A Quantitative Study of Alliance Structures in the Three Kingdoms of Ancient China, 208-280 AD

Shengliang Zhu[1], Liyang Mao[2], Yulin Chen[1], Xiaotong Chen[1]

1. Guangzhou Vocational University of Science and Technology, Guangzhou 510555, China.

2. Guangdong University of Finance & Economics, Guangzhou 510320, China.

Abstract: The Three Kingdoms period of ancient China (208-280 AD) refers to the period between Eastern Han (25–220 AD) and Jin dynasties (266–420), during which China was divided into Shu (221-263 AD), Wei (220-266 AD) and Wu (222-280 AD) kingdoms, and then united as Jin dynasty. This paper constructs the quarterly series of alliance structures between the Three Kingdoms. By collecting and analyzing a total of two hundred and eighty-nine quarterly observations, the paper shows that the three most frequent alliance structures are ρ0: 1) the finest partition or no-alliance structure with 192 partitions; 2) Three partitions with Shu-Jin alliance and Wu singletion with 57 partions; 3) Wei-Wu alliance and one singletion Shu with 12 partions. It also shows that the observed changes in alliance structures were the consequence of a total of fifteen major battles fought by the three kingdoms. Such results serve as a contribution to the studies of applied game theory, alliance study, and the economic and military histories in ancient China.

Keywords: Ancient China; The Three Kingdoms; Quantitative Research; State Power

1. Introduction

Academically, the Three Kingdoms period[1] has been recognized as the tripartite division of ancient China from 220 AD to 280 AD. In the board sense, the Three Kingdoms started from 184/190/208 AD., respectively. This paper picks 208 AD because it is the beginning of the three kingdoms' chaotic infighting form(Chen,2005)[1].

Figure 1: The Location of The Three Kingdoms States[2]

The Table 1 compared the census in Jin dynasty (280 AD) with the census in Han era. Almost fifty million people died during this

[1] Two alternative starting years of this period are 184 and 190 AD, both of which are not relevant for constructing our alliance structures. This paper picks 208 AD because it is the beginning of the three kingdoms' power formation.

[2] This paper focuses, as done by most historians, on Shu, Wu and Wei, ignoring other powers such as Xiongnu or Huns in the north.

period.

Table 1 The census between Jin and Han era

	Households	Individuals
Han era	10,677,960	56,486,856
Jin Dynasty	2,459,840	16,163,863

The primary purpose of this paper is to construct the series of alliance structure of the three kingdoms period from 208 to 280 AD.. Present study divided the years into quarters for the convenience of record and analysis. For the number of possible partitions, initially, there were six possible partitions that were observed. The results from quantitative analysis contribute to the future related studies, such as international relations, military research, and game theory.

2. Quantitative Analyses

Toward the end of Eastern Han dynasty, the empire collapsed. After nearly a quarter of century fighting (164-208 AD) between warlords, Liu Bei[3], Cao Cao[4] and Sun Quan emerged as leading contenders for the next ruler of China, who controlled respectively the Shu, Wei and Wu kingdoms.

Let N= {Shu, Wei, Wu} denote the set of kingdoms or players. There are a total of five partitions or regular alliance structures, which can be given in three groups: i) ρ^0 = {Shu, Wei, Wu}, the finest partition with no proper alliance; ii) ρ^1 = {Shu, Wei-Wu}, ρ^2 = {Wei, Shu-Wu}, and ρ^3 = {Wu, Shu-Wei}, three partitions with one-proper alliance and one singleton; and iii) ρ^* = {Shu-Wei-Wu}, the coarsest partition with no singleton or simply the grand coalition(Zhang, 2003)[2].

There are infinitely many inter-connected alliance structures (or collections of alliances with a positive weight attached to each alliance in the collection). Only one such collection, denoted by ρ^4 = {Shu-Wu, Wei-Wu}, is relevant in this study, which represents three components of the alliance structure: i) Shu puts one half of its relevant military resources into Shu-Wu, its alliance with Wu, and keeps the other half as defense against Wei; ii) Wei puts one half of its relevant military resources into its alliance Wei-Wu and keeps the other half as defense against Shu; and iii) Wu splits its relevant military resources equally between its two alliances: Shu-Wu and Wei-Wu.[5]

Table 2 List of Observed Alliance Structures

ρ	Singletons	Proper Alliances	Quarterly Observations
ρ^0	Shu, Wei, Wu	None	8
ρ^1	Shu	Wu-Wei	12
ρ^2	Wei	Shu-Wu	192
ρ^3	Wu	Shu-Wei *	8
ρ^4	None	Shu-Wu, Wu-Wei	11
ρ^J	Wu	Shu-Jin *	57
ρ^*	Shu-Wei-Wu*	Shu-Wei-Wu*	1

The common need to survive and to stop Wei's southward expansion forced the formation of Shu-Wu alliance at end of 208 AD. This change in alliance structure is recorded as $\rho^{Initial=}_{208q1}$ $\rho^0 \rightarrow \rho^{Final}_{208q4} = \rho^2$ = {Wei, Shu-Wu}.

