Fuzzy pricing of European options based on Liu process

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Abstract: Many studies have found that in addition to randomness, financial markets also have ambiguity. In order to better address this issue, this article introduces fuzziness into option pricing models and conducts in-depth research on existing fuzzy stock models.

Firstly, based on the generalized stock model of the standard Liu process, the corresponding option pricing formula was derived. Then, this article studied the generalized multi factor pricing model and derived the corresponding European option pricing formula. In order to make the research results more specific, this article provides specific examples of European options and corresponding chart analysis

This research achievement not only enriches the theory of option pricing, but also provides new methods and basis for option pricing in actual financial markets.

Keywords: credibility theory; Liu process; Fuzzy differential equation; Generalized multi factor price model

1. Introduction

Fuzzy option pricing is a cutting-edge research direction in the field of financial derivatives, mainly studying option pricing problems in fuzzy uncertainty environments. The study of fuzzy option pricing can provide more accurate and reliable information and guidance for investment decisions in financial markets, helping to reduce investment risks and improve market efficiency.

In real life, financial markets often have uncertainty and ambiguity, and traditional option pricing models cannot adapt well to this situation. Therefore, studying the pricing problem of fuzzy options has important theoretical and practical significance.

In recent decades, option pricing has developed rapidly. The Black Scholes model (abbreviated as B-S model) was first proposed by Black and Scholes, which obtained the price differential equation of financial derivatives without dividend and transaction cost payments. The B-S model is also the most classic option pricing model and has been widely applied in the financial field. Since then, option pricing has become a hot topic in financial market research. Merton first used Brownian motion to describe interest rates. Zadeh first proposed the concept of fuzzy sets, and then proposed possible metrics that can be used to quantify fuzzy events . Later, these theories gradually developed into today's fuzzy set theory, which was also the beginning of fuzzy mathematics. Although fuzzy set theory can be used to measure fuzzy events, there has been no effective method or theory for a long time. It was not until Liu proposed a credibility measure with self duality property that it developed into credibility theory and became an important theory for handling fuzzy events. Fuzziness is an important characteristic of option pricing. Based on credibility theory and Liu process, Zhang Yuanyuan established several types of fuzzy investment portfolio models and derived corresponding option pricing formulas using expected value method and actuarial method. Bo introduced the fuzzy interest rate model and fuzzy term structure, and derived the option pricing formula under fuzzy interest rates. Therefore, it is necessary to study option prices in fuzzy environments. As a common underlying asset in the financial market, if fuzziness can be added to the stock model, it can better explain the actual dynamic changes of stocks and be more in line with the actual situation

Firstly, many scholars have conducted in-depth research on fuzzy options and proposed a good model, such as the Liu stock model, the high stock model , and the general stock model. Most of these models are driven by geometric Liu processes. However, there is relatively little research on the stock model of mean regression, and the stock price model of mean regression can better explain the actual situation. Therefore, studying the mean regression model of the stock market is of great significance.

In addition, we can also consider the payment of dividends and transaction costs to make the resulting model more realistic. On this basis, this article studied a generalized stock model based on the standard Liu process and conducted numerical simulations.

2. basic concepts

Definition 1.1 Fuzzy variables are derived from the plausibility space () maps to the set of real numbersfunction on the Where, theis a non-empty set, thebethe power set ofis a plausibility measure.

An aggregate function is called a plausibility measure if it satisfies the following four conditions.

(1)$Cr\{\Theta\} = 1$;
(2)$Cr\{A\} \leq Cr\{B\}$"When$A \subset B$time; and
(3)$Cr\{A\} + Cr\{A^C\} = 1$, for any$A \in P$;
(4)$Cr\{\cup_i A_i\} = \sup_i Cr\{A_i\}$, for any$A_i$Both$\sup_i Cr\{A_i\} < 0.5$.

In order to describe fuzzy variables mathematically, the following definition of expected value of fuzzy variables is introduced. fuzzy variablesξ The expected value of is defined as that

$$E[\xi] = \int_0^{\infty} Cr\{\xi \geq r\}dr - \int_{-\infty}^{0} Cr\{\xi \leq r\}dr$$

At least one of the two integrals is finite.

