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Article

Effect of capital market on economic growth: An analysis using the autoregressive distributed lag (ARDL) approach

Tayo P. Ogundunmade

Department of Statistics, University of Ibadan, Ibadan 200001, Nigeria; ogundunmadetayo@yahoo.com

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Abstract: The limits testing method known as ARDL (Autoregressive Distributed Lag) examines Nigerian capital market improvement and monetary development. Relevant indicators about the capital market are taken into account during analysis while considering influencing factors as well. This study aims to explore the relationship between the capital market and financial development in Nigeria. The study's theoretical framework, which is based on the ARDL framework, includes both short- and long-term dynamics. The data used in this analysis was collected from the Central Bank of Nigeria (CBN), the World Bank database, and the Nigerian Stock Exchange (NSE). In this study, the Nigerian All-Share Index, foreign direct investment, currency exchange rates, and inflation rates are the free factors, while the increase in GDP is the dependent factor. Through cointegration analysis using the ARDL framework, it was discovered that in Nigeria there's a lengthy equilibrium correlation between economic expansion and financial development. The coefficients determined allow for a deeper understanding of how capital market factors affect economic growth over time. Furthermore, utilizing an error correction model derived from the ARDL analysis offers insight into both brief dynamics and modified speed toward reaching a lasting state of balance. By utilizing the ARDL approach, this study adds to the continuing discourse surrounding how capital markets impact growth in Nigeria. Its empirical evidence provides valuable knowledge for policymakers and stakeholders looking to utilize capital markets effectively toward achieving long-lasting economic development within the country.

Keywords: capital market; economic growth; auto regressive distributed lag model; granger causality

1. Introduction

The Nigerian capital market plays a significant and essential part in the expansion and development of the nation's economy. It is a crucial platform for raising long-term capital for investments and giving companies a venue to issue securities to raise money. The financial system is a vital source of finance for companies, and it also serves as an avenue for individuals to invest and grow their wealth. In recent times, the capital market in Nigeria has undergone significant expansion, and it has become a robust market that attracts domestic and foreign investors [1]. Nigeria's economy has grown significantly over time, and the capital market has played a vital role in this expansion. The Nigerian economy is largely dependent on the oil sector, but the government has been working to lessen the nation's reliance on oil and broaden the economy. The financial system permits companies to raise funds to finance their operations, and this has helped to create employment opportunities, increase production, and boost economic growth [2]. While investment in developing the nation's currency marketplace has been notable, their precise influence on the expansion of the economy remains debatable. While proponents advocate for its role in promoting investment, entrepreneurship, and innovation, others highlight potential drawbacks such as financial instability and increased inequality. This lack of clarity on the precise nature and strength of the relationship complicates policymakers' efforts to design effective strategies for harnessing the capital market's potential for boosting the Nigerian economy.

The financial industry is intricately intertwined with the expansion of the business, as it serves to allocate resources. In Nigeria, financing economic expansion depends on the crucial role played by its capital market in channeling funds towards investment projects pursued by governments and businesses across different sectors. Nonetheless, this association between economic progress and the capital market presents a multifaceted dynamic that requires statistical evaluation through tools like Auto Regressive Distributed Lag (ARDL) model analysis, which forms part of our study's objectives.

The ARDL method is an econometric tool commonly employed to explain both the immediate and far future components of many different factors. Given its capacity for detecting the enduring connection between financial market activity as well as financial performance in the nation, this approach proves helpful since it permits the calculation of said correlation across various timescales. The effectiveness of this model has been verified by previous studies [3,4], which have respectively assessed discrepancies within capital markets or reached a state of balance-related thereto but involving already developed economies. Evidence that is based on actual observations or experiences rather than theory or speculation. Studies based on empirical evidence reveal that the Nigerian financial market is essential in promoting economic expansion. Notably, Zhou et al. [5] suggest that firms' integrated reporting containing details of their performance in South Africa had positive effects on both the actual and financial markets. Similarly, according to Afolabi [6], businesses under market pressure tend to adhere to tax avoidance practices conducive to investing further in this same sector. All these data bolster claims about how imperative it is to look deeper at Nigeria's capital market as a vital source of funding for enhancing its economy even more effectively than before—given such encouraging signs already seen from previous analyses done thus far! Several studies have focused on the relationship between the nation's financial market and economic expansion. Badertscher et al. [7] highlight that the financial sector possesses a significant impact on the financial system, as it provides a platform for mobilizing savings and investing in productive ventures. Similarly, Briggs [8] posits that the capital market serves as a vital channel for resource allocation, which is essential for generating economic growth. Therefore, this research seeks to investigate how the country's financial system impacts the country's economic growth using an ARDL model.

The purpose of this research is to investigate the correlation between Nigeria's economic expansion and capital market through the application of this technique. The theoretical framework adopted in this framework comprises both temporal and permanent dynamics embedded within the ARDL framework. Previous empirical data affirm an affirmative relationship connecting Nigerian financial exchange trading with economic development [7,8]. This forthcoming investigation intends to provide new knowledge regarding how Nigerian stock exchanges influence its economy while adding to existing literature concerning said subject material.

2. Literature review

The capital market operates as a medium for enabling extended credit facilities, provisioning both fixed and operating cash reserves along with intermediate- to deep-rooted debt loans in aiding the monetary requirements of central, state, and regional administrations [9]. Acting as an instrumentality towards supporting long-term ventures amongst businesses, households, and governments alike [10], it involves transactions encompassing prolonged debts and equity instruments. The size and performance of a capital market are indicated by its market capitalization, which is an important factor in the financial industry. It represents the total market value of stocks within a given location [11], calculated by multiplying share price by shares outstanding for businesses [12]. Securities like Exchange Traded Funds (ETFs) and bonds fall under this metric as well, making it both an indication measure of development effectiveness [13,14].

The NSE ASI serves as a market index that evaluates the general direction and performance of the Nigerian stock market [15,16]. Consolidating stock values against a base value within a predetermined timeframe enables continual monitoring of market fluctuations [15,17], while also enabling comparative assessments among industries and companies alike [15]. Furthermore, the Nigerian Stock Exchange (NSE) quarterly report includes a comprehensive account of traded shares on the stock market. The total transaction volume reveals crucial data regarding exchange floor activities, with NGN 15.84bn recorded in Q3 2020, according to Emmanuel and Elizabeth's study findings. The connection between the capital market and economic expansion has always been a vital area of research, particularly when the economy is developing. Recent research has continued to accentuate the decisive role of well-functioning capital markets in promoting economic development. Notably, studies by Guo et al. [18] and Hu et al. [19] have emphasized the beneficiary link between the expansion of the stock market and economic expansion, citing mechanisms such as resource mobilization, investment efficiency, and risk diversification [20].

Lee et al. [21] have identified the financial market as a significant contributor to the expansion of the economy in Nigeria. By offering an avenue for firms to acquire long-term funds, it enables investment that stimulates further economic progress. Nonetheless, various factors can affect this dynamic relationship between the economy and the capital markets. Lin's [22] research highlights commercial banks' contribution towards small-scale enterprises, which significantly influence activity within Nigerian economics. They suggest that when these establishments access credit from such entities, they can create job opportunities while increasing their productivity of goods and services—all driving economic expansion forward.

Numerous investigations have analyzed how the Nigerian economy is affected by crude oil price shocks. Park's [23] research delves into viable legal and policy strategies to alleviate these impacts, whereas Sharpe [24] scrutinizes the consequences of capital flight on Nigeria's economic growth and development—indicating a detrimental effect. Ultimately, both studies conclude that external factors such as crude oil price fluctuations and capital outflow significantly impact the Nigerian economy.

3. Materials and methods

The research takes a quantitative study plan to look into the statistical connection between the commercial market and economic expansion. The focus is analyzing historical data through econometric modeling. This study will employ yearly time series data from 2003–2022 and will be retrieved from https://worldbankdata.org/indicator. To assess the influence of financial market indicators on Nigeria's economic expansion, it shall employ its surrogates as relevant parameters. That is, an increased rate of GDP will be employed as a stand-in for economic expansion. Nigerian Foreign Direct Investment (NFDI) is the chosen financial market indicator for this search, though inflation rates (IR) and exchange rates (ER) are control factors and financial indicators.

3.1. Data for the study

Data for this research are collected from the CBN, World Bank database, and NSE. The data is yearly data that spans from 2003 to 2022. The dependent variable for this research is GDP growth, while the independent variables include inflation rates, exchange rates, the Nigerian All-Share Index, and foreign direct investment. There are 32 observations, and this makes the choice of ARDL approach appropriate because of the high performance of ARDL on small samples [25].

3.2. Study model

This research will employ the approach of multiple linear regression, which is popular among authors because it emphasizes supplying more than two factors in the estimate. Consequently, we efficiently define the study's model using the specified parameters as follows:

$$\text{GGDP} = f\,(\text{INF, EXR, NFDI}) \tag{1}$$

This equation is linearly represented as follows;

$$\text{GGDP}_t = \beta_0 + \beta_1\ \text{INF}_{t-1} + \beta_2\ \text{EXR}_{t-1} + \beta_3\ \text{NFDI}_{t-1} + e_t \tag{2}$$

where GGDP = increase rate of GDP, NFDI = Nigerian Foreign Direct Investment, INF = Inflation Rates, EXR = Exchange Rates, and Et = Error term.

3.3. ARDL technique

The ARDL technique will be employed to investigate the connections between the prospective and temporary behavior of commercial and economic expansion in Nigeria. This model allows for the investigation of cointegration and short-run dynamics simultaneously. The factors that are autonomous and reliant in ARDL models are linear time series models where the relationship is not just contemporaneous but likewise over past (lagged) values. Specifically, given y_t as the dependent variable and $x1$, ..., x_k as the k explanatory factors, the general ARDL (p, q_1, ..., q_k) model may be obtained as follows:

$$y_t = a_0 + a_1 t + \sum_{i=1}^{p} \psi_i y_{t-1} + \sum_{j=1}^{k} \sum_{l_j=0}^{q_j} \beta_{j,} l_j x_{j,t-l_j} + \varepsilon_t \quad (3)$$

where a_0 is a constant term, ε_t denotes the usual innovations, and a1, φ_i, and β_j, l_j are the coefficients corresponding to a linear trend, y_t lags, and k regressor x_j, t for $j = 1$, … $k = 1$, … Alternatively, define $\psi(L)$ and $\beta_j(L)$ as the lag polynomials, and let L stand for the standard lag operator:

$$\psi(L) = 1 - \sum_{i=1}^{p} \psi_i L^i$$

$$\beta_j = \sum_{l_j=0}^{q_j} \beta_{j,l_j} L^{l^j}$$

ARDL is argued by Kim and Song [20] to be beneficial in its ability to manage cointegration with built-in resilience to incorrect integration orders of some pertinent variables.

3.4. Cointegration analysis

The study will conduct cointegration tests, such as the Engle-Granger and Johansen tests, to identify the existence of a sustained correlation between factors in the stock market and economic expansion. The test of Johansen cointegration will be chosen for this study because the approach may display more than two cointegrating associations if there are more than two variables, which is the Engle-Granger method's restriction. Its single equation model is another drawback. Cointegration is based on the concept that two or more non-stationary time series variables might be related in the long run, even if they individually display unit roots. Think of a k-system of non-stationary variables $y_t = (y_1t, y_2t, ..., y_kt)$'. If there is a straight-line combination of these factors β'yt that results in a static process, then yt is said to be cointegrated. Formally, if β'yt is I(0), then yt is cointegrated.

The classic representation of cointegration can be expressed in the framework of an error correction model (ECM). Let Δyt denote the first difference of yt, and Δd denote the d-th difference operator. The ECM takes the form:

$$\Delta y_t = \alpha + \sum_{i=1}^{p} \Gamma_i \Delta y_{t-1} + \beta(y_{t-1} - \sum_{i=1}^{p} \phi_i y_{t-1}) + \varepsilon_t \quad (4)$$

where:

α is a constant term.

Γ_i are coefficients for lagged differences of y_t.

β is the symbol for the error correction term's coefficient.

ϕ_i are coefficients for lagged values of y_t.

ε_t is the error term.

The term $(y_{t-1} - \sum_{i=1}^{p} \phi_i y_{t-i})$ captures the deviation from the equilibrium over time relationship, combining β representing the rate of modification towards equilibrium.

4. Results and discussion

The details of the data for this research are presented below, which is yearly data with a range from 2003 to 2022. It shows observations for the following macro-economic variables or indicators under study, which include the increase in GDP, which is based on the measure of monetary expansion, inflation (INF), currency rate (EXR), and foreign direct investment (FDI).

4.1. Descriptive statistics

The variables' descriptive figures findings are displayed in this section. **Table 1** displays the variables for inflation, exchange prices, foreign direct investment, and the total household income together with their range of minimum, maximum, and count values along with their mean, standard error, median, standard deviation, sample variance, kurtosis, and skewness. **Figures 1–4** show the line plots for the variables GDP, foreign direct investment, inflation rate, and exchange rate respectively.