From 209 to 215 Q1 AD, the alliance structures remained the same as in the Battle of Red Cliffs, this was recorded as $\rho^{Initial=}_{208q4}$ $\rho^2 \rightarrow \rho^{Final}_{215q1} = \rho^2$.

In May of 215 AD, Sun Quan wanted to take the Jin province back for limiting Shu's growing military power[3]. However, Liu Bei believed that there was no need to maintain the alliance, so he refused to return Jin province to Wu, which broke the Shu-Wu alliance (Chen, 2015)[4], this was recorded as $\rho^{Initial=}_{215q1}$ $\rho^2 \rightarrow \rho^{Final}_{215q2} = \rho^0$.

---

[3] Liu Bei (Shu) was a twenty-first generation decendent of the founding emperor of Han Dynasty.
[4] In 265 A.D., Sima Yan usurped Wei and founded Jin dynasty. This paper considers and records the data of Jin as the extend of Wei.
[5] Such inter-connected alliance structures are mathematically called balanced collections of coalitions in game theory. The precise definition of ρ^4 is ρ = {*Shu*, *Wei*, *Shu-Wu*, *Wei-Wu*} with a weight vector W = {½, ½, ½, ½}, or $W_{Shu} = W_{Wei} = W_{Shu\text{-}Wu} = W_{Wei\text{-}Wu}$ = ½. Note that the weight vector satisfies both feasiblity and no-waste condittions.

After a couple of weeks, at the end of May of 215 AD, Shu was indulged in an embarrassed situation. Firstly, Cao Cao attacked and took over the Hanzhong area. Secondly, Sun Quan raised a war to Shu because he was eager to recapture Jingzhou military area (Sun, 2007)[5]. Thereupon, Liu Bei worried about his army being harried from Wu and Wei, so he had to rebuild alliance with Wu. The detailed information was shown in Chapter 3. For this reason, the period between 215 q_4 AD and 216 q_4 AD was recorded as $\rho^{Initial}_{215q4} = \rho^2 \rightarrow \rho^{Final}_{216q4} = \rho^2 = \{S_1^*, S_2\}$, where S_1^*= Shu-Wu, S_2= Wei.

In order to recover from the loss of the battle of Ruxu and avoid future invasion, Sun Quan sent a letter to Cao Cao privately to ask for making an alliance. Sun Quan believed this alliance could help Wu get rid of the hostility of Wei and decrease Shu's power. Therefore, the pretended alliance started through Wu and Shu, which lasted for two years, recording as $\rho^{Initial}_{217\ q1} = \rho^4 = \{S1, S2\}$, where S_1= Shu-Wu, S_2= Wu-Wei.

On July 219 AD, Liu Bei declared himself the "King of Hanzhong". Besides, with the increasing morale of Shu, Guan Yu chose to make a difference and started to attack Wei's area. Then, Sun Quan decided to end up the Shu-Wu alliance and started to attack back. This period was recorded as $\rho^{Initial}_m = \rho^4 \rightarrow \rho^{Final}_m = \rho^1 = \{S1, S2\}$, where S_1= Wu-Wei, S_2= Shu, for m = 217 q_1 AD, 217 q_2 AD, 217 q_3 AD, …,219 q_3 AD.

Later, in the last quarter of 219 AD, $\rho^{Initial}_{219\ q4} = \rho^1$, due to the break of Shu-Wu alliance and the loss of Jinzhou area, Liu Bei decided to revenge to Sun Quan. In January 220 AD, after Cao Cao died, his son Cao Pi followed his father 's goal to unify China and established the state of Cao Wei with proclaimed himself emperor officiallyChina (Zhang, 2006)[6]. However, revenge road had not be stopped until 223 AD Accordingly, the period was recorded as $\rho^{Initial}_{mq1} = \rho^1$, where m = 220 q_1 to 222 q_4.

