Case Analysis of Teaching Design for High School Function Concept Courses under the Background of the New Curriculum——Taking "Monotonicity of Functions" as an Example

Xueni Sun, Xining Li
Ningxia University, Yinchuan 750021, China.

Abstract: Functions are the core of algebra, and the teaching of function concepts is also the main task of high school mathematics Students' learning of functions and their concepts shifts from understanding specific quantitative relationships to understanding abstract quantitative relationships The monotonicity of functions, as the property of the first function that students learn in high school, lays a certain foundation for learning function related knowledge in the future.
***Keywords**:* Function Concept; Instructional Design; Monotonicity of Function

Introduction

The learning of mathematical concepts is the essence of mathematical subject cognition, which lies in grasping what concepts explain, what their upper and lower levels of knowledge are Concepts progress layer by layer, constantly deriving new concepts through analogy and abstraction The study of concepts runs through the entire compulsory curriculum, demonstrating its extraordinary status The "Curriculum Standards for General High School Mathematics (2017 Edition, 2020 Revision)" points out that functions are the most basic concept in modern mathematics, the most basic mathematical language and tool for expressing variable relationships and laws in the objective world, and play an important role in solving practical problems. Functions are the main thread that runs through high school mathematics curriculum. Function monotonicity, as the first property that high school students encounter, is crucial for learning function knowledge well. However, the biggest difficulty of function monotonicity lies in abstracting the concept of function monotonicity using mathematical symbolic language Therefore, in the context of the new curriculum, this article takes the monotonicity of functions as an example, starting from students' existing cognition and utilizing visual perception of images, to explore how to enable students to abstractly express the concept of functions, thereby better improving students' mathematical thinking and abilities.

1. Analysis of learning content

This lesson is the first lesson of the "Basic Properties of Functions" in Chapter 3, Section 2, Volume 1 of the 2019 People's Education A Edition Compulsory Textbook for Ordinary High School Curriculum Standards. The main content is the formation and application of the concept of monotonicity of functions.

This lesson is based on students' understanding of the basic concepts of functions and studying the laws of function changes Function monotonicity is a property that students can easily discover from the functions they have already learned Mastering the relevant knowledge of monotonicity of functions not only lays the foundation for studying the extremum and parity of functions, but also lays the foundation for learning power functions, exponential functions, and logarithmic functions in the future At the same time, in the process of understanding the definition of function monotonicity, use function images to intuitively perceive the monotonicity of functions, and learn to use symbolic language to describe function monotonicity, improving students' abstract generalization ability; Improve students' logical reasoning ability in the process of using the definition method to prove the monotonicity of functions.

Keynote: Master the Concept of Monotonicity of functions and use definition methods to prove the monotonicity of simple functions.

2. The analysis of the students

Through the learning of function knowledge in junior high school, students have gained a certain foundation in function concepts, and have a visual perception and understanding of the changes in the function images of proportional functions, first-order functions, inverse proportional functions, and quadratic functions.

Students can describe the changes in functions in natural language and have the ability to preliminarily analyze function images However, students lack the ability to combine numbers and shapes and abstract generalization from special to general.

Difficulty: Can use mathematical language to describe the concept of dynamic monotonicity of functions.

3. Teaching objectives

Understand the concept of function monotonicity, be able to use mathematical symbolic language to describe the concept of function monotonicity, and be able to prove the monotonicity of functions using definitions.

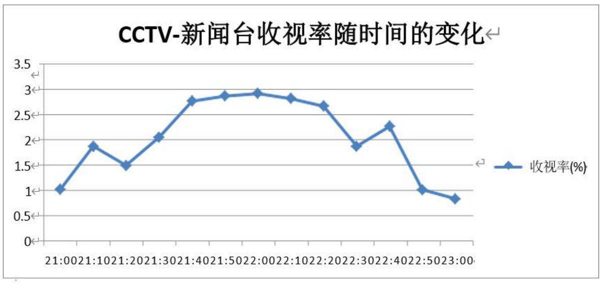
Through the study of graphs and tables, the concept of monotonicity in functions is formed through the integration of numbers and shapes, as well as the ability to generalize from special to general.