Table 1. Descriptive statistics of variables.

	Inflation rate	Exchange rate	Foreign Direct Investment	Gross Domestic Product
Mean	18.41967558	150.8797005	3,060,824,339	$2.72312 \times 10^{+11}$
Standard Error	2.872347607	20.46722074	465,519,213.4	29,825,528,540
Median	12.94177615	130.2483417	2,155,226,687	$2.58358 \times 10^{+11}$
Standard Deviation	16.24845177	115.7800846	2,633,374,341	$1.68719 \times 10^{+11}$
Sample Variance	264.0121848	13405.028	$6.93466 \times 10^{+18}$	$2.8466 \times 10^{+22}$
Kurtosis	4.473721983	0.124146206	−0.33472763	−1.48643578
Skewness	2.266855211	0.872433124	0.876396493	0.114363885
Range	67.44749433	416.0696664	9,027,854,480	$5.22126 \times 10^{+11}$
Minimum	5.388007969	9.909491667	186,792,428.9	52,058,181,854
Maximum	72.8355023	425.9791581	8,841,062,051	$5.74184 \times 10^{+11}$
Count	32	32	32	32

Source: Author's computation from MS-Excel (2024).

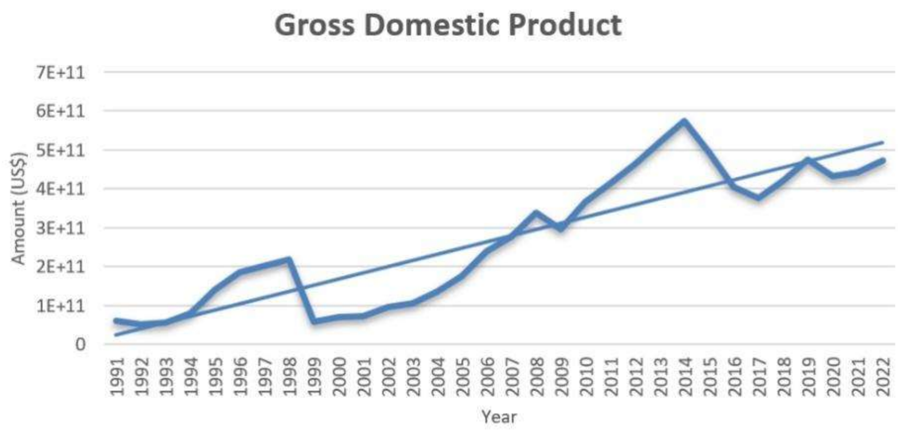

Figure 1. Line Graph for (GDP) growth.

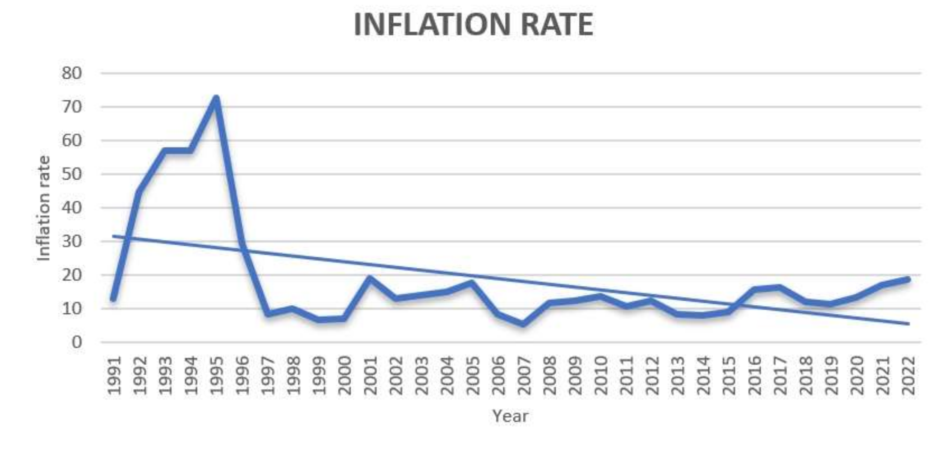

Figure 2. Line graph for inflation.

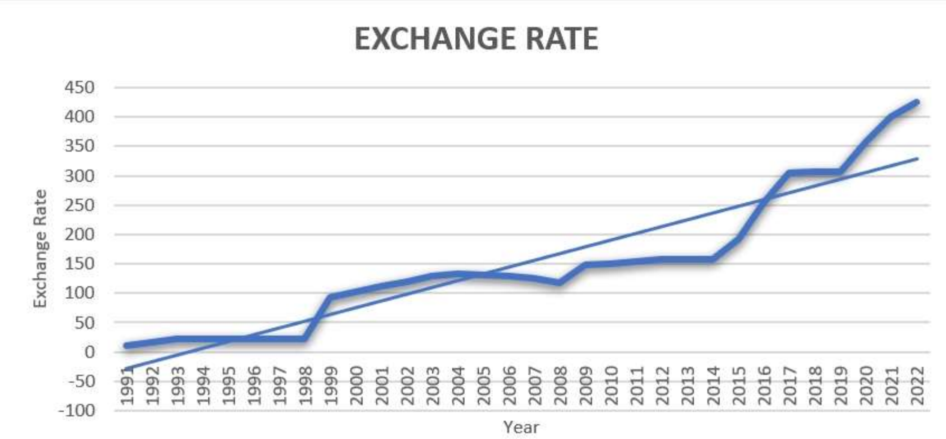

Figure 3. Line graph for official exchange rate.

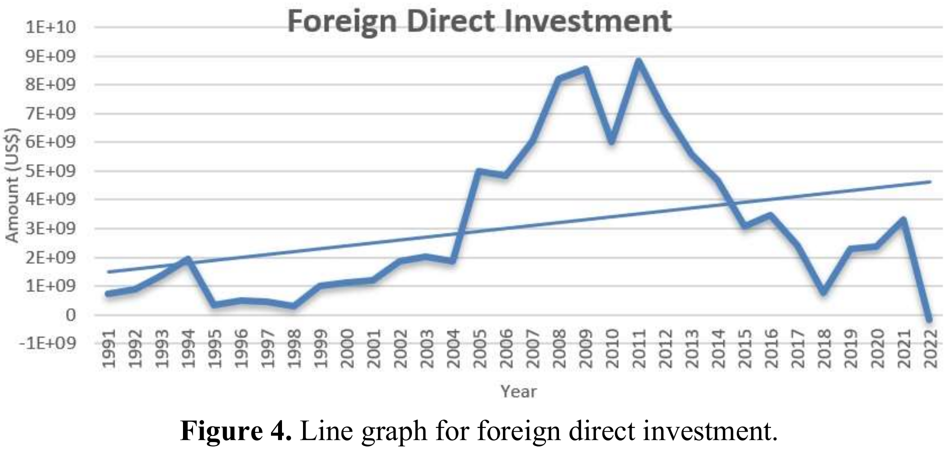

Figure 4. Line graph for foreign direct investment.

Table 1 displays the descriptive overview of different variables, each having 32 observations. It shows the mean, median, variance as well as the lowest and highest values of the selected indicators. From the table, the mean, standard error, kurtosis, and skewness of the inflation rate are 18.42, 2.87, 4.47, and 2.27, respectively. The mean, standard error, kurtosis, and skewness of the exchange rate are 150.88, 20.47, 0.12, and 0.87, respectively. The mean, standard error, kurtosis, and skewness of foreign direct investment are 30,60,824,339, 465,519,213.4, −0.33472763, and

0.876396493, respectively. The mean, standard error, kurtosis, and skewness of gross domestic product are $2.72 \times 10^{+11}$, 2,982,5528,540, −1.49, and 0.11, respectively.

4.2. Unit root test

The following shows the stationarity result for the factors of this study (see **Tables 2–9**)

Table 2. Unit root test on exchange rate.

Variable	Coefficient	Std. Error	*t*-Statistic	Prob.
C	−27.40068	14.74372	−1.858464	0.0729
@TREND	11.50196	0.817206	14.07473	0.0000

Table 3. Unit root test on exchange rate after first difference.

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		−4.253124	0.0111
Test critical values:	1% level	−4.296729	
	5% level	−3.568379	
	10% level	−3.218382	

Table 4. Unit root test on foreign direct investment.

Variable	Coefficient	Std. Error	*t*-Statistic	Prob.
C	$1.51 \times 10^{+09}$	$8.64 \times 10^{+08}$	1.748933	0.0905
@TREND	99,979,668	47,891,205	2.087642	0.0454

Table 5. Unit root test on foreign direct investment after first difference.

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		−6.082224	0.0001
Test critical values:	1% level	−4.296729	
	5% level	−3.568379	
	10% level	−3.218382	

Table 6. Unit root test on inflation rate.

Variable	Coefficient	Std. Error	*t*-Statistic	Prob.
C	31.42735	4.991015	6.296785	0.0000
@TREND	−0.839205	0.276639	−3.033574	0.0050

Table 7. Unit root test on inflation rate after first difference.

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		−5.235890	0.0014
Test critical values:	1% level	−4.356068	
	5% level	−3.595026	
	10% level	−3.233456	

Table 8. Unit root test on gross domestic product.

Variable	Coefficient	Std. Error	*t*-Statistic	Prob.
C	$2.52 \times 10^{+10}$	$2.74 \times 10^{+10}$	0.919093	0.3654
@TREND	$1.59 \times 10^{+10}$	$1.52 \times 10^{+09}$	10.48796	0.0000

Table 9. Unit root test on GDP after first difference.

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		−4.331182	0.0092
Test critical values:	1% level	−4.296729	
	5% level	−3.568379	
	10% level	−3.218382	

The exchange rate is only statistically important on trend at the 5% degree of significance when it is regressed on both constant and trend, as indicated in **Table 2**. Exchange rates are statistically significant ($p < 0.05$) at order 1, as **Table 3** demonstrates. It is found that foreign direct investment is statistically significant on trend only at the 5% level of significance when it is regressed on both constant and trend, as indicated in **Table 4.** Foreign direct investment at order 1 is statistically significant ($p < 0.05$), as **Table 5** demonstrates. Both the constant and trend inflation rates, as seen in **Table 6**, are found to be statistically significant when regressed. The statistical significance of the inflation rate ($p < 0.05$) at order 1 is shown by **Table 7**. Only at the 5% level of significance is GDP growth found to be statistically significant when it is regressed on both constant and trend, as indicated in **Table 8**. The GDP growth rate at order 1 is statistically significant ($p < 0.05$), as **Table 9** reveals.

4.3. Cointegration test

In order to build a lasting relationship, a cointegration test must be conducted if the series are integrated into distinct orders. The bounds test, which was proposed by Kim and Song [20], is a suitable cointegration test. The Johansen cointegration test is no longer appropriate for usage. The hypothesis is expressed as follows: H0: no cointegrating equation, and H1: H0 is false. Bounds Test decision criteria: Rejection at 10%, 5%, or 1% significance levels. We can determine that cointegration exists if the computed F-statistic exceeds the upper bound I(1) criteria value. In other words, the relationship is long-term. Don't accept the null hypothesis. Calculate the error correction model (ECM), which is the long-term model. We infer that there is no cointegration and, thus, no long-run relationship if the computed. The F-statistic for the lower bound I(0) is less than the critical value. Keep accepting the null hypothesis. Calculate the autoregressive distributed lag (ARDL) model, which is the short-run model. Should the F statistic lie between I(0), the lower bound, and I(1), the upper bound? The test yields inconclusive results in **Table 10**.

Table 10. Bounds test.

F-Bounds Test		Null Hypothesis: No levels of relationship		
Test Statistic	**Value**	**Signif.**	**I(0)**	**I(1)**
			Asymptotic: $n = 1000$	
F-statistic	5.004387	10%	2.25	2.8
k	3	5%	2.57	3.79
		2.5%	3.09	3.98
		1%	3.53	4.47
Actual Sample Size	28		Finite Sample: $n = 35$	
		10%	2.532	3.443
		5%	3.213	4.232
		1%	4.312	5.734

We reject the null hypothesis that there is no long-run connection in this instance because the F-statistic is 5.00, which is more than the I(1) bound value at a 5% level of significance. There is hence cointegration between the logs of an increased rate of GDP, inflation rate, exchange rate, and foreign direct investment. Consequently, it is necessary to test both short-term and long-term connections.