With the persisted rebelliousness of the Shanyue[6] terrorist, Sun Quan chose to continue to be the minister to Wei state in order for avoiding the possibility of being attacked by two sides. However, the state of Wei was not easy to be deceived (Sun, 2007)[5]. Then, Cao Pi sent envoys to Wu to ask Sun Quan to provide the prince of Wu as a hostage in Wei. However, Sun Quan dodged the issue by pretending cordiality and finally refused the requirement. Cao Pi was very angry, then he sent troops to attack the Wu in October 222 AD This period was recorded as $\rho^{Initial}_{222q3} = \rho^1 = \{S_1, S_2\} \rightarrow \rho^{Final}_{223q3} = \rho^0 = \{S_1^*, S_2^*, S_3^*\}$, where S_1 =Shu, S_2= Wu-Wei, and S_1^* = Shu, S_2^* =Wu, S_3^*= Wei.

Liu Shan inherited the king after Liu Bei died, and rebuilt the alliance with Wu under Zhuge Liang's suggestion in November 223 AD, recording as $\rho^{Initial}_{223q4} = \rho^0 \rightarrow \rho^2 = \{S_1, S_2\}$, where S_1= Shu-Wu, S_2=Wei. In ancient times, especially for the ear of agriculture, the population is the primary productivity. Due to the lost about 50,000 people in the battle of Yiling. Shu stepped into an awkward situation, which made them have to maintain the alliance with Wu. This period was recorded as $\rho_{224\ q1} = \rho^2$.

After Zhuge Liang died in 234 AD, Fei Yi, Jiang Wan, and Dong Yun became the imperial chancellor of the Shu (Zhao,1997)[7]. Jiang Wei, who was highly appreciated by Zhuge Liang, led another 11 military campaigns to continue the legacy of waging war against Wei between 240 AD and 262 AD. All of these wars dragged Shu's economy and the army force down (Sun, 2007)[5]. So that, in 263 AD, Deng Ai (the General of Wei) got through the Yinping mountains and took over the Mianzhu city, which led to Liu Shan gave up and surrounded (Ma, 2005)[8]. Finally, (Cao)Wei broke the three kingdoms balance and conquered (Shu) Han.

Figure 2.2: The Land of The Three Kingdoms States in 226 A.D.

[6] Shanyue: An ancient conglomeration of upland Yue hill (close to Wu state). The people who lived in there performs regular rebellions against Han citizens.

In December 265 AD, Sima Yan forced Cao Huan to abdicate to him and changed Wei to Jin. Thus, this period was recorded as $\rho^{Initial}_{265q4} = \rho^3 \rightarrow \rho^{Final}_{265q4} = \rho^J$, where J= Shu-Wei. This paper assumed Jin as the extended dynasty of Wei and recorded as J to replace Wei.

Later, in March 280 AD, the king of Wu, Sun Xin, was captured by Sima Yan. Therefore, the three kingdoms period ended, and the ancient China was united by Jin. This period was recorded as:

$\rho^{Initial}_{224q1} = \rho^2 \rightarrow \rho^{Final}_{263q3} = \rho^2 = \{S1, S2\}$, where S_1= Shu-Wu, S_2= Wei.

$\rho^{Initial}_{263q4} = \rho^2 \rightarrow \rho^{Final}_{265q3} = \rho^3 = \rho^J = \{S1, S2\}$, where S_1= Shu-Wei, S_2= Wu.

$\rho^{Initial}_{265q4} = \rho^J \rightarrow \rho^{Final}_{280\ q1AD} = \rho^* = \{S^*\}$, where S^*= Shu-Wei-Wu or the unified China

The following Table 1-5 show the statistical results of frequencies from ρ^0 -ρ^*. According to the results, the single alliance between Shu and Wu was the most frequent, which was ρ^2 with 191 observations. Furthermore, there were only eight observations showing that Shu was single with non-alliances, noted ρ^0, as the finest partition. There was a total of 65 observations for Wu as a singleton after Shu conquered by Wei (Jin) (Zhang,1987)[9]. The symbol * indicates that the alliance structure changed twice in one year.

Table 3.1: Annual Series of Coalition Structures (208 Q1-223 Q4AD.)