In order to compute the expectation value, it is necessary to introduce the following plausibility inversion theorem.

Hypothesisξ is a fuzzy variable and that is its affiliation function. Then for any set of real numbers B

$$Cr\{\xi \in B\} \quad = \quad \frac{1}{2}(\sup_{x \in B} \nu(x) + 1 - \sup_{x \in B^C} \nu(x)).$$

Liu ProcessC_t is a fuzzy process that satisfies the following four conditions

1.C_0=0;

2.C_tAbout anyBoth have smooth increments; and

3.C_tAbout anyBoth have independent increments; the

4.C_{k+t} -C_k is a fuzzy variable with normal distribution and a mean value of, with a variance of fσ^2 t^2, whose affiliation function is

$$\mu(x) = 2(1 + \exp(\frac{\pi|x - et|}{\sqrt{6}\sigma t}))^{-1}, \ \ x \in R$$

In particular, when *e* =0 and *e* =1 The Liu process becomes a standard Liu process The standard Liu process is a special case of Liu process The fractional Liu process, as an extension of the Liu process, is more in line with the actual situation. Therefore, it is very important to study the fractional Liu process The definition of fractional Liu process is given below

Definition 1.2 Assumptions Z_t is a fuzzy process. C_t It is a standard Liu process. In the interval [a,b] inserted arbitrarily in the *k*+1 The interval is divided by a number of points, and there are a=t_1<t_2<...<t_{k+1}=b, and assuming that

$$\Delta = \max_{1 \leq i \leq k} |t_{i+1} - t_i|.$$

If when$\Delta \to 0$， $\lim_{\Delta \to 0} \sum_{i=1}^{k} Z_{t_i} (C_{t_{i+1}} - C_{t_i})$

exists almost everywhere,then the limit is said to be a fuzzy process Z_t About C_t Liu integral of

$$\int_a^b Z_t \, dC_t = \lim_{\Delta \to 0} \sum_{i=1}^{k} Z_{t_i} (C_{t_{i+1}} - C_{t_i})$$

In turn, it is claimed thatAboutLiu integrable of

3. fuzzy stock models

Definition 1.3: Generalized multifactor price model

$$\begin{cases} dX_t = rX_t dt \\ dY_{it} = (m_i - \alpha_i Y_{it})dt + \sum_{j=1}^{n} \sigma_{ij} Y_{jt}^{\beta_j} dC_{jt}, i = 1,2,\cdots,m; j = 1,2,\cdots,n, \end{cases}$$

Among them r, m_i, α_i, β_j, σ_{ij} respectively, for the determination of the i=1,2,..., m,j=1,2,...,n is a definite real number. That is, the model has that n Independent Liu processes and m+1 Equations.

$$\begin{cases} X_t = X_0\exp(rt) \\ Y_{it} = \frac{m_i}{\alpha_i} + \exp(-\alpha_i t)(Y_{i0} - \frac{m_i}{\alpha_i}) + \sum_{j=1}^{n}\int_0^t \sigma_{ij} Y_{js}^{\beta_j} dC_{js}, i = 1,2,\cdots,m; j = 1,2,\cdots,n. \end{cases}$$

Proof:By $dX_t = rX_t dt$ Get $\frac{dX_t}{X_t} = rdt$, whereupon the two-sided integral yields $X_t = C\exp(rt)$. Order $t = 0$ Get $X_t = X_0\exp(rt)$.

$$\begin{aligned} d(\exp(\alpha_i t)Y_{it}) &= \alpha_i\exp(\alpha_i t)Y_{it}dt + \exp(\alpha_i t)dY_{it} \\ &= \alpha_i\exp(\alpha_i t)Y_{it}dt + \exp(\alpha_i t)[(m_i - \alpha_i Y_{it})dt + \sum_{j=1}^{n}\sigma_{ij} Y_{ij}^{\beta_j} dC_{jt}] \\ &= m_i\exp(\alpha_i t)dt + \exp(\alpha_i t)\sum_{j=1}^{n}\sigma_{ij} Y_{ij}^{\beta_j} dC_{jt} \end{aligned}$$