From these *p*-values, the variables that are statistically significant at the 5% significance level ($p < 0.05$) are GDP (−3) (lagged GDP), EXR (current exchange rate), EXR (−1) (lagged exchange rate), and EXR (−3) (exchange rate three periods ago). These are the variables to be considered including in the short-run ARDL model, as they have statistically significant relationships with the dependent variable. Based on the significant variables identified from the regression results, the ARDL model would include only the statistically significant variables. In this case, the significant variables incorporating the coefficient estimates from your regression results, the model can be specified as:

$$\Delta \mathrm{GDP} = -1.49 \times 1010 + 0.6459\Delta \mathrm{GDP}_{t-3} - 2.45 \times 109\Delta \mathrm{EXR}_{t} + 1.68 \times 109\Delta \mathrm{EXR}_{t-1} + 1.87 \times 109\Delta \mathrm{EXR}_{t-3} + ut$$

This model suggests that changes in GDP three periods ago and changes in the current, lagged, and three periods ago exchange rates have significant effects on the dependent factors. With an *R*-squared of 0.996782, the self-sustaining factors in the model take into consideration roughly 99.68% of the variance in the dependent variable. It's a high value, suggesting strong explanatory power of the model. An adjusted *R*-squared of 0.992102 suggests that approximately 99.21% of the variance in the dependent parameter is understood in terms of the self-sustaining variables, adjusted for the number of predictors. The Durbin-Watson statistic tests for the presence of autocorrelation in the residuals (errors). The values span from 0 to 4, with 2 indicating the absence of autocorrelation. Values closer to 2 are preferred. Here, the Durbin-Watson statistic is 1.985634, suggesting no significant autocorrelation. Overall, the regression model seems to have a high explanatory power, good fit, and statistical significance, as indicated by these statistics in **Table 11**.

Table 11. ARDL model.

Variable	**Coefficient**	**Std. Error**	***t*-Statistic**	**Prob.***
GDP(−1)	0.308896	0.263216	1.173547	0.2654
GDP(−2)	−0.124254	0.274928	−0.451953	0.6601
GDP(−3)	0.645942	0.298618	2.163102	0.0534
FDI	−2.080298	2.850402	−0.729826	0.4807
FDI(−1)	3.194763	2.874298	1.111493	0.2901
FDI(−2)	3.348029	3.469275	0.965052	0.3553
FDI(−3)	6.781183	3.692321	1.836564	0.0934
EXR	$-2.45 \times 10^{+09}$	$3.29 \times 10^{+08}$	−7.448342	0.0000
EXR(−1)	$1.68 \times 10^{+09}$	$6.02 \times 10^{+08}$	2.785575	0.0177
EXR(−2)	$-4.61 \times 10^{+08}$	$7.41 \times 10^{+08}$	−0.621679	0.5468
EXR(−3)	$1.87 \times 10^{+09}$	$7.83 \times 10^{+08}$	2.393305	0.0357
INF	$2.41 \times 10^{+08}$	$4.37 \times 10^{+08}$	0.551651	0.5922
INF(−1)	$3.27 \times 10^{+08}$	$4.42 \times 10^{+08}$	0.738085	0.4759
INF(−2)	$2.06 \times 10^{+08}$	$4.34 \times 10^{+08}$	0.474939	0.6441
INF(−3)	$6.09 \times 10^{+08}$	$4.86 \times 10^{+08}$	1.253430	0.2360
INF(−4)	$5.46 \times 10^{+08}$	$4.86 \times 10^{+08}$	1.123977	0.2850
C	$-1.49 \times 10^{+10}$	$1.64 \times 10^{+10}$	−0.907317	0.3837
R-squared	0.996782		Mean dependent var	$3.02 \times 10^{+11}$
Adjusted *R*-squared	0.992102		S.D. dependent var	$1.59 \times 10^{+11}$
S.E. of regression	$1.41 \times 10^{+10}$		Akaike info criterion	49.85739
Sum squared resid	$2.19 \times 10^{+21}$		Schwarz criterion	50.66622
Log-likelihood	−681.0034		Hannan-Quinn criteria.	50.10466
F-statistic	212.9810		Durbin-Watson stat	1.985634

4.4. Long-run (error correction model)

Approximately 97.25% within the variance of the reliant factor can be explained by the self-sustaining factors in the model, according to the *R*-squared value of 0.972485. Given the high value, the technique appears to have a significant explanatory capacity. A little less than the *R*-squared number, 0.950474, is the adjusted *R*-squared value. The model's modified *R*-squared considers the total count of forecasters and penalizes the overuse of several factors that do not necessarily increase the ability to explain the model. The mean shows how far apart on average the shown values are from the values that the regression model predicted. There is a 1.96 Durbin-Watson statistic. This examines whether the residuals have autocorrelation. There is no discernible autocorrelation when the value is near 2. In general, these statistical data offer insights about the model's explanatory capacity, autocorrelation in the residuals, and goodness of fit in **Table 12**.

Table 12. Autocorrelation verification.

Variable	Coefficient	Std. Error	*t*-Statistic	Prob.
C	$6.75 \times 10^{+10}$	$5.50 \times 10^{+10}$	1.225993	0.2304
INF	$-5.52 \times 10^{+08}$	$1.32 \times 10^{+09}$	−0.417801	0.6793
FDI	23.54488	7.642400	3.080823	0.0046
EXR	$9.47 \times 10^{+08}$	$1.75 \times 10^{+08}$	5.402133	0.0000
R-squared	0.647152	Mean dependent var		$2.72 \times 10^{+11}$
Adjusted *R*-squared	0.609347	S.D. dependent var		$1.69 \times 10^{+11}$
S.E. of regression	$1.05 \times 10^{+11}$	Akaike info criterion		53.71741
Sum squared resid	$3.11 \times 10^{+23}$	Schwarz criterion		53.90062
Log likelihood	−855.4785	Hannan-Quinn criter.		53.77814
F-statistic	17.11808	Durbin-Watson stat		0.585062
Prob(F-statistic)	0.000002			

The positive model's autocorrelation is evident, as indicated using the Durbin-Watson estimate of 0.59. The rationale is that Durbin-Watson's value needs to be between 1.5 and 2.0 in order for it to be considered acceptable and autocorrelation-free. 1.38 is outside of the range, indicating the existence of autocorrelation. Additionally, it is discovered that the model contains indications of serial autocorrelation (see **Table 13**).

Table 13. Autocorrelation removal.

Variable	Coefficient	Std. Error	*t*-Statistic	Prob.
C	$-2.41 \times 10^{+09}$	$2.99 \times 10^{+10}$	−0.080555	0.9364
INF	$4.75 \times 10^{+08}$	$6.69 \times 10^{+08}$	0.710323	0.4838
EXR	91642616	$1.25 \times 10^{+08}$	0.732702	0.4703
FDI	7.657523	4.137579	1.850726	0.0756
GDP(−1)	0.881993	0.092223	9.563738	0.0000
R-squared	0.917755	Mean dependent var		$2.79 \times 10^{+11}$
Adjusted *R*-squared	0.905102	S.D. dependent var		$1.67 \times 10^{+11}$
S.E. of regression	$5.14 \times 10^{+10}$	Akaike info criterion		52.31098
Sum squared resid	$6.87 \times 10^{+22}$	Schwarz criterion		52.54227
Log likelihood	−805.8202	Hannan-Quinn criter		52.38637
F-statistic	72.53249	Durbin-Watson stat		1.887246
Prob(F-statistic)	0.000000			

In the table above, the autocorrelation problem has been resolved by adding a lag of the reliant factor in the regression as a self-sustaining factor. Durbin-Watson test statistic of 1.89 shows this in **Table 13**.

4.5. Granger causality test

The outcome of the pairwise Granger Causality test of the variables is displayed in the **Table 14** below.

Table 14. Granger causality test between GDP and FDI.

Null Hypothesis:	Obs	F-Statistic	Prob.
FDI does not Granger Cause GDP	31	5.24891	0.0297
GDP does not Granger Cause FDI		0.95874	0.3359

Table 15 indicates that neither the GDP growth nor the exchange rate granger causes each other. **Table 16** indicates that neither the GDP growth nor the inflation rate granger causes each other.

Table 15. Granger causality test between GDP and EXR.

Null Hypothesis:	Obs	F-Statistic	Prob.
EXR does not Granger Cause GDP	31	1.85197	0.1844
GDP does not Granger Cause EXR		2.33626	0.1376

Table 16. Granger causality test between GDP and INF.

Null Hypothesis:	Obs	F-Statistic	Prob.
INF does not Granger Cause GDP	31	0.71896	0.4037
GDP does not Granger Cause INF		2.04281	0.1640

Table 14 above indicates that the FDI granger causes the GDP growth, but on the other hand, the GDP growth fails to provide the amount of foreign direct investment at the 5% level of significance.

4.6. Discussion of study

The values of *R* square, standard error, adjusted *R* square, and multiple *R* were displayed when the regression analysis and model modification of the variables were undertaken. The chosen variables' coefficient, standard error, and *P*-value. The logs of increase in GDP, inflation rate, currency rate, and foreign direct investment cointegrate; however, tests of the short- and long-term links are necessary. Only the statistically significant variables are included within the ARDL model built from the based regression findings. These data in a short-run connection suggest that the regression model appears to have strong explanatory power, a good fit, and statistical significance. The long-term relationship's goodness of fit, explanatory power, and autocorrelation in the residuals are all indicated by *R* squared, modified *R* squared, the dependent variable's mean and its standard deviation of the dependent variables, Durbin-Watson statistic, and regression standard error. Additionally, it was shown that the model contains indications of serial autocorrelation. By incorporating a lag of the dependent factor in the regression as a self-sustaining factor, the autocorrelation issue was handled.

5. Conclusion

In conclusion, the result of this research shows the existence of a long-term association involving capital market expansion together with Nigeria's economic expansion. The autoregressive distributed lag limits test indicated that the financial market's development includes a positive and considerable impact on the country's financial expansion. This proves that enhancing the financial system's efficiency as well as depth intends to help Nigeria's economy expand. The findings show that a well-developed capital market can serve as a catalyst for economic growth by allocating resources efficiently and encouraging investment. This demonstrates the significance of improving the operation and efficiency of Nigeria's financial market to boost financial expansion. To fully harness the benefits of capital sector development for long-term economic growth in Nigeria, regulatory frameworks must be strengthened, market transparency promoted, and investor trust built. As a result, policymakers should prioritize policies that encourage the advancement of the capital sector to fuel the nation's economic growth and progress. More research is needed to determine the particular routes via which the financial market influences economic expansion in Nigeria. Future investigation can be done using machine learning models to establish relationships between the financial system and GDP expansion in Nigeria [26–31].

Conflict of interest: The author declares no conflict of interest.

References

1. Han J, He J, Pan Z, et al. Twenty Years of Accounting and Finance Research on the Chinese Capital Market. Abacus. 2018; 54(4): 576-599. doi: 10.1111/abac.12143
2. Teytelboym A. Natural Capital Market Design. SSRN Electronic Journal. 2018. doi: 10.2139/ssrn.3250413
3. Hubbard RG. Capital-Market Imperfections and Investment. National Bureau of Economic Research; 1997. doi: 10.3386/w5996
4. Merton RC. A Simple Model of Capital Market Equilibrium with Incomplete Information. The Journal of Finance. 1987; 42(3): 483-510. doi: 10.1111/j.1540-6261.1987.tb04565.x
5. Zhou S, Simnett R, Green W. Does Integrated Reporting Matter to the Capital Market? SSRN Electronic Journal. 2015. doi: 10.2139/ssrn.2600364
6. Afolabi A. Impact of the Nigerian capital market on the economy. European Journal of Accounting Auditing and Finance Research. 2015; 3(2): 88-96.
7. Badertscher BA, Katz SP, Rego SO, et al. Conforming Tax Avoidance and Capital Market Pressure. The Accounting Review. 2019; 94(6): 1-30. doi: 10.2308/accr-52359
8. Briggs AP. Capital Market and Economic Growth of Nigeria. Research Journal of Finance and Accounting. 2015; 6, 82-93.
9. Aobdia D, Lin CJ, Petacchi R. Capital Market Consequences of Audit Partner Quality. Financial Accounting eJournal. 2015.
10. Baker MP, Wurgler JA. Market Timing and Capital Structure. SSRN Electronic Journal. 2001. doi: 10.2139/ssrn.267327
11. Radhakrishnan S, Wang Z, Zhang Y. Customers' Capital Market Information Quality and Suppliers' Performance. Production and Operations Management. 2014; 23(10): 1690-1705. doi: 10.1111/poms.12211
12. Brochet F, Naranjo P, Yu G. The Capital Market Consequences of Language Barriers in the Conference Calls of Non-U.S. Firms. The Accounting Review. 2016; 91(4): 1023-1049. doi: 10.2308/accr-51387
13. Adepoju AA, Ogundunmade TP. A Longitudinal and Cross-Regional Analysis of Economic Growth and its Determinants based on Bayesian Model Averaging. Journal of the Nigerian Association of Mathematical Physics. 2018; 46: 45-154.
14. Ayansola OA, Ogundunmade TP, Adedamola AO. Modelling Willingness to Pay of Electricity Supply Using Machine Learning Approach. Modern Economy and Management. 2022; 1: 9. doi: 10.53964/mem.2022009
15. Cho YJ. Segment Disclosure Transparency and Internal Capital Market Efficiency: Evidence from SFAS No. 131. Journal of Accounting Research. 2015; 53(4): 669-723. doi: 10.1111/1475-679x.12089
16. Edame GE, Okoro U. The Impact of Capital Market on Economic Growth in Nigeria. Journal of Poverty, Investment and Development. 2013; 1: 45-56.
17. Chen M, Cheng S, Hwang Y. An empirical investigation of the relationship between intellectual capital and firms' market

