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A λ-c- genetic Algorithm for Integrals with Fuzzy Measure

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ABSTRACT

In this paper, beginning we define a fuzzy Parametric measure, with having values of a weight function on n points. Afterwards, we obtain one equation by use from properties of fuzzy measure that with solving equation, we define parameters of fuzzy measure. For solving equation, we design a genetic algorithm and hereby we provide the facility of solving integrals.

***Keywords*:** Fuzzy Measure; Fuzzy Integral; Genetic Algorithm

1. Introduction

In some cases, from a weight function is used for ranking, comparison and selection[3,6,9,12,14,18,20]. Ratings, comparison and selection can be done in many different ways[12,14,16–20]. Since data is fuzzy in many systems that in them should be done process ranking, comparison and selection, therefore it is necessary that we design types of methods which with implementation them on fuzzy data, be done a good selection [3,6,9]. One of these methods is construct a fuzzy measure and fuzzy integral. In this paper, beginning we define a parametric-fuzzy measure, afterwards we solve a type of fuzzy integral by use from above fuzzy measure[1,2,5,10,11,13]. We need to amount of parameter of fuzzy measure for solving above fuzzy integrals. For find the amount of above parameter, we obtain one nonlinear equation with two unknown by use from properties of fuzzy measure. At the end, we design a genetic algorithm for solving the above nonlinear equation and by implementation it, obtain suitable approximation solutions.

2. Basic Concepts

This section introduces of the required methodologies.

2.1 Fuzzy measure

Sugeno defined fuzzy measure[1,8,11,19]. This concept is widely in various fields, especially in the field of collecting and summarizing information.

Definition 1. [7]Assume that $X=\{x_1,x_2,...,x_n\}$ and $P(X)$ is the power set of X, the set function $g:P(X)\longrightarrow[0,1]$ is called a fuzzy measure, which is non-additive and preserves the following properties

1. $g(\emptyset)=0$,
2. $g(X)=1$,
3. if $A,B\in P(X)$ and $A\subset B$ then $g(A)\le g(B)$ (monotonicity),
4. in $P(X)$, if $A_1\subset A_2\subset A_3\subset A_4\subset ...$ and $\bigcup_{i=1}^{\infty}A_i\in P(X)$ then $\lim_{i\to\infty} g(A_i)=g(\bigcup_{i=1}^{\infty}A_i)$ (continuity from below),
5. in $P(X)$, if $A_1\supset A_2\supset A_3\supset A_4\supset ...$ and $\bigcap_{i=1}^{\infty}A_i\in P(X)$, then $\lim_{i\to\infty} g(A_i)=g(\bigcap_{i=1}^{\infty}A_i)$ (continuity from below).

Now we can give the definition of fuzzy parametric-measure as following.

Defination 2. [7]The set function $g_\lambda:P(X)\longrightarrow[0,1]$ is called a fuzzy λ -measure if and only if there be a parameter λ such that $\lambda\in\left(\frac{-1}{\sup g},\infty\right)$ which $\sup g=\sup_{A\in P(X)} g(A)$ and

$$g_\lambda(A\cup B)=g_\lambda(A)+g_\lambda(B)+\lambda g_\lambda(A)g_\lambda(B) \quad (1)$$

that $A,B\in P(X)$, $A\cap B=\emptyset$. This measure is λ -additive.

2.2 Fuzzy Integrals

Assume that $X=\{x_1,x_2,...,x_n\}$ and g is a fuzzy measure on X. Integral of a function $h:P(X)\longrightarrow[0,1]$ with respect to g is defined by $\int f\,dg:=\sum_{i=1}^{n}(h(x_i)-h(x_{i+1}))g(H_i)$ where $0\le h(x_n)\le h(x_{n-1})\le ...\le h(x_1)\le 1$, $h(x_{n+1})=0$ and $H_i=\{x_1,x_2,...,x_i\}$.

In literature, the fuzzy integral defined by $\int h\,dg$ is called "Choquet integral". The basic concept can be illustrated in **Figure 1**[13].