Experience the inherent beauty of mathematics through the combination of numbers and shapes, and perceive the charm of mathematical rigor through the transformation of graphic language, natural language, and mathematical symbolic language.

4. Teaching process

4.1 Scenario Introduction

4.1.1 Did everyone watch the opening ceremony on TV? Presumably, the ratings of the program on that day should not be low. Please take a look at the large screen:



Q1: What trends can you observe？

4.1.2 Draw the following functions and observe the pattern of changes in their images

$$① f(x)=x \qquad ② f(x)=-x+2 \qquad ③ f(x)=\frac{1}{x} \qquad ④ f(x)=x^2$$

Design Intention: Through function images, students can intuitively experience the trend of function values' increasing 'or' decreasing 'within the defined domain Students can observe that the changing trends of images for different functions are different; The changing trend of the same function varies across different intervals The change pattern of the function image is a reflection of the properties of the function, which leads to the topic of this lesson.

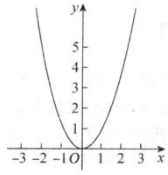
4.2 New Course Teaching

4.2.1 Discuss and study $f(x)=x^2$ images and $f(x)=x$ images

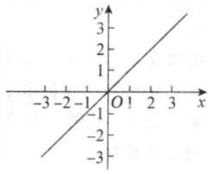
Teacher and student activities: The teacher guides students to draw images of $f(x)=x^2$ and $f(x)=x$,complete a table

(PPT) for the corresponding values of x and $f(x)$, and explore what changes are reflected in the two function images.

x	$\cdots$	-4	-3	-2	-1	0	1	2	3	4	$\cdots$
$f(x)=x^2$	$\cdots$	16	9	4	1	0	1	4	9	16	$\cdots$



x	$\cdots$	-3	-2	-1	0	1	2	3	$\cdots$
$f(x)=x$	$\cdots$	-3	-2	-1	0	1	2	3	$\cdots$



Design Intention: From form to number, from sensibility to rationality, students can preliminarily perceive that the points on the function image are infinite, and can then induce the arbitrariness of guessing the monotonicity of the function within the interval range.

4.2.2 Constructing the Definition of Increasing Function

Teacher-student activities: Guide students to observe and discuss the function image of the right half of the quadratic function $y=x^2$, identify the two points on the image and their corresponding coordinates, and think about why they need to find the two points.

Q2: How to accurately describe the monotonicity of functions using mathematical symbolic language?

1）Symbolization of 'change'

The function value y increases with the increase of x - when $x_1<x_2$, $f(x_1)<f(x_2)$.

2）Symbolization of 'intervals'

Monotonicity describes the local properties of a function. For example, studying the image of the right half of a quadratic function actually limits $x>0$, so it is inseparable from intervals.

Therefore, let the definition domain of function $f(x)$ be D, interval $I\in D$, and emphasize $x_1,x_2\in I$.

For the two values x_1,x_2 in interval I, when $x_1<x_2$, $f(x_1)<f(x_2)$ - $x_1,x_2\in I$, and when $x_1<x_2$, $f(x_1)<f(x_2)$.

3）Symbolization of 'Any'

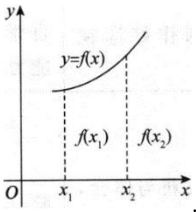
Teacher-student activities: Teacher guides students to list the points on function $y=x^2(x>0)$, take $x=1,2,3,4\text{L}$, and corresponding $y=1,4,9,16\text{L}$.

Q3: Can the monotonicity of a function be demonstrated through individual numerical values? Can individual points indicate that the function value y increases with the increase of x?

The finite or infinite independent variables on interval I satisfy $x_1<x_2$, and $f(x_1)<f(x_2)$ cannot reflect the essence of "the function value y increases with the increase of x Therefore, it is necessary to emphasize the arbitrariness of x_1,x_2 in order to accurately express the feature of monotonic increase.