value and financial performance. Journal of Intellectual Capital. 2005; 6(2): 159-176. doi: 10.1108/14691930510592771
18. Guo L, Li FW, Wei KC. Security Analysts and Capital Market Anomalies. SSRN Electronic Journal. 2018. doi: 10.2139/ssrn.3101672
19. Hu GX, Pan J, Wang J. Chinese Capital Market: An Empirical Overview. SSRN Electronic Journal. 2017. doi: 10.2139/ssrn.3095056
20. Kim H, Song J. Filling institutional voids in emerging economies: The impact of capital market development and business groups on M&A deal abandonment. Journal of International Business Studies. 2016; 48(3): 308-323. doi: 10.1057/s41267-016-0025-0
21. Lee LF, Hutton AP, Shu S. The Role of Social Media in the Capital Market: Evidence from Consumer Product Recalls. SSRN Electronic Journal. 2015. doi: 10.2139/ssrn.2557212
22. Lin L. Engineering a Venture Capital Market: Lessons from China. Columbia Journal of Asian Law. 2017; 30(2): 160-220. doi: 10.52214/cjal.v30i2.9251
23. Park CY. Asian Capital Market Integration: Theory and Evidence. SSRN Electronic Journal. 2013. doi: 10.2139/ssrn.2282305
24. Sharpe WF. Capital asset prices: a theory of market equilibrium under conditions of risk. The Journal of Finance. 1964; 19(3): 425-442. doi: 10.1111/j.1540-6261.1964.tb02865.x
25. Civcir I, Akkoc U. Non-linear ARDL approach to the oil-stock nexus: Detailed sectoral analysis of the Turkish stock market. Resources Policy. 2021; 74: 102424. doi: 10.1016/j.resourpol.2021.102424
26. Ogundunmade TP, Abidoye M, Olunfunbi OM. Modelling Residential Housing Rent Price Using Machine Learning Models. Modern Economy and Management. 2023; 2: 14. doi: 10.53964/mem.2023014
27. Ogundunmade TP, Daniel AO, M. Awwal A. Modelling Infant Mortality Rate using Time Series Models. International Journal of Data Science. 2023; 4(2): 107-115. doi: 10.18517/ijods.4.2.107-115.2023
28. Ogundunmade TP, Adepoju AA. Predicting the Nature of Terrorist Attacks in Nigeria Using Bayesian Neural Network Model. In: Awe OO, Vance EA. (editors). Sustainable Statistical and Data Science Methods and Practices. STEAM-H: Science, Technology, Engineering, Agriculture, Mathematics & Health. Springer, Cham; 2023.
29. Ogundunmade TP, Adepoju AA. Modelling Liquefied Petroleum Gas Prices in Nigeria Using Time Series Machine Learning Models. Modern Economy and Management. 2022; 1: 5. doi: 10.53964/mem.2022005
30. Adepoju AA, Ogundunmade TP, Adebayo KB. Regression Methods in the presence of heteroscdesticity and outliers, Academia Journal of Scientific Research. 2017; 5(2): 776-783.
31. Ogundunmade TP, Adepoju AA. Prior Specification in Bayesian Modelling: An application to Economic Growth, Anale. Seria Informatică. 2018.

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Article

Structural corporate ownership links to corporate cash and financial earnings: New evidence from non-financial firms

Waheed Ullah Shah[1,*], Ibtissem Missaoui[2], Wajid Khan[3], Fakhrullah[1]

[1] Business School, Shandong Normal University, Jinan 250358, Shandong, PR China
[2] Higher Institute of Management of Sousse (University of Sousse), Sousse BP763-4000, Tunisia
[3] Department of Business Management, University of Baltistan, Skardu 16100, Pakistan
*** Corresponding author:** Waheed Ullah Shah, shahfin01@gmail.com, fakharkhan64@yahoo.com

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Abstract: Institutional and executive shares account for the majority of ownership and have a considerable impact on the firm's future financing, investment, earnings, and other corporate decision-making activities. This study aims to investigate the statistical links of institutional ownership and executive ownership on financial earnings and the corporate cash level of non-financial firms using a balanced dataset of 200 non-financial listed firms in PSX (Pakistan Stock Exchange) during the period 2013 to 2018. Among many advanced econometric methods, fixed effect models with pooled ordinary least square (OLS) estimation were found more appropriate to our investigation. Our main findings are twofold. The outcome of our analysis indicates that institutional ownership and executive ownership are significantly related to financial earnings. Further, our results suggest that there is a significant relationship between executive ownership and corporate cash level, as well as a positive and significant relationship between institutional ownership and finance. The cash level of firms can only be identified and predicted through executive ownership in a developing economy like Pakistan. This study provides insightful information for non-financial industry shareholders and policymakers in Pakistan.

Keywords: institutional ownership; executive ownership; financial earnings; corporate cash holding

1. Introduction

Strong corporate governance systems are becoming more and more necessary in businesses worldwide, especially in emerging economies, as a result of many financial crises and corporate scandals. Ownership has mainly affected the decisions of various companies as an important determinant in corporate governance [1–3]. The corporate ownership structure is a dynamic area of research that has attracted significant attention in corporate finance. While the ownership structure is crucial for understanding corporate accounting practices, existing literature provides limited evidence of how ownership structure affects actual earnings management within governance practices [3,4]. The present empirical literature related to the relation between ownership structure, corporate cash holding, and financial earnings has mostly come up with mixed outcomes. For instance, Attia et al. [5] have found weak or negative correlations between institutional ownership and firm financial performance. Further, Afifa et al. [6], Valent and Yanti [4] have found an insignificant association between institutional ownership and firm performance. These studies suggest that institutional investors are primarily focused on their short-term trading profits and not improving corporate governance or firm performance. However, Zhang et al. [7] have found that institutional ownership has a significant and positive effect on firm performance. Prior researchers [1,2,8,9] confirm the relationship between the two main variables. The corporate ownership structure and corporate cash holding are the key factors that can influence the decisions of the management and investors [10]. There are countless purposes behind enterprises holding money, and unquestionably one of them is related to lessening exchange costs and avoiding loss of under-investment deficiency of assets [11]. Specialists argued that high current assets are as often as possible related to little returns of speculation [12].

Specifically, a study identifying links between a CEO's authority and ability of firm earnings found that managers' ability affects firm earnings, and managers' skill can increase returns [13]. Dinh et al. [14] found that family companies perform better than non-family companies unless non-family firm CEOs have political links and family firms have either improved non-family leadership or connected political boards of directors. Ngatno et al. [15] identify that ownership structure has not moderated the links between financing and firm returns. Board independence adversely influences cash holding, showing that governance has a functioning influence in privately owned companies, though board size emphatically impacts cash holding and exhibits wasteful governance [4,16,17]. Corporate governance fundamentally affects cash holdings, but corporate governance essentially affects firm execution [6]. Firms with great governance spend less abundance of cash on interior ventures, profits, and expansion in cutthroat enterprises. The purpose of this paper is to explore the effect of structural corporate ownership on the two most important strategic decisions concerning cash and financial earnings.

Corporate ownership structure plays a crucial role in determining the governance practices of firms, particularly in influencing decision-making processes. While existing literature has extensively explored the impact of ownership concentration on firm value, there is limited evidence on how ownership structure affects actual earnings management within governance practices. This research gap is significant, as the correlation between corporate ownership structure, cash reserves, and financial performance remains largely unexplored. Therefore, the primary aim of the present study is to investigate the impact of corporate ownership structure on financial earnings and cash holdings in non-financial firms in Pakistan. The current study investigates the effect of different ownership structures, i.e., institutional ownership and executive ownership, on financial earnings and the corporate cash level on a panel of 200 non-financial Pakistani firms from 2013 to 2018.

The relationship between structural corporate ownership and financial outcomes has been widely studied, yet findings are mixed regarding its impact on cash holdings and earnings management. This paper investigated the relationship between ownership structure—particularly institutional and executive ownership—and corporate cash holdings and financial earnings among non-financial firms. Using data from emerging markets, we explore whether specific ownership patterns can influence cash management and financial performance. Non-financial firms are chosen as they offer a unique perspective on cash flow management practices in sectors that do not rely heavily on financial services. Ownership structure is a critical factor here, as non-financial firms typically manage cash with different strategic objectives, balancing between operational liquidity and investment opportunities. This paper also focuses on emerging markets, where corporate governance and ownership structure are evolving and may uniquely affect cash and earnings management. Given these factors, understanding the ownership structure's role in optimizing cash and enhancing earnings is essential for investors and policymakers aiming to progress corporate performance. We aim to provide new insights into how structural corporate ownership influences cash and earnings management within these firms, shedding light on the balance of institutional and executive ownership in driving optimal financial outcomes.

Our study contributes to existing literature in several ways. Firstly, the study contributes to the growing number of studies [2,4,9,16–21] on ownership structure, corporate cash holdings, and financial earnings by adding emerging market non-financial firms and their corporate structure effect on cash and financial earnings. However, our study also offers a further understanding of the contribution of governance to reducing the agency cost to boost value by assuring that firms' assets are implemented efficiently in the best interests of stakeholders based on agency theory. Secondly, this study is one of the few studies that scrutinizes the issue of institutional monitoring from the perspective of the quality of financial reporting, represented by financial earnings and cash holding in Asia, especially in Pakistan. Finally, this study links the empirical findings with them for a better understanding of the role of governance in determining the future performance and future cash holdings of the non-financial firms of Pakistan. Therefore, this study aims to explore the statistical relationship of corporate ownership structure with financial earnings and corporate cash holdings of the non-financial sector in Pakistan.

Our main results found that institutional ownership (IO) has a positive impact on financial earnings (FE), while executive ownership (EO) also positively affects financial earnings. However, institutional ownership (IO) has a negative effect but is statistically insignificant on corporate cash holdings (CCH), while executive ownership (EO) has a negative statistically significant effect on corporate cash holdings (CCH).

Significance of the study

Our findings indicate that a strategic balance of institutional and executive ownership can lead to optimal financial performance and cash management. Insightful for institutional investors to divert their expertise and skills to the non-financial sector for higher outcomes. Non-financial firms' management can utilize cash to generate positive cash flow through operating activities efficiently and design a diversified investment plan for a higher return with the help of this study.

This study provides valuable insights for stakeholders by revealing how institutional and executive ownership impact financial performance and cash holdings in non-financial firms. Supported by agency theory, the positive effect on financial earnings suggests that both ownership types prioritize profitability, potentially lowering agency costs. However, the negative impact of executive ownership on cash, in line with the Trade-Off Theory, indicates a preference to minimize idle funds for greater firm value. The insignificant influence of institutional ownership on cash holdings suggests a passive stance, guiding stakeholders in evaluating ownership structures' effects on financial stability and liquidity management.

The rest of this paper unfolds as follows: Section 2 covers literature review and hypothesis development. Section 3 demonstrates data and methodology. Section 4 discusses the results. Lastly, section 5 presents the conclusion.

2. Theories and literature review

2.1. Relationship between institutional ownership and financial earnings

Previous research has shown mixed and inconclusive evidence on the relationship between institutional ownership and financial earnings. Numerous studies highlight the positive influence of institutional ownership on financial earnings through effective corporate governance mechanisms. According to agency theory, institutional investors play a crucial role as monitors of management, reducing agency conflicts and improving firm performance. For instance, Jensen and Meckling [22] documented that an increase in institutional stakeholders can lead to progress in the firm's performance by improving oversight and accountability. Institutional stockholders, through their monitoring and oversight abilities, can influence financial earnings positively by ensuring that management decisions align with stakeholder interests [9]. The efficient monitoring hypothesis shows that institutional investors have more abilities and incentives to effectively monitor managers and individual investors, leading to better financial results [23]. This increased level of oversight can lead to improved corporate governance practices and increased transparency, which in turn can result in higher financial earnings [2,8,24]. This notion is proposed by the work of Shleifer and Vishny [25], who argue that institutional investors, due to their significant holdings, have the ability to investigate information and monitor managers in a way that is not possible for smaller stakeholders. Using the data of the companies in Finland, Bhattacharya and Graham [26] examined the effect of institutional ownership on firm performance. The results documented a statistically significant and positive effect of institutional ownership on firm performance. By employing data from India, Kansil and Singh [27] reported a significant and positive correlation between institutional investors and firm performance. Institutional investors boost companies to adopt good governance practices and are responsible for protecting the interests of corporate principals, leading to enhanced firm performance [8,10]. Using data of Malaysian companies, Bhattacharya and Graham [26] investigated the connection between institutional ownership, firm performance, and capital structure. The findings reveal a positive effect of institutional stakeholders on the firm's performance.