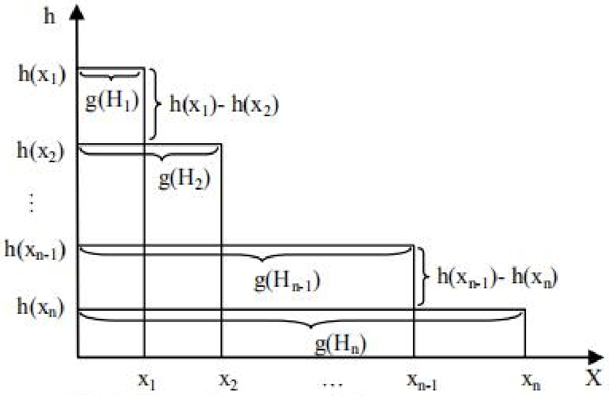

Figure 1; The basic concept of Choquet integral.

Lemma 1. Suppose $X = \{x_1, x_2, \ldots, x_n\}$, therefore for any fuzzy measure g on X and for any function $h: X \longrightarrow [0,1]$, it is verified that $\int h\,dg = \sum_{i=1}^{n} h_i\,dg_i$.

Proof. According to the definition of the Choquet integral, we have

$$\int f\,dg \coloneqq \sum_{i=1}^{n} (h(x_i) - h(x_{i+1}))g(H_i).$$

Now since $h(x_{n+1}) = 0$, $g_0 = g(\emptyset) = 0$ and $g(X) = g(H_n) = 1$, we have

$$\sum_{i=1}^{n} (h_i - h_{i+1})g_i = \sum_{i=1}^{n} h_i g_i - \sum_{i=1}^{n} h_{i+1} g_i$$

$$= \sum_{i=1}^{n} h_i g_i - \sum_{i=2}^{n+1} h_i g_{i-1}$$

$$= h_1 g_1 + \sum_{i=2}^{n} h_i g_i - \sum_{i=2}^{n} h_i g_{i-1} - h_{n+1} g_1$$

$$= h_1 g_1 - h_1 g_0 + \sum_{i=2}^{n} h_i (g_i - g_{i-1}) = \sum_{i=1}^{n} h_i (g_i - g_{i-1})$$

$$= \sum_{i=1}^{n} h_i\,dg_i .\square$$

3. Calculating the fuzzy integral

We obtain parameter of fuzzy measure for this purpose, for any $i = 1,2,\ldots,n$ we set

$$g_i = g(\{x_i\}) = cw_i \qquad (2)$$

which c is a constant real number and w_i is amount of a weight function on x_i[2,5,13].

Now according to (1), (2) and $g(X) = 1$, we have

$$c\sum_{\alpha_1=1}^{n} w_{\alpha_1} + c^2\lambda \sum_{\substack{\alpha_1=1,\alpha_2=1 \\ \alpha_1 \neq \alpha_2,\, \alpha_1 < \alpha_2}}^{n} w_{\alpha_1} w_{\alpha_2} + c^3\lambda^2 \sum_{\substack{\alpha_1=1,\alpha_2=1,\alpha_3=1 \\ \alpha_1 \neq \alpha_2 \neq \alpha_3,\, \alpha_1 < \alpha_2 < \alpha_3}}^{n} w_{\alpha_1} w_{\alpha_2} w_{\alpha_3}$$

$$+ \ldots + c^n\lambda^{n-1} \prod_{\alpha_i=1}^{n} w_{\alpha_i} - 1 = 0 \qquad (3)$$

Equality (3) is a nonlinear equation with two unknown λ, c and can be re-written as following

$$F(c,\lambda) = \sum_{i=1}^{n} a_i c^i \lambda^{i-1} - 1 = 0 \qquad (4)$$

Which

$a_1 = \sum_{\alpha_1=1}^{n} w_{\alpha_1}$, $a_2 = \sum^{n}_{\substack{\alpha_1=1,\alpha_2=1 \\ \alpha_1 \neq \alpha_2,\, \alpha_1 < \alpha_2}} w_{\alpha_1} w_{\alpha_2}$, $a_3 = \sum^{n}_{\substack{\alpha_1=1,\alpha_2=1,\alpha_3=1 \\ \alpha_1 \neq \alpha_2 \neq \alpha_3,\, \alpha_1 < \alpha_2 < \alpha_3}} w_{\alpha_1} w_{\alpha_2} w_{\alpha_3}$, … , $a_n = \prod_{\alpha_i=1}^{n} w_{\alpha_i}$.

Now we design a λ -c-genetic algorithm for solving equation (4). By solving equation (4), we obtain approximation amount of c, λ with the accuracy of pre-defined.

