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Based GeoGebra Software to Explore the Fixed Value Problem in Conic Curves

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Abstract: With the dynamic demonstration function of GeoGebra software, a class of fixed value problems in conic curves is dynamically explored, students are inspired to find out the nature of "unchanging" from "changing", and they are guided to observe, generalize,conjecture and prove the generalized mathematical conclusions, and analogies from parabola to ellipse and hyperbola.

Keywords: GeoGebra Software; Fixed Value Problems; Dynamic Exploration

Introduction

GeoGebra is a powerful dynamic demonstration software, precisely provides an ideal platform for students' inquiry activities, which helps students get rid of complicated arithmetic operations in order to save more time and energy for exploring and discovering mathematical laws.

1. Presentation of the topic

It is known that the parabola $C: y^2 = 2px\ (p > 0)$ has its focus at F and a line *l* through N (2,0) intersects C at two points A and B. When *l* perpendicular to the x-axis, the $|AB| = 4$.

(1) Find the equation of C.

(2) If there exists a point P on the x-axis,let the slope of the line PA and the line PB are k_{PA} and k_{PB} respectively, the $k_{PA} + k_{PB} = 0$ constantly, find the coordinates of point P.

The solution: (1) When *l* is perpendicular to the x-axis, we have from the question $|AB| = 4\sqrt{p}$ and thus $4\sqrt{p} = 4$ and solve for $p = 1$, so the equation of the parabola C is $y^2 = 2x$.

(2) From the question, it is clear that the slope of the *l* is not 0.Let the line $l: x = my + 2$,the $P(x_0, 0)$, $A(x_1, y_1)$, $B(x_2, y_2)$. The association of $\begin{cases} x = my + 2 \\ y^2 = 2x \end{cases}$ and elimination of x organizes to give $y^2 - 2my - 4 = 0$. We have that $\Delta = 4m^2 - 4 \times 1 \times (-4) = 4(m^2 + 4) > 0$,$y_1 + y_2 = 2m$,$y_1 \cdot y_2 = -4$, so

$$\begin{aligned} k_{PA} + k_{PB} &= \frac{y_1}{x_1 - x_0} + \frac{y_2}{x_2 - x_0} = \frac{y_1}{my_1 + 2 - x_0} + \frac{y_2}{my_2 + 2 - x_0} \\ &= \frac{2my_1y_2 + (2 - x_0)(y_1 + y_2)}{(my_1 + 2 - x_0)(my_2 + 2 - x_0)} = \frac{-4m - 2mx_0}{(my_1 + 2 - x_0)(my_2 + 2 - x_0)} = 0 \end{aligned}$$

i.e.$-4m - 2mx_0 = 0$ and thus solves for $x_0 = -2$.

In summary, when $k_{PA} + k_{PB} = 0$ constantly, the coordinates of point P are (-2,0).

Observing point P (-2,0) and point N (2,0), you can find that they are symmetric about the y-axis, which leads to the conjecture that if $k_{PA} + k_{PB} = 0$ constantly, then the transverse coordinates of point P and point N are opposite to each other. So with the help of GeoGebra software to carry out dynamic investigation, to verify the conjecture.

1.1 Dynamic investigation of parabolas

It is known that the parabola $C: y^2 = 2px\ (p > 0)$ and a line through the point $N(x_N, 0)$ $(x_N > 0)$ intersects C at points A and B. There exists a point P on the x-axis, and the slopes of the line PA and the line PB are k_{PA}, k_{PB} respectively. If $k_{PA} + k_{PB} = 0$ constantly, ask: Are the coordinates of the point $P(-x_N, 0)$.

Dynamic Exploration of Results:

(1) As shown in Figure 1, under the condition of $k_{PA} + k_{PB} = 0$, change the *p* and the k_{PA}, the coordinates of point N(3,0) remain un-

changed, at this time the coordinates of point P is always (-3,0).

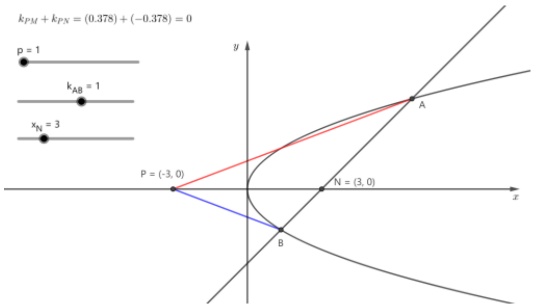

Figure 1

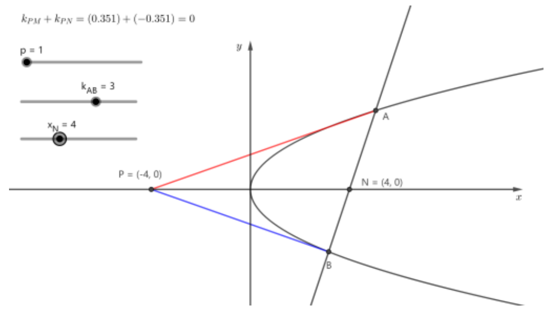

Figure 1

(2) As in Figure 2, under the condition of $k_{PA} + k_{PB} = 0$, take the $p = 1$, the $k_{AB} = 3$, change the x_N, at this time the coordinates of the point P are changed and always $(-x_N, 0)$.