However, some studies offer opposing evidence, highlighting a negative or insignificant effect of high institutional ownership on firm performance. In the American context, Tsouknidis [28] found a negative connection between institutional ownership and firm performance, which appears mostly attributed to non-strategic rather than strategic institutional stockholders. Along the same lines, Widhiadnyana and Dwi Ratnadi [29] discovered that agency conflicts can increase with higher institutional ownership. The largest part of institutional stakeholders' holds a significant amount of shares in a company, leading them to strictly monitor and supervise the firm's performance to protect their interests and investment.

However, it is important to note that the relationship between institutional ownership and financial earnings is not always straightforward. Some studies have found that a high level of institutional ownership can also lead to increased pressure on company management to meet short-term financial targets, potentially sacrificing long-term value creation in the process. Additionally, conflicts of interest between institutional investors and company management can also arise, potentially leading to suboptimal decision-making and negative impacts on financial earnings. Nevertheless, some works documented that institutional ownership does not significantly affect financial earnings. For instance, Loderer and Martin [30] do not find any significant relationship between the level of institutional ownership and firm performance.

The present study seeks to address this gap by exploring the impact of institutional ownership on financial earnings in the context of an emerging market, where institutional ownership and governance structures may differ significantly from those in developed markets. By focusing on non-financial firms, this research provides insights into whether institutional ownership serves as an effective governance mechanism in sectors less reliant on external financing. Thus, the first hypothesis is developed as follows:

H1: Institutional ownership significantly influences the financial earnings of non-financial firms.

2.2. Relationship between executive ownership and financial earnings

In terms of the linkages between executive ownership and firm financial earnings, the evidence is also mixed. The ownership stake can affect their decision-making processes and the overall firm financial performance. Jensen and Murphy [31] found that companies with higher levels of executive ownership tend to have higher stock returns and better financial performance. This is because when executives have a significant ownership stake in the company, they are more incentivized to make good decisions that will benefit the company in the long term, rather than focusing on short-term gains. Furthermore, Aggarwal and Samwick [32] showed that executive ownership has a positive connection with firm value. They found that CEOs who hold a larger percentage of company stock tend to make decisions that are in the best interest of shareholders, leading to higher firm value and financial performance. According to Elsayed and Elbardan [33], there is a positive correlation between corporate performance and the percentage of equity ownership and equity-based compensation of managers. However, other studies have documented a negative correlation between ownership held by managers or executives and firm performance [34]. For instance, Yermack [35] found that firms with high levels of executive ownership tend to have lower financial performance. This is explained by the reason how executives may focus on personal gain rather than the long-term success of the company, leading to making decisions contradictory to the interests of shareholders. Therefore, the relationship between executive ownership and financial earnings is complex and may depend on various factors, such as the level of ownership, the industry in which the company operates, and the specific behavior of individual executives. These mixed outcomes point to a complex association between executive ownership and financial earnings, which may be influenced by determinants such as market context, firm size, and sector of activity.

The present study intends to illuminate this relationship by examining how executive ownership affects financial earnings in emerging markets, such as Pakistan. By focusing on the non-financial sector, this study investigates whether executive ownership consistently aligns managerial decisions with shareholder interests. Hence, the second hypothesis is established as follows:

H2: Executive ownership has a statistically significant impact on enhancing financial earnings.

2.3. Relationship between institutional ownership and corporate cash holdings

Institutional ownership's effect on corporate cash holding is mixed. Agency theory suggests that institutional ownership mitigates agency costs by monitoring management's use of cash resources, as managers may prefer to hoard cash for personal gain rather than shareholder value [36]. Al-Najjar and Clark [37] examined the effect of institutional stakeholders on the cash holdings of companies in the MENA region. Their findings showed that institutional ownership has a significant and positive impact on cash holdings, implying that these shareholders intend to upsurge their private profits and receive high cash. According to the pecking order theory, internal funds are more preferred to finance the investment than external funds [38]. In the Egyptian context, Basiouny et al. [39] investigate the linkage between institutional ownership and corporate cash holdings. Their result showed that institutional ownership appeared to have a significant positive impact on corporate cash holdings. Similarly, Brown et al. [40] found that short-term institutional stockholders have a positive impact on cash reserves, while long-term institutional investors have an adverse effect on cash holdings. Jebran et al. [41] demonstrated a significant and positive correlation between institutional ownership and corporate cash holdings. Also, Harford et al. [42] found a positive but non-significant relationship between institutional ownership and corporate cash holdings and offer institutional monitoring as the explication.

Some studies found a negative effect of institutional ownership on cash holdings. By analyzing a sample of 61 Egyptian-listed companies on the Egyptian Stock Exchange, Elsayed and Elbardan [33] investigated the effect of institutional ownership on cash holdings. Their result revealed that institutional ownership has a significant negative influence on cash holdings level. Further, the firms with higher levels of institutional ownership prefer to receive more cash. Similarly, by analyzing a sample of 15 listed companies on the Pakistan stock exchange, Khalil et al. [43] investigate the impact of managerial ownership on cash holdings. Their results indicate that institutional ownership has a significant and negative correlation with cash holdings, which means that if institutional ownership goes up, the cash holdings level will drop as a result of institutional investors increase and vice versa.

According to agency theory, Brown et al. [40] suggest that there is a negative relationship between cash holdings and institutional ownership. Lee and Lee [44] found a negative and significant relationship between cash holdings level and firm performance. However, previous research confirms the idea that there is no significant relationship with corporate cash holding. For instance, Alghadi et al. [21] investigated the impact of ownership structure on the cash holdings of 100 listed companies in the Saudi financial market (TADAWUL) between 2011 and 2019. Their main findings showed that institutional ownership appeared to have no direct effect on cash holdings. Thus, they indicated that institutional ownership is not a main indicator of cash holdings. Similarly, in the Jordanian context, Al-Najjar and Clark [37] demonstrated an insignificant correlation between institutional ownership and corporate cash holdings. Darma et al. [45] examined the effect of corporate governance on cash holdings of companies listed in the Indonesian stock exchange during 2015–2017. Their result revealed that institutional ownership does not have a significant relationship with corporate cash holding. A recent study conducted by Valent and Yanti [4] showed that institutional ownership has an insignificant positive effect on cash holding. A few investigations, giving evidence concerning dynamic firms of outer directors of the market response, are more emphatic than inside directors [46]. Wajid and Safi [20] reveal that institutional shareholders have no impact on cash holdings. This disagreement highlights the multifaceted role of institutional ownership in cash management, mainly in emerging markets where governance practices and investor aims may differ.

The current study adds to the literature by examining whether institutional ownership influences cash holdings in non-financial firms within an emerging market, where cash management and financing policies change from developed economies. Hence, the third hypothesis is developed as follows:

H3: Institutional ownership has a significant impact on corporate cash holdings.

2.4. Relationship between executive ownership and corporate cash holdings

The relationship between executive ownership and firm cash holdings is complex and multifaceted. Academic literature has extensively examined the relationship between executive equity ownership and corporate cash holdings, specifically through the implications of agency theory [16]. A lot of literature also described an insignificant [12,42] and significant connection between ownership executives and corporate cash holdings [16,17,47]. Ownership of executives is expected as a way to address diverging interests between owners and executives [25]. Prior research supports a positive relationship between firm performance and executive ownership [48]. This is because executives with a higher ownership stake in the company are likely to have a long-term vested interest in the firm's success. Thus, they may prefer to hold onto excess cash rather than pay dividends to shareholders or use it for investments in risky investments. Therefore, executive ownership would incentivize employees to report misconduct and serve as whistle-blowers, reducing the overall information asymmetry with outsiders [49]. Liu and Mauer [50] scrutinized the connection between CEO compensation, corporate cash holdings, and corporate firm value. They found a positive relationship between CEOs' risk-taking incentives and corporate cash holdings. Further, the theory of trade-off supports the idea that companies set their optimal level of cash holdings by weighting the marginal costs and marginal benefits of cash holdings [51].

One of the main advantages of holding cash is the ability to offer a safety net for firms, allowing them to avoid the expenses related to obtaining outside financing or liquidating current assets. This can support companies in funding their expansion opportunities [52]. On the other hand, holding onto cash can also diminish the risk of financial difficulties and empower companies to continue with their investment policy despite financial restrictions [53]. One of the key costs linked with cash holdings is the director's ability to increase and create the stockholder's wealth. If the manager fails to serve the interests of shareholders, the growth in resources under their supervision will upsurge their managerial autonomy, leading to agency costs linked to managerial discretion. The conflict between proprietors and managers, mainly regarding payout policies, can create tensions within a firm, especially for those with high cash flow [53].

Some studies have also found a negative relationship between executive ownership and firm cash holdings [54]. This may be because executives with a large ownership stake may have a personal incentive to extract cash from the company for their own benefit, rather than reinvest it back into the firm. This can lead to a reduction in cash reserves and increase the firm's risk of financial distress in the long run. Moreover, the relationship between executive ownership and firm cash holdings may also be influenced by other factors, such as firm size, industry characteristics, and corporate governance mechanisms. For example, larger firms may have higher cash holdings regardless of executive ownership levels, as they may require more liquid assets to fund their operations and investments [54]. Similarly, firms operating in industries with high levels of uncertainty or volatility may also hold more cash as a precautionary measure. In terms of corporate governance, the presence of independent directors on the board and effective monitoring mechanisms may mitigate the potential agency conflicts that can arise from high executive ownership levels. This can lead to a more optimal level of cash holdings that stabilizes the interests of executives and shareholders in maximizing firm value. This relationship is further complicated by factors like firm size and industry volatility, which may alter executives' cash management preferences.

The present study develops the subsequent hypotheses founded on the assertion supported by agency theory and trade-off theory that large stockholders do not need more comprehensive information disclosure. Thus, we tested the following hypothesis:

H4: Executive ownership significantly affects the corporate cash levels of non-financial firms.

3. Data and methodology

3.1. Data and descriptive statistics

Our research issues evaluate whether ownership structure affects financial earnings and corporate cash holding in Pakistan stock exchange-listed firms. To achieve our goals, we use a sample of top companies (which account for approximately 90% of Pakistan's stock exchange market capitalization) and corporate non-financial firms' data listed in PSX (Pakistan stock exchange) for each of the six years from 2013–2018 (based on the availability of data). We exclude financial companies due to differences in disclosure requirements of financial and non-financial sectors. Also, the choice of the sample period was driven by the availability of the dataset.

Therefore, we selected 200 companies (1200 firm-year observations). We investigate the effect of ownership structure on corporate cash and financial earnings of these 200 non-financial listed firms of PSX by using a linear regression model. Panel regression analysis, including random effect (REM) and fixed effect (FEM) models, was conducted, with the Hausman test indicating the better model for analysis. We performed a variance inflation factor test to check the multi-collinearity problem among the independent variables in our study. The multi-collinearity test checked if the independent variables of the study were highly correlated to one another. The validity of the models was verified with the help of the F-test and P-value.

Table 1 shows detailed variable measurements. Specifically, the dataset was obtained from two sources: we manually collected data from the firms' annual reports. We extracted the ownership structure data of companies by locating each firm's financial statements for each year of the sample period from their respective websites.

Table 1. Variables measurement.

Variables	Measurements
Dependent variables:	
Financial earnings (FE)	Measured by calculating the percentage of after-tax income generated by a company's investment in assets.
Corporate Cash Holdings (CCH)	Measured by calculating the proportion of cash and cash equivalent to total assets.
Independent variables:	
Institutional Ownership (IO)	Calculated as the number of shares held by institutional investors divided by the number of outstanding shares.
Executive Ownership (EO)	Measured by calculating the proportion of shares owned by company executives out of the total number of shares outstanding.
Control variables:	
Size (SZ)	Measured by the natural logarithm of total assets.
Sales Growth (SG)	Percentage increase in sales from the previous year.

Table 2 provides descriptive statistics and correlation analysis for all variables under study. The mean values show the average for each variable, with EO and CCH having negative averages, whereas EF, IO, SG, and SZ have higher positive averages. The median values are close to the means for most variables, indicating relatively symmetric distributions for some. However, the maximum and minimum values reveal substantial ranges, especially for IO and EO, indicating significant variability. Standard deviations confirm this, particularly for IO (2.002082) and EO (2.811241), which have high variability. Skewness values indicate asymmetry in the distributions, with FE, CCH, IO, and EO being negatively skewed and SG and SZ positively skewed. Kurtosis values indicate that CCH and SG have heavy tails (leptokurtic), especially SG (11.57615), while other variables are closer to a normal distribution.