3.1 The λ -c-genetic algorithm

```
begin
Best_c = 0;
Best_lamda = 0;
alfa = Producealfa(n,w);
for i = 1 : k
        c(i) = i/k ;
end
t = k/2 ;
For i = 1 : t
        lamda(i) = -i/t;
        lamda(i + t) = i/t;
        lamda(i + 2 * t) = i^2 + 1;
end
Term = 0;
nk = 3*k;
Gen = 1;
while Term ~= 1 && Gen <= 100
for i = 1 : k
        X = rand i(k,1,2,);
        l = X(1);
        f = X(2);
       c(k + i) = (c(l)+c(f))/2;
       c(2 * k + i) = c(l) * c(f);
       lamda(k + i) = (lamda(l)+lamda(f))/2;
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        lamda(2 * k + i) = abs(c(l) * c(f));
    End
    Fit = Fitness_FCL(n,nk,alfa,c,lamda);
    for i = 1 : nk
        for j = 1 : nk
            if abs(Fit(i,j)) <= (10^(-Ep))
    Term = 1;
    Best_c = c(i);
    Best_lamda = lamda(j);
            end
        end
    end
    i = nk;
    while i > k
            [SX,SY] = size(Fit);
            Max = Max(Max(abs(Fit) ));
            [X,Y] = find(abs(Fit) = Max);
            X = X(1);
            Y = Y(1);
            c(X) = c(SX);
            lamda(Y) = lamda(SY);
            Fit(X,:) = Fit(SX,:);
            Fit(:,Y) = Fit(:,SY);
            Fit = Fit(1 : SX-1,1 : SY-1);
    end
    Gen = Gen + 1;
    function Fitness = Fitness_FCL(n,k,alfa,lamda)
    for X = 1 : k
        for Y = 1 : k
            Fitness(X,Y) = 0;
            for i = 1 : n
    Fitness(X,Y)=Fitness(X,Y)+
alfa*(c(x)^i)*(lamda(Y)^(i-1));
            end
            Fitness(X,Y) = Fitness(X,Y) – 1;
        end
    end
    function alfa = Producealfa(n,w)
    alfa(1 : n) = 0;
    for X = 1 : n
        start_index(1 : X) = 1 : X;
        stop_index(1 : X) = n - X + 1 : n;
        while start_index(1) <= stop_index(1)
    c = 0;
    P = 1;
    for z = 1 : X;
        P = P*W(start_index(z));
    end
    alfa(X) = alfa(X) + P;
    z = 1;
    while z <= X && c = 0
            if start_index(z) = stop_index(z)
    if z = 1 && isempty(find(start_index = stop_index) = 0))
        start_index(z) = start_index(z) + 1;
        c = 1;
        else
        start_index(z-1) = start_index(z-1) + 1;
        start_index(z:x) = start_index(z-1) + 1 : start_index(z-1) + X – z + 1;
    c = 1;
      end
            end
            z = z + 1;
        end
        z = z -1;
        if z = X && start_index(z) < stop_index(z) && c=0
            start_index(z) = start_index(z) + 1;
        end
    end
    end
```

3.2 Example

Consider $X = \{1,2,3\}$, $g_\lambda : P(X) \longrightarrow [0,1]$, $h : X \longrightarrow [0,1]$, with $h(x) = e^{-x}$ and $w_1 = w_2 = w_3 = \frac{1}{3}$. So, we have $w_1 + w_2 + w_3 = 1$, $w_1w_2 + w_1w_3 + w_2w_3 = \frac{1}{3}$, $w_1w_2w_3 = \frac{1}{27}$, $c + \frac{1}{3}c^2\lambda + \frac{1}{27}c^3\lambda^2 - 1 = 0$.

By implementation λ-c-genetic algorithm, we obtain that $c = \frac{1}{2}$ and $\lambda = 5$. Thus we have $g_1 = \frac{1}{6}$, $g_2 = \frac{1}{6}$, $g_3 = \frac{1}{6}$, $g\{x_1,x_2\} = \frac{17}{36}$ and $g\{x_1,x_2,x_3\} = \frac{223}{216}$ which imply that

$$\int h\,dg := \sum_{i=1}^{3} (h(x_i) - h(x_{i+1}))g(H_i) = 0.128.\square$$

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