_{i \in \Lambda} (f_i, E_i) \),

(iii) \( \lambda_{i \in \Lambda} f_i, E_i \),

(iv) \( \prod_{i \in \Lambda} (f_i, E_i) \) are fuzzy soft generalized \( LA-(m,n)\)-\( \Gamma \)-bi-ideals of \( S \).

**Proof.** Let \( a, b \in S \) and \( u \in (a^{m} B) \Gamma c^{n} \).

(i) Let \( \cap_{i \in \Lambda} (f_i, E_i) = (f, E) \). Then \( E = \cap_{i \in \Lambda} E_i \) and 
\[ f(x) = \lambda_{i \in \Lambda} f_i(x) \text{ for all } x \in E. \]
Since 
\[ f(x)(u) = (\lambda_{i \in \Lambda} f_i(x))(u) = \lambda_{i \in \Lambda} f_i(x)(u) \geq \lambda_{i \in \Lambda} f_i(x)(a) \wedge f_i(x)(c) = \lambda_{i \in \Lambda} f_i(x)(a) \wedge \lambda_{i \in \Lambda} f_i(x)(c) = (\lambda_{i \in \Lambda} f_i(x)(a)) \wedge (\lambda_{i \in \Lambda} f_i(x)(c)) = f(x)(a) \wedge f(x)(c) \]
therefore \( \cap_{i \in \Lambda} (f_i, E_i) \) is a fuzzy soft generalized \( LA-(m,n)\)-\( \Gamma \)-bi-ideals of \( S \).

(ii) Let \( \cap_{i \in \Lambda} (f_i, E_i) = (f, E) \). Then \( E = \cap_{i \in \Lambda} E_i \) and 
\[ f((x_i)_{i \in \Lambda}) (u) = (\bigvee \text{finite } \lambda_{ij} f_i(x_j))(u) = \bigvee \text{finite } \lambda_{ij} (\lambda_{ij} f_i(x_j))(u) = \bigvee \text{finite } \lambda_{ij} (\lambda_{ij} f_i(x_j))(u) \geq \bigvee \text{finite } \lambda_{ij} (\lambda_{ij} f_i(x_j)) \geq (\bigvee \text{finite } \lambda_{ij} (\lambda_{ij} f_i(x_j)))(b) = (\bigvee \text{finite } \lambda_{ij} (\lambda_{ij} f_i(x_j)))(b) = f((x_i)_{i \in \Lambda}) (b) \]

Therefore \( \prod_{i \in \Lambda} (f_i, E_i) \) is a fuzzy soft generalized \( LA-(m,n)\)-\( \Gamma \)-bi-ideals of \( S \).
Theorem 3.13 Let \((f_i, E_i)\) be a family of fuzzy soft LA-\((m,n)\)-\(\Gamma\)-interior ideal of \(S\). Then

(i) \(\bigcap_{i \in \Lambda} f_i E_i\) if \(\bigcap_{i \in \Lambda} E_i \neq \emptyset\),

(ii) \(\bigcap_{i \in \Lambda} f_i E_i\),

(iii) \(\bigcup_{i \in \Lambda} f_i E_i\),

(iv) \(\bigcup_{i \in \Lambda} f_i E_i\),

(v) \(\bigcap_{i \in \Lambda} f_i E_i\),

(vi) \(\bigcap_{i \in \Lambda} (f_i E_i)\),

(vii) \(\bigcap_{i \in \Lambda} (f_i E_i)\) are fuzzy soft LA-\((m,n)\)-\(\Gamma\)-interior ideals of \(S\).

Proof. Let \(a, b, c \in S\) and \(u \in (a^m \Gamma b)\Gamma c^n\).

(i) Let \(\bigcap_{i \in \Lambda} f_i E_i = (f E)\). Then \(E = \bigcap_{i \in \Lambda} E_i\) and \(f(x) = \bigcap_{i \in \Lambda} f_i(x)\) for all \(x \in E\). Since \(f(x)(u) = (\bigcap_{i \in \Lambda} f_i(x))(u) = \bigcap_{i \in \Lambda} f_i(x)(u) \geq \bigcap_{i \in \Lambda} f_i(x)(b) = (\bigcap_{i \in \Lambda} f_i(x))(b) = f(x)(b)\), therefore \(\bigcap_{i \in \Lambda} f_i E_i\) is a fuzzy soft LA-\((m,n)\)-\(\Gamma\)-interior ideal of \(S\) if \(\bigcap_{i \in \Lambda} E_i \neq \emptyset\).