Two sides of the score

$$\exp(\alpha_i t)Y_{it} - Y_{i0} = m_i\int_0^s \exp(\alpha_i s)ds + \exp(\alpha_i t)\sum_{j=1}^{n}\int_0^s \sigma_{ij} Y_{js}^{\beta_j} dC_{js}$$

Processing the first integral yields that

$$\exp(\alpha_i t)Y_{it} - Y_{i0} = \frac{m_i}{\alpha_i}\exp(\alpha_i t) - \frac{m_i}{\alpha_i} + \exp(\alpha_i t)\sum_{j=1}^{n}\int_0^s \sigma_{ij} Y_{js}^{\beta_j} dC_{js}$$

So $Y_{it} = \frac{m_i}{\alpha_i} + \exp(-\alpha_i t)(Y_{i0} - \frac{m_i}{\alpha_i}) + \sum_{j=1}^{n}\int_0^s \sigma_{ij} Y_{js}^{\beta_j} dC_{js}$.

Therefore, the proposition is proved.

Theorem 1.5 Suppose that X_t is the price of the bond, and Y_t is the stock price,and X_t and Y_t satisfies model (2.1),then the European power call option price equation is given by

$$\begin{cases} f = \theta\exp(-rT)(\int_J^{+\infty}(1 + \exp(\frac{\pi v}{\sqrt{6}(\sum_{j=1}^{n}\int_0^T \sigma_{ij} Y_{js}^{\beta_j}(js)^{-\alpha}djs)})))^{-1}U_1 dv, \ J \geq 0 \\ f = \theta\exp(-rT)U_2, \ J < 0, \end{cases}$$

Among them

$$\begin{aligned} U_1 &= [v + \frac{m_i}{\alpha_i} + \exp(-\alpha_i t)(Y_{i0} - \frac{m_i}{\alpha_i})]^{\theta-1} \\ U_2 &= \int_J^0 (1 - (1 + \exp(\frac{\pi v}{\sqrt{6}(\sum_{j=1}^{n}\int_0^t \sigma_{ij} Y_{js}^{\beta_j}(js)^{-\alpha}djs)}))^{-1}) \\ &\times [v + \frac{m_i}{\alpha_i} + \exp(-\alpha_i t)(Y_{i0} - \frac{m_i}{\alpha_i})]^{\theta-1}dv \\ &+ \int_0^{\infty}(1 + \exp(\frac{\pi v}{\sqrt{6}(\sum_{j=1}^{n}\sum_0^t \sigma_{ij} Y_{ij}^{\beta_j}(js)^{-\alpha}djs)}))^{-1} \\ &\times [v + \frac{m_i}{\alpha_i} + \exp(-\alpha_i t)(Y_{i0} - \frac{m_i}{\alpha_i})]^{\theta-1}dv. \end{aligned}$$

Proof:According to the definition of expected value of fuzzy variables,there are

$$\begin{aligned} f(Y_{i0}, K, m_i, \alpha_i, \beta_j, \sigma_{ij}, r) &= \exp(-rT)E[(Z_T^{\theta} - K)^+] \\ &= \exp(-rT)\int_0^{\infty} Cr\{(Z_T^{\theta} - K)^+ \geq x\}dx \\ &= \exp(-rT)\int_0^{\infty} Cr\{(\frac{m_i}{\alpha_i} + \exp(-\alpha_i t)(Y_{i0} - \frac{m_i}{\alpha_i}) + \sum_{j=1}^{n}\int_0^t \sigma_{ij} Y_{js}^{\beta_j} dC_{js} \geq (K + \frac{1}{x})^{\frac{1}{\theta}}\}dx \\ &= \exp(-rT)\int_0^{\infty} Cr\{\sum_{j=1}^{n}\int_0^t \sigma_{ij} Y_{js}^{\beta_j} dC_{js} \geq (K + \frac{1}{x})^{\frac{1}{\theta}} - \frac{m_i}{\alpha_i} - \exp(-\alpha_i t)(Y_{i0} - \frac{m_i}{\alpha_i})\}dx \end{aligned}$$