Table 2 reports the pairwise correlation. The table shows that the coefficients of corporate cash holding (CCH) 0.323 and institutional ownership (IO) 0.1439 are positively associated with financial earnings. While executive ownership (EO) is negatively correlated with financial earnings (EF) and corporate cash holding (CCH). With coefficients of −0.128 and −0.413, institutional ownership (IO) and executive ownership (EO) are negatively correlated with corporate cash holding (CCH). Size and sales growth positively correlate with Pakistani firms' cash holding levels and financial earnings. Besides, no correlation coefficient spreads the level of 0.6. Thus, the findings do not present any collinearity problem for multivariate analyses.

Table 2. Descriptive statistics and correlation matrix.

	FE	**CCH**	**IO**	**EO**	**SG**	**SZ**
Mean	0.025187	−1.052424	0.247741	−1.944603	4.630373	6.951394
Median	0.028517	−0.97355	0.727549	−2.302585	4.63837	6.952347
Maximum	0.199046	−0.231214	2.870169	2.709915	6.402686	8.158862
Minimum	−0.21018	−3.223554	−4.60517	−9.21034	3.006331	5.654631
Std. Dev.	0.073403	0.478	2.002082	2.811241	0.39152	0.550968
Skewness	−0.601267	−1.201907	−0.801977	−0.208029	0.679947	0.171378
Kurtosis	3.686081	5.526516	2.927552	2.663328	11.57615	2.287019
Correlation Matrix						
	FE	CCH	IO	EO	SG	SZ
FE	1					
CCH	0.323027206	1				
IO	0.143981769	−0.128167392	1			
EO	−0.243226948	−0.413176236	−0.118290105	1		
SG	0.267006763	0.084056066	0.07349192	−0.104233623	1	
SZ	0.39667527	0.161492955	0.04721428	−0.130131188	0.143061501	1

Table 3. Collinearity statistics.

	Tolerance-FE	**VIF**	**Tolerance-CCH**	**VIF**
Institutional Ownership	.991	1.010	.991	1.010
Executive Ownership	.995	1.005	.995	1.005
Size	.905	1.106	.905	1.106
Sales Growth	.905	1.106	.905	1.106

Table 4. Heteroskedasticity test: Breusch-Pagan-Godfrey: FE.

F-statistic	1.355173	Prob. *F* (4,118)	0.2537
Obs*R-squared	5.402214	Prob. Chi-square (4)	0.2485
Scaled explained SS	8.577065	Prob. Chi-square (4)	0.0726
Heteroskedasticity test: Breusch-Pagan-Godfrey: CHH			
F-statistic	1.9434	Prob. *F* (4,118)	0.1077
Obs*R-squared	7.602174	Prob. Chi-square (4)	0.1073
Scaled explained SS	19.5902	Prob. Chi-square (4)	0.0006

Tables 3 and **4** report collinearity statistics to check for any collinearity among the independent variables of the study. Further, according to Ringle and Sarstedt [55], the tolerance and VIF (less than 10) values of independent variables are normal, indicating that these variables have no collinearity issue and statistics are consistent.

The (p-values > 0.05) suggests that we fail to reject the null hypothesis of homoscedasticity at the 5% significance level. This implies that there is no significant evidence of heteroscedasticity based on the R-square and F-statistic.

3.2. Ordinary least square (OLS) estimation regression

In this study, we used the Hausman test to select a random or fixed effect model. Its null hypothesis is that the preferred model is a random effect. Ordinary Least Squares (OLS) estimation for Fixed Effects Model (FEM), and Random Effects Model (REM) has been utilized in numerous recent studies. These methods implied a model with a varying interception in each cross-section, whereas the slope remains constant over time [56]. However, some recent studies show that the OLS estimations, fixed, and random effect models outperform cross-sectional time series and other models and gained popularity in numerous recent studies as effective tools for analyzing panel datasets [2,3,17]. The random effect model, in particular, is useful for panel data analysis as it accounts for correlated residual variables between subjects and time points, overcoming the limitations of fixed effects models by incorporating dummy variables [57]. In the context of panel data regression, OLS and fixed effects models use the standard least squares method for estimation, whereas the random effects model utilizes the generalized least squares (GLS) method. Recent research [2,3,17] has shown that OLS, fixed effects, and random effects models outperform cross-sectional time series and other GARCH family models [42] in terms of statistical significance and accuracy.

The OLS estimations (estimated common effect) framework used in our study provides a comprehensive analysis of ownership structure links to corporate cash and financial earnings. This highlights the importance of using OLS estimations, fixed effects, and random effects models in panel data analysis to draw robust and efficient conclusions from complex datasets. OLS estimation analysis has been utilized for investigation while the firm has assigned numbers from 01 to 200, both big and small firms, and regression results are obtained with the help of the given equations:

$$FE_{it} = \beta_0 + \beta_1 IO_{it} + \beta_2 EO_{it} + \beta_3 SZ_{it} + \beta_4 SG_{it} + \varepsilon_{it} \quad (1)$$

$$CCH_{it} = \beta_0 + \beta_1 IO_{it} + \beta_2 EO_{it} + \beta_3 SZ_{it} + \beta_4 SG_{it} + \varepsilon_{it} \quad (2)$$

Here β_0 intercept and β_1, β_2, β_3, β_4 are coefficients. FEit, is financial earnings (ROA), CCH_{it} corporate cash holdings, IO_{it} institutional ownership, and EO_{it} executive ownership. SZ (log of total assets) is the size and SG sales growth of the firm while ε_{it} is the error term.

4. Empirical analysis

4.1. Ordinary least square—Fixed effect models

From **Table 5** below, the result of the Hausman test displays that the chi-square statistics value is 20.048 and the p-value is 0.000. This indicates that there is no systematic difference between the two models. Therefore, the most consistent and efficient estimation for the research is the fixed effect cross-sectional model. The outcome proposes that the fixed effect model is better for the sample data because the

Hausman test statistics as represented by the corresponding probability value are less than 5%. It can be seen from the table that R-squared is 0.7786. This indicates that the independent variable explains 77.86% of the variations of the model. The F-statistic is significant at the 1 percent level. Institutional ownership (IO) has a positive and statistically significant effect on financial earnings (p-value < 0.05). Executive ownership (EO) has a positive and statistically significant effect on financial earnings. Size (SZ) has a negative and statistically significant effect on financial earnings (p-value < 0.05). Sales growth (SG) is not statistically significant at the 5% level (p-value > 0.05). This statistic tests for autocorrelation in the residuals; a value close to 2 suggests no autocorrelation.

Table 5. Hausman test and fixed effect model-dependent variable: Financial Earnings FE.

Correlated random effects—Hausman test				
Test summary		**Chi-Sq. statistic**	**Chi-Sq. d.f.**	**Prob.**
Cross-section random		20.048047	4	0.0005
Dependent variable: financial earnings FE				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.504130	0.987854	1.522623	0.1291
IO	0.011879	0.004534	2.619685	0.0093
EO	0.092257	0.036109	2.554969	0.0112
SZ	−3.953105	1.162937	−3.399244	0.0008
SG	0.190513	0.134561	1.415813	0.1581
	Effects Specification			
Cross-section fixed (dummy variables)				
R-squared	0.778672	Mean dependent var		−1.354229
Adjusted R-squared	0.665768	S.D. dependent var		0.542326
S.E. of regression	0.313534	Akaike info criterion		0.782459
Sum squared resid	24.28098	Schwarz criterion		2.115027
Log likelihood	−19.31983	Hannan-Quinn criter.		1.311550
F-statistic	6.896763	Durbin-Watson stat		1.954337
Prob (F-statistic)	0.000000			

Overall, the Hausman test indicates that the fixed effects model is appropriate. The fixed effects model results show significant positive impacts of IO and EO on financial earnings, a significant negative impact of SZ, and an insignificant effect of SG. The overall model fits the data well, explaining a substantial portion of the variance in financial earnings. Hence, it shows that the financial earnings (FE) of non-financial firms can be predicted through executive shares held (EO) as well as with institutional shares held (IO) plus the size of the firm (SZ) (control variable) in the industry.

Further, in **Table 6**, the fixed effect model has been accepted again because the null hypothesis is rejected (p < 0.05). The value (R-square 82.59%) indicates the variance in corporate cash holdings is explained by the model and adjusted R-square for the number of predictors, indicating a good fit of the model to the data. The model is statistically significant overall (the p-value for the F-statistic is 0.000). Here our independent variable IO is insignificant, and EO is significantly related to CCH ($p < 0.05$). So, it indicated that the corporate cash levels of non-financial organizations can be analyzed through shares held by executive EOs and with sales growth SG (control variable) of the firms in the market. Management and the board of the firm can efficiently manage its corporate cash holdings for higher returns. The fixed effects model shows significant negative impacts of the constant term and EO on corporate cash holdings and a significant positive impact of SG. IO and SZ are not statistically significant. A similar study observed that SZ is insignificant in financial earnings [58]. The model explains a substantial portion of the variance in corporate cash holdings.

Finally, the robust least squares highlight the significance level of all the study variables in both the models in **Table 7**.

Table 6. Hausman test and fixed effect model-dependent variable: Corporate cash holdings CCH.

Correlated random effects—Hausman test				
Test summary		**Chi-Sq. statistic**	**Chi-Sq. d.f.**	**Prob.**
Cross-section random		16.978404	4	0.0020
Dependent Variable: Corporate Cash Holdings CCH				
Variable	Coefficient	Std. Error	t−Statistic	Prob.
C	−0.861345	0.347209	−2.480766	0.0135
IO	−0.001692	0.001542	−1.097398	0.2731
EO	−0.020407	0.007150	−2.854303	0.0045
SZ	0.305465	0.423014	0.722116	0.4706
SG	0.085672	0.025104	3.412680	0.0007
	Effects Specification			
Cross-section fixed (dummy variables)				
R-squared	0.825853	Mean dependent var		−0.455874
Adjusted R-squared	0.759344	S.D. dependent var		0.255223
S.E. of regression	0.125204	Akaike info criterion		−1.087676
Sum squared resid	6.239047	Schwarz criterion		0.109593
Log likelihood	452.6548	Hannan-Quinn criter.		−0.619840
F-statistic	12.41723	Durbin-Watson stat		0.966196
Prob (F-statistic)	0.000000			

Table 7. Robust least squares (robustness tests).

Dependent variable: FE				
Variable	**Coefficient**	**Std. error**	**z-statistic**	**Prob.**
C	−0.46676	0.084839	−5.50169	0
IO	0.003154	0.002617	1.204991	0.2282
EO	−0.00461	0.001881	−2.45215	0.0142
SG	0.062593	0.013472	4.646209	0

Table 7. (*Continued*).

Dependent variable: FE				
Variable	**Coefficient**	**Std. error**	***z*-statistic**	**Prob.**
SZ	0.028858	0.009589	3.009363	0.0026
	Robust statistics			
R-squared	0.224889	Adjusted *R*-squared		0.198614
*R*w-squared	0.381035	Adjust *R*w-squared		0.381035
Akaike info criterion	166.2445	Schwarz criterion		181.0386
Deviance	0.345272	Scale		0.046899
Rn-squared statistic	49.87638	Prob (Rn-squared stat.)		0
	Non-robust statistics			
Mean dependent var	0.025187	S.D. dependent var		0.073403
S.E. of regression	0.066814	Sum squared resid		0.526766
Dependent variable: CCH				
Variable	Coefficient	Std. error	*z*-statistic	Prob.
C	−1.40297	0.552458	−2.53951	0.0111
IO	−0.02457	0.017043	−1.44148	0.1495
EO	−0.08651	0.012247	−7.06393	0
SG	−0.03611	0.087727	−0.41162	0.6806
SZ	0.056668	0.062444	0.907507	0.3641
	Robust statistics			
R-squared	0.22034	Adjusted *R*-squared		0.193911
*R*w-squared	0.354064	Adjust *R*w-squared		0.354064
Akaike info criterion	132.1506	Schwarz criterion		148.4237
Deviance	15.2329	Scale		0.349982
*R*n-squared statistic	53.37412	Prob (Rn-squared stat.)		0
	Non-robust statistics			
Mean dependent var	−1.05242	S.D. dependent var		0.478
S.E. of regression	0.437571	Sum squared resid		22.59322

4.2. Results and discussion

Tables 5 and **6** report the results of our models. The empirical tests of the main hypotheses examine the relationship between ownership structure, cash holdings, and financial earnings.