1.2 Conclusion of the parabolic generalization

It is known that the parabola $C: y^2 = 2px\ (p > 0)$ and a line through the point $N(x_N, 0)\ (x_N > 0)$ intersects C at points A and B. There exists a point P on the x-axis, and the slopes of the line PA and the line PB are k_{PA}, k_{PB} respectively. If $k_{PA} + k_{PB} = 0$ constantly, then the coordinates of point P are $(-x_N, 0)$.

The proof: from the condition, the line *l* has a slope that is not 0. Let $l: x = my + x_N$, $P(x_P, 0)$, $A(x_1, y_1)$, $B(x_2, y_2)$. The association $\begin{cases} x = my + x_N \\ y^2 = 2px \end{cases}$ and elimination of x organizes to give $y^2 - 2pmy - 2px_N = 0$.We have that $\Delta = 4p^2m^2 - 4 \times 1 \times (-2px_N) = 4p(m^2 + 2x_N) > 0$, $y_1 + y_2 = 2pm$, $y_1 \cdot y_2 = -2px_N$, so

$$\begin{aligned} k_{PA} + k_{PB} &= \frac{y_1}{x_1 - x_P} + \frac{y_2}{x_2 - x_P} = \frac{y_1}{my_1 + x_N - x_P} + \frac{y_2}{my_2 + x_N - x_P} \\ &= \frac{2my_1y_2 + (x_N - x_P)(y_1 + y_2)}{(my_1 + x_N - x_P)(my_2 + x_N - x_P)} = \frac{-2pmx_N - 2pmx_P}{(my_1 + x_N - x_P)(my_2 + x_N - x_P)} \\ &= 0 \end{aligned}$$

i.e. $-2pmx_N - 2pmx_P = 0$ and thus solves for $x_P = -x_N$.

In summary, when $k_{PA} + k_{PB} = 0$ constantly, the coordinates of point P are $(-x_N, 0)$.

2. Variant Exploration

Are there similar properties in ellipses and hyperbolas? Use GeoGebra software to explore variations.

2.1 Dynamically Exploring Ellipses

It is known that the ellipse $C: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ (a > 0, b > 0)$ and the straight line *l* (slope not 0) through the point $N(x_N, 0)$ intersects C at points A and B. There exists a point P on the x-axis, and the slopes of the line PA and the line PB are k_{PA}, k_{PB} respectively. If $k_{PA} + k_{PB} = 0$ constantly, ask: How do the coordinates of point P change.

Dynamic Inquiry Results:

(1) As in Figure 3, if the $k_{PA} + k_{PB} = 0$, take $a = 4$ and the coordinates of the point N are (2,0), change the value of *b* and the slope of the line *l*, when the coordinates of point P are always (8,0).

(2) As in Figure 4, if $k_{PA} + k_{PB} = 0$, take $b = 5$, change the x_N and *a* values, the transverse coordinate of point P changes accordingly.

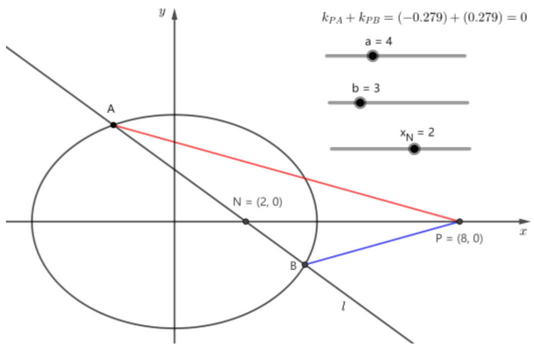

Figure 3

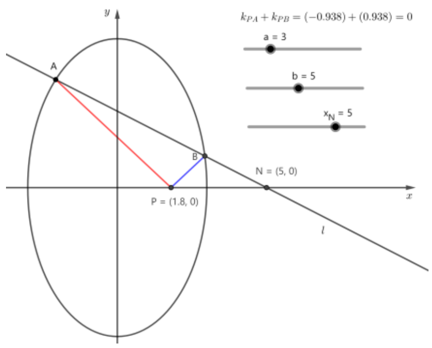

Figure 4

Let the horizontal coordinates of the point P be x_P, use the controlled variable method to investigate the relationship between x_N, *a* and x_P.

Take $a = 5$, change x_N value while recording the x_P value to get Table 1. Take $x_N = 3$, change the value of *a* and record the value of x_P to obtain Table 2. Observing Table 1 and Table 2, it is easy to find that x_N,x_P and *a* are satisfied $x_N \cdot x_P = a^2$.