make $v = (K + x)^{\frac{1}{\theta}} - \frac{m_i}{\alpha_i} - \exp(-\alpha_i t)(Y_{i0} - \frac{m_i}{\alpha_i})$,

rule

$$x = [v + \frac{m_i}{\alpha_i} + \exp(-\alpha_i t)(Y_{i0} - \frac{m_i}{\alpha_i})]^{\theta} - K,$$

$$dx = \theta[v + \frac{m_i}{\alpha_i} + \exp(-\alpha_i t)(Y_{i0} - \frac{m_i}{\alpha_i})]^{\theta-1}dv, \ x \in (0, +\infty).$$

$$J = K^{\frac{1}{\theta}} - \frac{m_i}{\alpha_i} - \exp(-\alpha_i t)(Y_{i0} - \frac{m_i}{\alpha_i}),$$

make

$$v \in (K^{\frac{1}{\theta}} - \frac{m_i}{\alpha_i} - \exp(-\alpha_i t)(Y_{i0} - \frac{m_i}{\alpha_i}), +\infty) = (J, +\infty),$$

So there is

$$\begin{cases} f = \theta \exp(-rT)(\int_J^{+\infty} (1 + \exp(\frac{\pi v}{\sqrt{6}(\sum_{j=1}^n \sum_0^t \sigma_{ij} Y_{js}^{\beta_j} (js)^{-\alpha} djs)}))^{-1} U_1 dv, J \geq 0 \\ f = \theta \exp(-rT) U_2, J < 0. \end{cases}$$

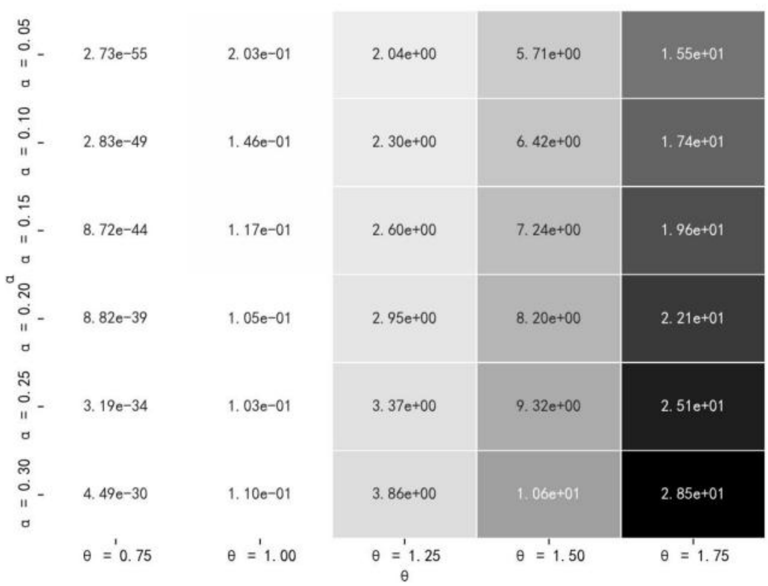
Among them

$$\begin{aligned} U_1 &= [v + \frac{m_i}{\alpha_i} + \exp(-\alpha_i t)(Y_{i0} - \frac{m_i}{\alpha_i})]^{\theta - 1}, \\ U_2 &= \int_J^0 (1 - (1 + \exp(\frac{\pi v}{\sqrt{6}(\sum_{j=1}^n \int_0^t \sigma_{ij} Y_{js}^{\beta_j} (js)^{-\alpha} djs)})))^{-1} \\ &\times [v + \frac{m_i}{\alpha_i} + \exp(-\alpha_i t)(Y_{i0} - \frac{m_i}{\alpha_i})]^{\theta - 1} dv \\ &+ \int_0^{\infty} (1 + \exp(\frac{\pi v}{\sqrt{6}(\sum_{j=1}^n \sum_0^t \sigma_{ij} Y_{ij}^{\beta_j} (js)^{-\alpha} djs)}))^{-1} \\ &\times [v + \frac{m_i}{\alpha_i} + \exp(-\alpha_i t)(Y_{i0} - \frac{m_i}{\alpha_i})]^{\theta - 1} dv. \end{aligned}$$