(ii) Let \(\bigcap_{i \in \Lambda} f_i E_i = (f E)\). Then \(E = \bigcap_{i \in \Lambda} E_i\) and \(f(x) = \bigcap_{i \in \Lambda} f_i(x)\) for all \(x \in E\). Since \(f(x)(u) = (\bigcap_{i \in \Lambda} f_i(x))(u) = \bigcap_{i \in \Lambda} f_i(x)(u) \geq \bigcap_{i \in \Lambda} f_i(x)(b) = (\bigcap_{i \in \Lambda} f_i(x))(b) = f(x)(b)\), therefore \(\bigcap_{i \in \Lambda} f_i E_i\) is a fuzzy soft LA-\((m,n)\)-\(\Gamma\)-interior ideal of \(S\).

(iii) Let \(\bigcap_{i \in \Lambda} f_i E_i = (f E)\). Then \(E = \bigcap_{i \in \Lambda} E_i\) and \(f(x) = \bigcap_{i \in \Lambda} f_i(x)\) for all \(x \in E\). Since \(f(x)(u) = (\bigcap_{i \in \Lambda} f_i(x))(u) = \bigcap_{i \in \Lambda} f_i(x)(u) \geq \bigcap_{i \in \Lambda} f_i(x)(b) = (\bigcap_{i \in \Lambda} f_i(x))(b) = f(x)(b)\), therefore \(\bigcap_{i \in \Lambda} f_i E_i\) is a fuzzy soft LA-\((m,n)\)-\(\Gamma\)-interior ideal of \(S\).

(iv) Let \(\bigcup_{i \in \Lambda} f_i E_i = (f E)\). Then \(E = \bigcup_{i \in \Lambda} E_i\) and \(f(x) = \bigcup_{i \in \Lambda} f_i(x)\) for all \(x \in E\). Since \(f(x)(u) = (\bigcup_{i \in \Lambda} f_i(x))(u) = \bigcup_{i \in \Lambda} f_i(x)(u) \geq \bigcup_{i \in \Lambda} f_i(x)(b) = (\bigcup_{i \in \Lambda} f_i(x))(b) = f(x)(b)\), therefore \(\bigcup_{i \in \Lambda} f_i E_i\) is a fuzzy soft LA-\((m,n)\)-\(\Gamma\)-interior ideal of \(S\).

(v) Let \(\bigcup_{i \in \Lambda} f_i E_i = (f E)\). Then \(E = \bigcup_{i \in \Lambda} E_i\) and \(f(x) = \bigcup_{i \in \Lambda} f_i(x)\) for all \(x \in E\). Since \(f(x)(u) = (\bigcup_{i \in \Lambda} f_i(x))(u) = \bigcup_{i \in \Lambda} f_i(x)(u) \geq \bigcup_{i \in \Lambda} f_i(x)(b) = (\bigcup_{i \in \Lambda} f_i(x))(b) = f(x)(b)\), therefore \(\bigcup_{i \in \Lambda} f_i E_i\) is a fuzzy soft LA-\((m,n)\)-\(\Gamma\)-interior ideal of \(S\).

(vi) Let \(\bigcap_{i \in \Lambda} f_i E_i = (f E)\). Then \(E = \bigcap_{i \in \Lambda} E_i\) and \(f(x) = \bigcap_{i \in \Lambda} f_i(x)\) for all \(x \in E\). Since \(f(x)(u) = (\bigcap_{i \in \Lambda} f_i(x))(u) = \bigcap_{i \in \Lambda} f_i(x)(u) \geq \bigcap_{i \in \Lambda} f_i(x)(b) = (\bigcap_{i \in \Lambda} f_i(x))(b) = f(x)(b)\), therefore \(\bigcap_{i \in \Lambda} f_i E_i\) is a fuzzy soft LA-\((m,n)\)-\(\Gamma\)-interior ideal of \(S\).

4. Conclusions

In this paper, we introduce the definitions of some certain fuzzy soft concepts in an LA-\((m,n)\)-\(\Gamma\)-semigroup and investigate some algebraic properties of fuzzy soft sets in LA-\((m,n)\)-\(\Gamma\)-semigroups. To extend this work, one could define fuzzy soft LA-\((m,n)\)-\(\Gamma\)-quasi, -prime and -semiprime ideals of an LA-\((m,n)\)-\(\Gamma\)-semigroup and examine algebraic properties of them.

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References