Any $x_1, x_2 \in I$, when $x_1 < x_2$, has $f(x_1) < f(x_2)$ - $\forall x_1, x_2 \in I$, and when $x_1 < x_2$, has $f(x_1) < f(x_2)$.

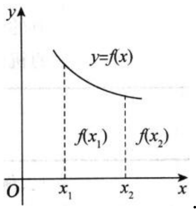
Definition: Generally, let the definition domain of function $f(x)$ be D, and the interval $I \subseteq D$ .If $\forall x_1, x_2 \in I$, then $x_1 < x_2$, all have $f(x_1) < f(x_2)$, then function $f(x)$ is said to monotonically increase on the interval

I 

Design Intention: The solution to Problem 2 is a three-step process that allows students to fully participate and use strict mathematical symbolic language to define the entire process of monotonicity of functions. Students can experience firsthand how mathematical concepts go from text to symbols, from intuition to abstraction, and fully understand the construction principles of symbolization of mathematical concepts At the same time, make students realize that the essence of the concept of monotonicity in functions lies in the impossibility of exhausting independent variables, and guide students to take any two independent variables x_1, x_2 within a given interval.

4.3 Students independently explore the definition of subtraction function

Definition: Generally, let the definition domain of function $f(x)$ be D, and the interval $I \subseteq D$ .If $\forall x_1, x_2 \in I$, then $x_1 < x_2$, all have $f(x_1) > f(x_2)$, then function $f(x)$ is said to monotonically decrease on the interval

I 

Design Intention: By using analogical methods, students can independently derive the definition of a decreasing function. On the one hand, the definition of an increasing function has been strengthened, and the differences between the definitions of increasing and decreasing functions have been identified to experience the expansion and improvement of mathematical concepts; On the other hand, it can reflect the subjectivity of students and emphasize their independent thinking and meaning construction.

4.3.1 Consolidation exercises

1. Research on Function Monotonicity and Monotonic Interval Based on Definition.

1） $y = |x|$　　　　2） $y = \dfrac{1}{x}$

2. Prove monotonically increasing function $y = x + \dfrac{1}{x}$ on interval $(1, +\infty)$ according to the definition.

3. Study the monotonicity of function $y = x^2$ according to its definition.

Design Intention: The three questions go from shallow to deep, progressing layer by layer The first question can deepen students' understanding of the concept of monotonicity of functions; The complete answer process for question 2 was jointly completed by teachers and students, and the teacher played a good demonstration role; The third question was independently discussed and completed by students, reflecting the people-oriented student view. This question was also classified and discussed, which can improve students' mathematical logical thinking In addition, after students have mastered the use of mathematical symbolic language to describe the monotonicity of a function, the third question immediately proves the monotonicity of the function image using the definition method. This corresponds to the natural language description of the function change pattern of $y = x^2$ before and after.

4.3.2 Brief summary

1） Talk about the knowledge you gained in this class?

2） What methods and processes did you use to learn today's knowledge?

3） Are there any unresolved questions?

Design Intention: Firstly, students summarize this lesson, construct the knowledge structure independently, and then the teacher summarizes it to make the knowledge more complete and systematic; At the same time, students raise questions, and the teacher triggers students to think and expand, laying the groundwork for the next class.

4.3.3 Assignment

Exercise 2, 3

What other methods can be used besides observing images and defining functions to determine their monotonicity?

Conclusion: The teaching of mathematical concept courses is an important component of high school mathematics teaching. Teachers should pay special attention to the use of teaching methods and innovative use of textbooks, and design the teaching of concept courses based on students' cognitive patterns, in order to achieve the goal of improving students' mathematical thinking.

References

[1] Lu MT. Research on Teaching the Concept and Properties of High School Mathematical Functions[D]. Jiangsu: Yangzhou University, 2021:19-20.

[2] MoE. General High School Mathematics Curriculum Standards (2017 Edition)[M]. Beijing: People's Education Press, 2017:18-19.