Year/A quarter (AD.)	Final Count of Proper Alliances	Final Count of Alliances
208 Q1	ρ^0	3
208 Q2	ρ^0	3
208 Q3	ρ^2	2
208 Q4	ρ^2	2
|	ρ^2	2
215 Q1	ρ^2	3
215 Q2	ρ^0	3
215 Q3	ρ^0	3
215 Q4	ρ^2	2
216 Q1	ρ^2	2
216 Q2	ρ^2	2
216 Q3	ρ^2	2
216 Q4	ρ^2	2
217 Q1*	ρ^4	0
|	ρ^4	0
219 Q3*	ρ^4	0
219 Q4	ρ^1	2
|		
222 Q4	ρ^0	3
223 Q1	ρ^0	3
223 Q2	ρ^0	3
223 Q3	ρ^0	3
223 Q4	ρ^2	2

Table 3.2: Annual Series of Coalition Structures (224 Q1-239 Q4 AD.)

Year/Quarter (AD.)	Final Count of Proper Alliances	Final Count of Alliances
224 Q1	ρ^2	2

|	ρ^2	2
239 Q4	ρ^2	2

Table 3.3: Annual Series of Coalition Structures (240 Q1-255 Q4AD.)

Year/Quarter (AD.)	Final Count of Proper Alliances	Final Count of Alliances
240 Q1	ρ^2	2
|	ρ^2	2
255 Q4	ρ^2	2

Table 3.4: Annual Series of Coalition Structures (256 Q1-271 Q4 AD.)

Year/Quarter (AD.)	Final Count of Proper Alliances	Final Count of Alliances
256 Q1	ρ^2	2
|	ρ^2	2
263 Q3	ρ^2	2
263 Q4	ρ^3	2
|	ρ^3	2
265 Q3	ρ^3	2
265 Q4	ρ^J	2
|	ρ^2	2
271 Q4	ρ^J	2

Table 3.5: Annual Series of Coalition Structures (272 Q1-280 Q1 AD.)

Year/Quarter (AD.)	Final Count of Proper Alliances	Final Count of Alliances
272 Q1	ρ^J	2
|	ρ^J	2
280 Q1	ρ^*	0

3. Conclusion And Discussion

This paper provided and quantified some details about the alliance structures during the Three Kingdoms Period of ancient China. An alliance was built mainly because of two significant reasons: a) defending for the strongest one and avoiding the sneak attack from another kingdom; b) enlarging the exiting advantages and territory.

The primary conclusion is concluded briefly though the project. Wei had always been the most powerful kingdom with the absolute military force and enough talented managers. It is also the reason why they could remain to the end and then unified China (Although Jin dynasty took over Wei later, it was still recognized as the extend and inhered of Wei.).

Secondly, sometimes, the weather still had an impact on ancient wars and people cannot control the final result if you ignore the nature. Therefore, an excellent general would observe the weather and check the geography before the war, then try to make use of the weather and geographical conditions so as to serve his own campaign and tactics.

All the quantitative analysis of alliance structures might provide insights for two potential future research topics. The alliance structures changed along with the changes of the strategies of the three kingdoms. These data and differences can be applied in the game theory about the cooperative or symmetric and international economics. Besieds, because of the large amount of observations, this paper could be helpful for the researchers who are interested in theoretical statistical analysis.

References

[1] Chen, J. (2005). A Talk on the Wu Kingdom's Strategy at Yizhou. Military Historical Research, (4), pp. 86-93.

[2] Zhang, D. (2003). The History of The Three Kingdoms Period. Beijing, China: Huawen Press.

[3] Chen, Shou (233–297), Records of the Three Kingdoms, with annotations by Pei Songzhi (372–451). Beijing, China: Zhonghua shuju (2011).

[4] Chen, H. (2015), A Quantitative Study of Alliance Structures in the Warring States of Ancient China, 453-221 B.C. MA Thesis, Economics Department. Saskatoon, SK: University of Saskatchewan.

[5] Sun, Z. (2007). Introduction to Diplomatic Activities of the Three Kingdoms. Shidai Wenxue, (6), p. 147.

[6] Zhang, C. (2006). Sanguo Dawaijiao. Chongqin, China: Chongqing Press.

[7] Zhao, G. (1997). The relation between Wu and Shu. Journal of Central China Normal, University (Philosophy and Social Sciences), 36(1).

[8] Ma, Q. (2005). On the geographic environments & the Historical Course of the Three Kingdoms at the end of the Han Dynasty. Journal of Chengdu University (Social Science), (2), pp. 53-57..

[9] Zhang, D. (1987). Brief Discussion of Diplomatic Activities of the Three Kingdoms. Academic Monthly, (9), pp. 65-70.