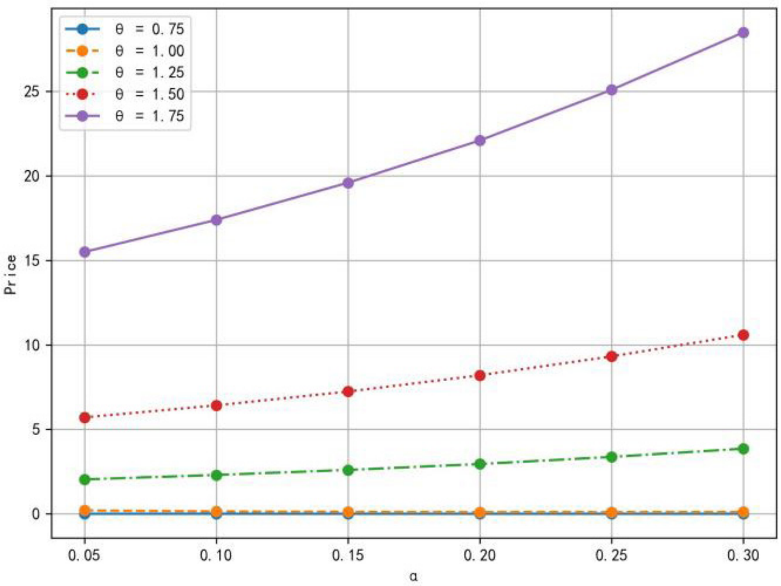
Rowe 2018 Research on Option Pricing Methods in Fuzzy Environments Suppose there is a one-factor European power call option, current stock price, strike price, annual risk-free rate, time to expiration, drift coefficient of the stock, diffusion coefficient, and the rest of the constants, respectively

$$Y_0 = 29, k = 30, r = 0.08, T = 0.25, \sigma = 1, m = 1, a = 1$$

, it follows from the theorem that the option price increases withAnd all the parameters are substituted into the European power call option formula and solved by python, as shown in Figure 3.1 below



and table 3.1



From Table 3.1,Figure 3-1,it can be seen that when θ=0.75 and θ=1.00 When, the range of option price changes is not large, and the color is lighter in Figure 3-1; when, the $\theta \in [0.75,1.75]$ When fixed α, option prices along with θ The colors in Table 3-1 are progressively darker from left to right.

Theorem 3.1 Assume that X_t is the price of the bond, and Y_t is the stock price,and X_t and Y_t Meet the model (2.1),then the formula for the price of a European power put option is

$\begin{cases} f=\theta\exp(-rT)U_1, J\geq 0 \\ f=\theta\exp(-rT)U_2, J<0. \end{cases}$ Among them Among them

$$U_1 = \int_0^J \left(1-\left(1+\exp\left(\frac{\pi v}{\sqrt{6}(\sum_{j=1}^n \int_0^t \sigma_{ij} Y_{js}^{\beta_j}(js)^{-\alpha} djs)}\right)\right)^{-1}\right)$$
$$\times [v+\frac{m_i}{\alpha_i}+\exp(-\alpha_i t)(Y_{i0}-\frac{m_i}{\alpha_i})]^{\theta-1}dv,$$
$$U_2 = \int_J^0 \left(1+\exp\left(\frac{-\pi v}{\sqrt{6}(\sum_{j=1}^n \sum_0^t \sigma_{ij} Y_{js}^{\beta_j}(js)^{-\alpha} djs)}\right)\right)^{-1}$$
$$\times [v+\frac{m_i}{\alpha_i}+\exp(-\alpha_i t)(Y_{i0}-\frac{m_i}{\alpha_i})]^{\theta-1}dv.$$