Table 5 presents the results of our model (1), which also analyzes whether a firm's ownership structure affects the levels of financial earnings, controlling for the impact of other relevant variables. We find that institutional ownership (IO) is expected to be positive and statistically significant at the 1 percent level in the case of the Financial Earnings variable (FE). This result suggests that there is a significant and positive relationship between institutional ownership and financial earnings. It suggests that as the level of institutional ownership increases by 1%, the financial earnings of the firm tend to also rise by 0.93%. This indicates that institutional ownership has a significant impact on a firm's financial performance. According to the efficient monitoring hypothesis, institutional investors possess the resources and incentives to actively monitor management, thereby reducing agency problems and contributing to higher transparency and performance. This can contribute to increasing oversight and improving corporate governance practices, which in turn can result in higher financial earnings. Studies by Koh and Jang [24]; Ali et al. [8], Din et al. [10] reported similar results, showing that institutional ownership correlates positively with financial earnings, attributed to these investors' impact on governance quality and managerial accountability. Also, this observation supports the view of agency theory, which argues that the more institutional ownership a company has, the larger the transparency level becomes. Therefore, agency problems, such as information asymmetry, are reduced, forcing managers to make decisions beneficial to shareholders rather than prioritizing their interests. Interestingly, these outcomes contrast with studies such as [4], which report an insignificant relationship between institutional ownership and financial earnings. relationship between institutional ownership (IO) and financial earnings (FE). Therefore, Hypothesis one (H1), the relationship between institutional ownership and financial earnings is statistically positively significant, was supported.

This study assumes that there is a positive and statistically significant relationship between executive ownership (EO) and financial earnings (FE) at the 5% threshold. This indicates that the increase of executive ownership is associated with higher financial earnings levels. The result is in line with our empirical studies [32,33], which illustrate that executive ownership was significantly related to financial earnings. Aggarwal and Samwick [32] found that CEOs who hold a larger percentage of company stock tend to make decisions that are in the best interest of shareholders, leading to higher earnings growth and financial performance. Therefore, the second hypothesis (H2), the relationship between Executive Ownership (EO) and financial earnings is statistically positively significant, was supported. Regarding the other control variables, we find that the size of the firm is significantly higher for firms with greater political costs (size). Only the Sales Growth (SG) has a positive and significant effect on Corporate Cash Holdings at the 1% level, whereas the firm size (SIZE) has a positive impact on Corporate Cash Holdings but is statistically non-significant.

This research projected a negative relationship between institutional ownership (IO) and corporate cash holdings. As shown in **Table 6**, institutional ownership (IO) has a negative and non-significant relationship with corporate cash holdings. The result is incoherent with our expectation and empirical study of [33,39,43]. Therefore, there are also contrary results to the works of Alghadi et al. [21] and Kusnadi and Wei [59], which found a positive and significant relationship between institutional ownership and corporate cash holdings. Their findings suggest that the presence of institutional investors in a firm indicates the possibility of an effective governance mechanism. Hence, our outcomes show that in an Asian country like Pakistan, where corporate governance is weak, institutional investors are the reason for the firms to hold less cash. Thus, hypothesis three (H3), the relationship between Executive Ownership (EO) and corporate cash holdings is not statistically significant, was rejected.

Executive ownership (EO) has a negative and significant effect on corporate cash holdings at a 1% level. It suggests that when executives have significant ownership stakes in the firms they work for, it tends to negatively impact the amount of cash that the firms hold. On the other hand, the increase in executive ownership leads to lower cash holdings. A reasonable explanation can be that executive ownership (EO) operates as a two-edged sword where elevated levels are linked with bigger short-term relative performance but also a higher probability of failure. Consequently, small business managers need to balance these trade-offs, and then little is known about what encourages managers to opt for higher or lower levels of executive ownership (EO). This result is supported by research conducted by Wang et al. [16]. The results indicate that firms subject to regulated executive compensation exhibit lower cash holdings.

Hence, hypothesis four (H4), the relationship between executive ownership (EO) and corporate cash holdings, which is statistically negatively significant, was accepted. For controlled variables, only the sales growth (SG) has a positive and significant effect on corporate cash holdings at the 1% level, whereas the firm size (SIZE) has a positive impact on corporate cash holdings but is statistically non-significant.

5. Conclusion and policy implications

This paper tests the impact of ownership structure (presented by institutional ownership and executive ownership) on firm financial earnings and corporate cash of 200 non-financial firms listed on the stock market exchange of Pakistan PSX during the period 2013 to 2018. Our dataset is balanced for firms and years. For the empirical analysis, the study employs a fixed effect model with pooled ordinary least squares (OLS) regression to encounter any endogeneity problem between ownership structure, cash holding, and financial earnings.

Our results of regression tests and analysis prove that institutional ownership (IO) and executive ownership (EO) are positively and statistically significantly related to executive ownership (EO) dependent variables, which are financial earnings (FE). Thus, the presence of institutional ownership and executive ownership has a direct impact on financial earnings. This significant relationship underlines the monitoring and governance mechanisms within the company, leading to enhanced financial performance. This suggests that institutional investors, under their stake in the firm, exert a positive impact on executive decision-making and alignment with shareholders’ interests, ultimately contributing to increased financial earnings. Further, institutional ownership (IO) has a negative and statistically insignificant connection with corporate cash holdings (CCH). Thus, we agree that in the case of an Asian country like Pakistan, the presence of institutional shareholders is active in reducing cash holdings. Furthermore, the detection of the negative relationship between institutional ownership and cash holding indicates that these investors aim to improve the value of firms and maximize their financial health. Thus, executive ownership (EO) has a negative and significant effect on corporate cash holdings (CCH). Consequently, the involvement of executive shareholders in a company’s management could result in decreased levels of cash reserves within the corporation. Additionally, executive shareholders tend to have lower corporate cash holdings because they prioritize their interests over the liquidity requirements of the company and those shareholders.

Our study contributes to the research on the relationship between ownership structure within firms and their impact on their cash holding and financial earnings in emerging economies. It has significant implications for different imminent usage of cash by shareholders, analysts, and regulators. Also, this research is highly relevant in the field of corporate finance and has practical and social implications that need to be understood and explored. The practical implication of this research can help companies make informed decisions about their capital structure and financing choices. By analyzing the ownership structure, companies can detect potential agency conflicts and managerial entrenchment that may affect cash flow. Different ownership structures may lead to differences in the concentration of power and decision-making processes within firms. So, firms and policymakers need to consider how ownership structures affect financial earnings and align them with the interests of both shareholders and other stakeholders (e.g., employees, customers, and communities). From a social perspective, understanding the implications of structural corporate ownership on cash flow and financial earnings is crucial for maintaining a fair and sustainable system.

Different ownership structures can result in variations in wealth distribution, income inequality, and corporate responsibility. For instance, the ownership concentration leads to a focus on short-term financial goals at the expense of long-term sustainability and social impact. In other words, the dispersion of ownership can result in a greater focus on shareholder value and accountability. The outcomes highlight for managers the significance of ownership in managing liquidity and financial performance, showing that executive ownership can result in reduced cash reserves to increase firm value. In order to strike a balance between immediate profitability and long-term financial stability, managers should evaluate cash allocation practices while taking ownership structures into account.

Therefore, policymakers, regulators, and stakeholders need to consider the social implications of ownership structures and their impact on cash flow and financial earnings. This may involve implementing regulations and frameworks that promote transparency, accountability, and long-term value creation. This research highlights that both institutional and executive ownership positively impact profitability, suggesting policies to encourage responsible shareholder engagement. Policymakers could foster institutional involvement while balancing governance to ensure sufficient liquidity. It also requires promoting a balance between the interests of shareholders, employees, and society as a whole. These implications are essential for companies to make informed decisions, optimize cash flow, and align ownership structures with the interests of shareholders and society. By elucidating the relationships between ownership, earnings, and cash holdings, this study provides analysts and investors with a clearer understanding of how ownership patterns impact financial results in non-financial organizations. Gaining insight into these patterns enables analysts to assess corporate governance's contribution to Pakistani firm stability and profitability more effectively and aids investors in making well-informed decisions.

Given the social implications acknowledged, future studies could examine the effect of ownership concentration on firms' CSR activities and their alignment with long-term sustainable value creation. Future research could examine cash management strategies and financial performance across diverse regulatory environments.

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Abbreviation

IO	Institutional Ownership
EO	Executive Ownership
FE	Financial Earnings
CCH	Corporate Cash Holdings
SZ	Firm Size
SG	Sales Growth
PSX	Pakistan Stock Exchange
OLS	Ordinary Least Squares
FEM	Fixed Effects Model
REM	Random Effects Model

References

1. Attia EF, Khémiri W, Mehafdi M. Does ownership structure reduce earnings manipulation practice of Egyptian listed firms? Evidence from a dynamic panel threshold model. Future Business Journal. 2023; 9(1). doi: 10.1186/s43093-023-00213-4
2. Missaoui I, Brahmi M, Rajeb JB. Ownership structure management and its effect on dividend policy in the Tunisian stock exchange enterprises: an empirical study. International Journal of Technology Transfer and Commercialisation. 2022; 19(1): 83. doi: 10.1504/ijttc.2022.123084
3. Dong N, Wang F, Zhang J, et al. Ownership structure and real earnings management: Evidence from China. Journal of Accounting and Public Policy. 2020; 39(3): 106733. doi: 10.1016/j.jaccpubpol.2020.106733
4. Valent C, Yanti Y. Institutional Ownership, Board Size, Growth Opportunities, Net Working Capital and Cash Holding. International Journal of Application on Economics and Business. 2023; 1(1): 705–715. doi: 10.24912/ijaeb.v1i1.705-715
5. Attia EF, Ismail TH, Mehafdi M. Impact of board of directors attributes on real-based earnings management: further evidence from Egypt. Future Business Journal. 2022; 8(1). doi: 10.1186/s43093-022-00169-x
6. Afifa MA, Saleh I, Haniah F. Direct and Mediated Associations Among Ownership Structure, Cash Holdings and Firm Value: The Case of Jordanian Insurance Firms. Vision: The Journal of Business Perspective. 2021; 25(4): 471–482. doi: 10.1177/0972262920983989
7. Zhang D, Ma S, Pan X. Institutional investors, controlling shareholders and CEO pay-performance relationship: evidence from China. Accounting & Finance. 2019; 61(1): 467–498. doi: 10.1111/acfi.12580
8. Ali MJ, Biswas PK, Chapple L, et al. Institutional ownership and earnings quality: Evidence from China. Pacific-Basin Finance Journal. 2024; 84: 102275. doi: 10.1016/j.pacfin.2024.102275
9. Ward C, Yin C, Zeng Y. Institutional investor monitoring motivation and the marginal value of cash. Journal of Corporate Finance. 2018; 48: 49–75. doi: 10.1016/j.jcorpfin.2017.10.017
10. Din SU, Arshad Khan M, Khan MJ, et al. Ownership structure and corporate financial performance in an emerging market: a dynamic panel data analysis. International Journal of Emerging Markets. 2021; 17(8): 1973–1997. doi: 10.1108/ijoem-03-2019-0220
11. Akhtar T. Are the motives of holding cash differing between developed and emerging financial markets? Kybernetes. 2023;