Table 1 Changing values

x_N	x_P	a	a^2
-10	-5/2	5	25
-8	-25/8	5	25
-6	-25/6	5	25
-4	-25/4	5	25
-2	-25/2	5	25
2	25/2	5	25
4	25/4	5	25
6	25/6	5	25
8	25/8	5	25
10	5/2	5	25

Table 2 Changing values

x_N	x_P	a	a^2
3	1/3	1	1
3	4/3	2	4
3	3	3	9
3	16/3	4	16
3	25/3	5	25
3	12	6	36
3	49/3	7	49
3	64/3	8	64
3	27	9	81
3	100/3	10	100

2.2 Conclusion of elliptic generalization

It is known that the ellipse $C\colon \frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>0, b>0)$ and the point $N(x_N, 0)$ on the x-axis, a line *l* (slope not 0) through point N intersects C at points A and B. On the x-axis there exist points $P(x_P, 0)$, the slopes of the line PA and the line PB are k_{PA}, k_{PB} respectively. If $k_{PA}+k_{PB}=0$ constantly, then x_N, x_P and *a* are satisfied $x_N \cdot x_P = a^2$.

The proof: Let the line $l\colon x = my + x_N$, $A(x_1, y_1)$, $B(x_2, y_2)$.

The association $\begin{cases} x = my + x_N \\ \frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \end{cases}$ and elimination of x organizes to give $(b^2m^2+a^2)y^2 + 2mb^2x_Ny + b^2(x_N^2-a^2)=0$. We have that $\Delta = 4m^2b^4x_N^2 - 4b^2(b^2m^2+a^2)(x_N^2-a^2) > 0$, $y_1+y_2 = -\frac{2mb^2x_N}{b^2m^2+a^2}$,, $y_1 \cdot y_2 = \frac{b^2(x_N^2-a^2)}{b^2m^2+a^2}$, so

$$k_{PA}+k_{PB} = \frac{y_1}{x_1-x_P}+\frac{y_2}{x_2-x_P} = \frac{y_1}{my_1+x_N-x_P}+\frac{y_2}{my_2+x_N-x_P}$$

$$= \frac{2my_1y_2+(x_N-x_P)(y_1+y_2)}{(my_1+x_N-x_P)(my_2+x_N-x_P)} = 0$$

Namely $2my_1y_2+(x_N-x_P)(y_1+y_2) = \frac{2mb^2(x_N^2-a^2)}{b^2m^2+a^2} - \frac{(x_N-x_P)2mb^2x_N}{b^2m^2+a^2} = 0$,

i.e. $2mb^2(x_N^2-a^2) = (x_N-x_P)2mb^2x_N$ and thus solves for $x_N \cdot x_P = a^2$.

Analogous to ellipses, does the conclusion still hold in hyperbolas? Verify the conjecture using GeoGebra software.

2.3 Dynamic exploration of hyperbola

A known hyperbola $C:\frac{x^2}{a^2}-\frac{y^2}{b^2}=1\ (a>0,b>0)$ and a line *l* (slope not 0) through the point $N(x_N,0)$ intersects C at points A and B. On the x-axis there exist points $P(x_P,0)$, the slopes of the line PA and the line PB are k_{PA}, k_{PB} respectively. If $k_{PA}+k_{PB}=0$ constantly, ask: $x_N\cdot x_P=\ a^2$ does it hold?

Dynamic exploration results: as shown in Figure 5, changing the x_N and *a* values while recording the x_P values, it is easy to find that $x_N\cdot x_P=\ a^2$ still holds.

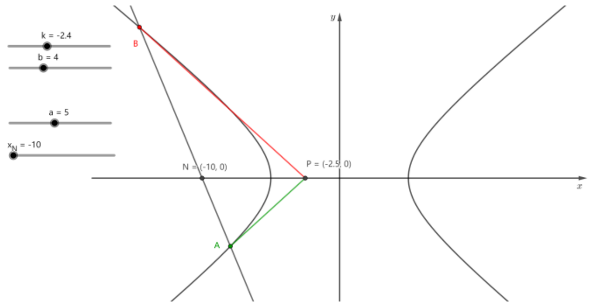

Figure 5

Limiting space, the proof of this conclusion in hyperbolas is left to the interested reader.

As the mathematician Polya said, good problems are somewhat similar to certain mushrooms, they all grow in heaps, after finding one, you should look around, there are probably several nearby[1] . Teachers in the teaching process, should be good at guiding students to a good problem to carry out variations of the investigation, so as to achieve the effect of the point to bring about the surface, less than more than the effect.

3. Conclusion

In this paper,we use GeoGebra software to conduct a dynamic investigation of the problem of slope summed to zero in conic curves, thus conjecturing and arguing generalized mathematical conclusions.Through demonstration, observation, discovery, conjecture, proof, and analogy from parabola to ellipse and hyperbola, students are directly involved in the whole process of inquiry and construction of conclusions, which is conducive to the development of the students' ability of independent inquiry, analogy and reasoning, and enables students to master the method of researching the problem from the particular to the general[2].

In the era of "Internet +", the wide application of information technology is having a profound impact on mathematics education[3]. With the help of GeoGebra software, it is easy to explore mathematical problems in depth intuitively and from multiple perspectives, giving full play to students' initiative, enthusiasm and creativity, effectively enhancing students' interest in learning, expanding students' mathematical thinking, and providing an effective way of exploring and understanding new knowledge .

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