Proof:According to the definition of expected value of fuzzy variables,there are

$$\begin{aligned} f(Y_{i0},K,m_i,\alpha_i,\beta_j,\sigma_{ij},r) &= \exp(-rT)E[(Z_T^\theta-K)^+] \\ &= \exp(-rT)\int_0^\infty Cr\{(Z_T^\theta-K)^+\geq x\}dx \\ &= \exp(-rT)\int_0^\infty Cr\{Z_T\leq(K-x)^{\frac{1}{\theta}}\}dx \\ &= \exp(-rT)\int_0^\infty Cr\{(\frac{m_i}{\alpha_i}+\exp(-\alpha_i t)(Y_{i0}-\frac{m_i}{\alpha_i})+\sum_{j=1}^n\int_0^t\sigma_{ij}Y_{js}^{\beta j}dC_{js}\leq(K-x)^{\frac{1}{\theta}}\}dx \\ &= \exp(-rT)\int_0^\infty Cr\{\sum_{j=1}^n\int_0^t\sigma_{ij}Y_{js}^{\beta j}dC_{js}\leq(K-x)^{\frac{1}{\theta}}-\frac{m_i}{\alpha_i}-\exp(-\alpha_i t)(Y_{i0}-\frac{m_i}{\alpha_i})\}dx \end{aligned}$$

make $v=(K-x)^{\frac{1}{\theta}}-\frac{m_i}{\alpha_i}-\exp(-\alpha_i t)(Y_{i0}-\frac{m_i}{\alpha_i})$,

Rule
$$x=K-[v+\frac{m_i}{\alpha_i}+\exp(-\alpha_i t)(Y_{i0}-\frac{m_i}{\alpha_i})]^\theta,$$
$$dx=-\theta[v+\frac{m_i}{\alpha_i}+\exp(-\alpha_i t)(Y_{i0}-\frac{m_i}{\alpha_i})]^{\theta-1}dv,\ x\in(0,K).$$

Make
$$J = K^{\frac{1}{\theta}} - \frac{m_i}{\alpha_i} - \exp(-\alpha_i t)(Y_{i0} - \frac{m_i}{\alpha_i}),$$
$$v \in (K^{\frac{1}{\theta}} - \frac{m_i}{\alpha_i} - \exp(-\alpha_i t)(Y_{i0} - \frac{m_i}{\alpha_i}),0) = (J,0).$$

So there is

$$\begin{cases} f = \theta \exp(-rT)U_1, J \geq 0 \\ f = \theta \exp(-rT)U_2, J < 0 \end{cases}$$

Among them
$$U_1 = \int_0^J (1 - (1 + \exp(\frac{\pi v}{\sqrt{6}(\sum_{j=1}^n \int_0^t \sigma_{ij} Y_{js}^{\beta_j}(js)^{-\alpha} djs)})))^{-1}$$
$$\times [v + \frac{m_i}{\alpha_i} + \exp(-\alpha_i t)(Y_{i0} - \frac{m_i}{\alpha_i})]^{\theta-1} dv,$$
$$U_2 = \int_J^0 (1 + \exp(\frac{-\pi v}{\sqrt{6}(\sum_{j=1}^n \sum_0^t \sigma_{ij} Y_{js}^{\beta_j}(js)^{-\alpha} djs)}))^{-1}$$
$$\times [v + \frac{m_i}{\alpha_i} + \exp(-\alpha_i t)(Y_{i0} - \frac{m_i}{\alpha_i})]^{\theta-1} dv.$$

Theorem proved.

the price of zero-coupon bonds P^T The following fuzzy term structure equation is satisfied

$$\begin{cases} [\epsilon(t,r_t) - \varsigma_t \eta(t,r_t)] \frac{\partial P^T}{\partial r_t} + \frac{\partial P^T}{\partial t} - r_t P^T = 0 \\ P^T(T, r_T) = 1, \end{cases}$$

Among them r_t is the fuzzy interest rate that satisfies model (3.1), the ϵ and η It's about t and r_t function, the ς_t is the market price of risk.

If the price of a zero-coupon bond satisfies that

$$P^T(t,r_t) = \exp(M(t,T) - N(t,T)r_t)$$

Among them M，N about, respectively t,T of a definite real function, the r_t If it is a fuzzy interest rate satisfying the model (3.1), then the zero coupon bond price has a fuzzy affine term structure (FATS)

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