53(5): 1653–1681. doi: 10.1108/k-11-2022-1527
12. Opler T, Pinkowitz, L, Stulz, R, et al. The determinants and implications of corporate cash holdings. Journal of Financial Economics. 1999; 52(1): 3–46.
13. Salehi M, Bazrafshan A, Hosseinkamal M. The relationship between supervision quality, CEO's ability and authority with firm performance. Journal of Facilities Management. 2021; 19(2): 150–173. doi: 10.1108/jfm-08-2020-0056
14. Dinh TQ, Calabrò A, Campopiano G, et al. The Impact of Politically Connected CEOs and Boards of Directors on Firm Performance: A Study of Vietnamese Family and Nonfamily Firms. Entrepreneurship Theory and Practice. 2021; 46(5): 1284–1316. doi: 10.1177/1042258720985477
15. Ngatno, Apriatni EP, Youlianto A. Moderating effects of corporate governance mechanism on the relation between capital structure and firm performance. Cogent Business & Management. 2021; 8(1). doi: 10.1080/23311975.2020.1866822
16. Wang P, Zhang X, Wang T, et al. Executive compensation controls and corporate cash holdings. PLOS ONE. 2023; 18(9): e0285387. doi: 10.1371/journal.pone.0285387
17. Sun W, Yin C, Zeng Y. Precautionary motive or private benefit motive for holding cash: Evidence from CEO ownership. International Review of Financial Analysis. 2023; 90: 102820. doi: 10.1016/j.irfa.2023.102820
18. Gupta CP, Bedi P. Corporate cash holdings and promoter ownership. Emerging Markets Review. 2020; 44: 100718. doi: 10.1016/j.ememar.2020.100718
19. Zhou M, Cao J, Lin B. CEO organizational identification and firm cash holdings. China Journal of Accounting Research. 2021; 14(2): 183–205. doi: 10.1016/j.cjar.2021.03.001
20. Wajid A, Safi UK. Corporate Governance and Cash Holdings: Evidence from Family Controlled and Non-Family Business in Pakistan. Pakistan Journal of Applied Economics: Special Issue. 2016.
21. Alghadi MY, Al Nsour IR, Aizyadat A. Ownership Structure and Cash Holdings: Empirical Evidence from Saudi Arabia. Journal of Asian Finance, Economics and Business. 2021; 8(7): 0323–0331.
22. Jensen MC, Meckling WH. Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure. Journal of Financial Economics. 1976; 3: 305–360.
23. Shleifer A, Vishny R. Large Shareholders and Corporate Control. Journal of Political Economy. 1986; 94: 461–488.
24. Koh JH, Jang S. Cash Holdings of Hotel Firms: The Financial Constraints Perspective. In: Massachussets U (editor). Research Conference in Hospitality and Tourism; 2011.
25. Shleifer A, Vishny RW. A Survey of Corporate Governance. The Journal of Finance. 1997; 52(2): 737–783. doi: 10.1111/j.1540-6261.1997.tb04820.x
26. Bhattacharya PS, Graham MA. On institutional ownership and firm performance: A disaggregated view. Journal of Multinational Financial Management. 2009; 19(5): 370–394. doi: 10.1016/j.mulfin.2009.07.004
27. Kansil R, Singh A. Institutional ownership and firm performance: evidence from Indian panel data. International Journal of Business and Emerging Markets. 2018; 10(3): 250. doi: 10.1504/ijbem.2018.092993
28. Tsouknidis DA. The effect of institutional ownership on firm performance: the case of U.S.-listed shipping companies. Maritime Policy & Management. 2019; 46(5): 509–528. doi: 10.1080/03088839.2019.1584408
29. Widhiadnyana IK, Dwi Ratnadi NM. The impact of managerial ownership, institutional ownership, proportion of independent commissioner, and intellectual capital on financial distress. Journal of Economics, Business, & Accountancy Ventura. 2019; 21(3): 351–60. doi: 10.14414/jebav.v21i3.1233
30. Loderer C, Martin K. Executive Stock Ownership and Performance Tracking Faint TRACES. Journal of Financial Economics. 1997; 45: 223–255.
31. Jensen MC, Murphy KJ. Performance Pay and Top-Management Incentives. Journal of Political Economy. 1990; 98(2): 225–264. doi: 10.1086/261677
32. Aggarwal R, Samwick A. The Other Side of the Tradeoff: The Impact of Risk on Executive Compensation. National Bureau of Economic Research; 1998. doi: 10.3386/w6634
33. Elsayed N, Elbardan H. Investigating the associations between executive compensation and firm performance. Journal of Applied Accounting Research. 2018; 19(2): 245–270. doi: 10.1108/jaar-03-2015-0027
34. Shah SZA, Hussain Z. Impact of Ownership Structure on Firm Performance Evidence from Non-Financial Listed Companies at Karachi Stock Exchange. International Research Journal of Finance and Economics. 2012; 84: 6–13.
35. Yermack D. Higher market valuation of companies with a small board of directors. Journal of Financial Economics. 1996; 40(2): 185–211.

36. Jensen MC. Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers. American Economic Review. 1986; 76: 323–329.
37. Al-Najjar B, Clark E. Corporate governance and cash holdings in MENA: Evidence from internal and external governance practices. Research in International Business and Finance. 2017; 39: 1–12. doi: 10.1016/j.ribaf.2016.07.030
38. Yıldırım D, Çelik AK. Testing the pecking order theory of capital structure: Evidence from Turkey using panel quantile regression approach. Borsa Istanbul Review. 2021; 21(4): 317–331. doi: 10.1016/j.bir.2020.11.002
39. Basiouny MM, Mohamed E, Tammam AA. The Effect of Institutional Ownership on Corporate Cash Holdings: Evidence from Egypt. Alexandria Journal of Accounting Research. 2023; 7.
40. Brown JR, Martinsson G, Petersen BC. Do financing constraints matter for R&D? European Economic Review. 2012; 56(8): 1512–1529. doi: 10.1016/j.euroecorev.2012.07.007
41. Jebran K, Chen S, Zhang R. Board diversity and stock price crash risk. Research in International Business and Finance. 2020; 51: 101122. doi: 10.1016/j.ribaf.2019.101122
42. Harford J, Mansi SA, Maxwell WF. Corporate governance and firm cash holdings in the US. Journal of Financial Economics. 2008; 87(3): 535–555. doi: 10.1016/j.jfineco.2007.04.002
43. Khalil MS, Nadeem MA, Shahzad S. The Impact of Managerial Ownership on the Cash Holdings in the Cement Sector of Pakistan. Journal of Business & Tourism. 2016; 2: 97–105. doi: 10.34260/jbt.v2i1.32
44. Lee KW, Lee CF. Cash Holdings, Corporate Governance Structure and Firm Valuation. Review of Pacific Basin Financial Markets and Policies. 2009; 12(03): 475–508. doi: 10.1142/s021909150900171x
45. Darma NS, Lukviarman N, Setiany E. The Effect of Corporate Governance on Cash Holdings. Jurnal Dinamika Akuntansi. 2021; 13(2): 158–167. doi: 10.15294/jda.v13i2.32896
46. Fadhillah R, Burhany DI. The Mechanism of Corporate Governance, Financial Performance and Corporate Values in Sharia Companies in Indonesia. In: Proceedings of the International Conference on Creative Economics, Tourism and Information Management. doi: 10.5220/0009857500220034
47. Abdioğlu N. Managerial Ownership and Corporate Cash Holdings: Insights from an Emerging Market. Business and Economics Research Journal. 2016; 7(2): 29–29. doi: 10.20409/berj.2016217494
48. Mehran H. Executive compensation structure, ownership, and firm performance. Journal of Financial Economics. 1995; 38: 163–184.
49. Iona A, Leonida L, Ventouri A. Does executive ownership lead to excess target cash? The case of UK firms. Corporate Governance: The International Journal of Business in Society. 2017; 17(5): 876–895. doi: 10.1108/cg-02-2017-0028
50. Liu Y, Mauer DC. Corporate cash holdings and CEO compensation incentives. Journal of Financial Economics. 2011; 102(1): 183–198. doi: 10.1016/j.jfineco.2011.05.008
51. Gill A, Shah C. Determinants of Corporate Cash Holdings: Evidence from Canada. International Journal of Economics and Finance. 2011; 4(1). doi: 10.5539/ijef.v4n1p70
52. Myers S, Majluf N. Corporate Financing and Investment Decisions When Firms Have InformationThat Investors Do Not Have. National Bureau of Economic Research; 1984.
53. Ferreira MA, Vilela AS. Why Do Firms Hold Cash? Evidence from EMU Countries. European Financial Management. 2004; 10(2): 295–319. doi: 10.1111/j.1354-7798.2004.00251.x
54. Ozkan A, Ozkan N. Corporate cash holdings: An empirical investigation of UK companies. Journal of Banking & Finance. 2004; 28(9): 2103–2134. doi: 10.1016/j.jbankfin.2003.08.003
55. Ringle CM, Sarstedt M. Gain more insight from your PLS-SEM results. Industrial Management & Data Systems. 2016; 116(9): 1865–1886. doi: 10.1108/imds-10-2015-0449
56. Gujarati DN, Porter DC, Gunasekar S. Basic Econometrics. McGraw-Hill Education (India) Private Limited; 2012.
57. Widarjono A. Econometrics: Theory and Applications for Economics and Business, 2nd ed. Yogyakarta: Econisia FE Universitas Islam Indonesia; 2007.
58. Shah WU, Afridi FK. Impact Of Investment and Financial Risks on Financial Indicators of Investment and Securities Firms in Pakistan. City University Research Journal. 2021; 11(1): 108–123.
59. Kusnadi Y, Wei KCJ. The determinants of corporate cash management policies: Evidence from around the world. Journal of Corporate Finance. 2011; 17(3): 725–740.

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Article

Impact of ownership concentration on the auditor switching with modified audit opinion as mediation variable

Omid Farhad Touski

Department of Accounting, Khorramabad Branch, Islamic Azad University, Khorramabad 6817816645, Iran; O.tosski@khoiau.ac.ir

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Abstract: Motivated by recent studies that show that ownership characteristics have an effect on auditor opinion and auditor change, this research examines the effect of ownership concentration on the auditor switching with modified audit opinion as a mediation variable. The explanatory variable is ownership concentration, and the explained variable is auditor change and modified audit opinion as mediating variables. The method used for analysis is called logistic regression. The data is related to manufacturing companies listed on the Tehran Stock Exchange from 2013 to 2022. Research findings show that ownership concentration has a positive and significant effect on auditor change. Ownership concentration has a significant negative effect on the modified audit opinion. Modified audit opinion mediates between ownership concentration and auditor change. Empirical findings show that high ownership concentration may increase the probability of auditor change and decrease the probability of modified audit opinions.

Keywords: auditor switching; modified audit opinion; ownership concentration; Tehran stock exchange

1. Introduction

One of the important mechanisms of corporate governance is external audit, which has a significant impact on the quality of accounting information of companies listed on the stock exchange. Auditors complete the audit process and then comment on the companies' financial statements. Auditors' comments reduce the quality of accounting information. One of the effective external control mechanisms for corporate governance, which is increasingly important globally, is the emergence of the ownership concentration phenomenon. In terms of concentration, ownership can be dispersed (numerous small shareholders) or concentrated (a few major shareholders). Ownership concentration can be measured through the level of common stock held by majority shareholders [1]. Ownership concentration creates the authority to exercise control over the entire set of assets and objectives of the company [2]. A centralized control system is created when the ownership of the company is in the hands of major shareholders, and a decentralized system is when the ownership of the company is distributed. Ownership concentration in a company occurs when a major percentage of the shares is owned by a specific person, group, or institution. These shareholders have the potential to influence the activities of managers directly through ownership and indirectly by exchanging their shares. Auditors play a vital role in determining the validity of information, and this person should not be influenced or influenced by others under any circumstances. By studying agency theory, you can logically understand the concept of auditor change. According to Eisenhart [3], the agency theory states that human actions are based on their own interests (personal interests). In order to minimize the conflict between the agents and principals and to strengthen their trust in each other, the auditor transfer mechanism can be used. Because it is possible that by not changing the auditor, a dependency is created between the auditor and the managers, and the auditors act according to the opinion of the managers, and in this case, the interests of the shareholders are neglected. According to the signal theory, the change of auditor by clients is a signal of the quality and reliability of financial statements [4]. Lennox [5] states that companies change auditors to get rid of the current auditor's opinion and hire a new auditor with a new opinion, which is called opinion shopping. Opinion shopping is conducted by companies to avoid going-concern audit opinions. Companies can change auditors (auditor switching) to avoid receiving a going-concern audit opinion. According to the research of Chen et al. [6] and Dodgson et al. [7], it was found that opinion shopping leads to changing the auditor, and opinion shopping has a positive and significant effect on changing the auditor. Pereira [8] found out that there is a significant relationship between the existence of a board of directors with financial expertise and institutional owners with the provision of an acceptable opinion by external auditors. Hu et al. [9] suggest that in listed companies with a concentrated ownership structure, there is a positive and significant relationship between the level of earnings management and the willingness to issue modified audit opinions by auditors. However, in this situation, the issuance of revised audit opinions does not lead to a change of auditor. This is because in companies with concentrated ownership, the majority or large shareholders can control the selection of accounting firms, as the large shareholders have a greater influence on the management of the company. Therefore, auditors are more likely to issue modified opinions based on actual accounting information to meet the expectations of the majority or large shareholders. Conversely, for listed companies with a dispersed ownership structure, there is no significant relationship between profitable management and the willingness to issue a modified audit opinion. Moreover, issuing a modified audit opinion may increase the likelihood of auditor change. Carey et al. [10] conclude that a revision of the auditor's opinion on going concern may lead to a change in business policy. Chow and Rice [11], Croswell [12], Citron and Toffler [13], in their research, concluded that there is a positive relationship between an auditor's modified opinion and auditor change. However, the research of Schwartz and Menon [14], Hoskins and Williams [15] shows that there is no significant relationship between the auditor's modified opinion and auditor change. D'Angelo [16] argues that this relationship can be bidirectional, so it is possible that "qualified opinions lead to auditor changes, or auditor changes lead to qualified opinions." However, the auditor's opinion can help attract investors to invest in the tire company. If the company's expectations are not met through the auditor's opinion, the company's concern about the auditor's opinion will lead to the opinion shopping. The study of Newton et al. [17] states that opinion shopping has a negative and significant effect on changing auditors. Chen et al. [6] argue that there is a positive and significant relationship between opinion purchase and auditor change. As can be seen, there is still no consensus between the results of previous studies regarding the subject of this research, including the research of Fauziyyah et al. [18], Faradilla and Yahya [19], and Putra and Suryanava [20], that the audit opinion has an effect on the change of auditors, while the research findings of Hartono and Roman [21], Vinata and Anisikorlila [22], Pavitri and Yadnyana [23] show that there is no significant relationship between the audit opinion and the change of auditors [21]. In addition, in Iran, if a company or shareholder submits a proposal to change the external auditors and statutory auditors before the expiration of the maximum term of the external auditors and auditors, it must be submitted with the reasons. The opinion of the audit committee must be notified to the Tehran Stock Exchange Organization at least 10 days before the meeting. The organization will consider the reasons for the change and publish its opinion for or against it at least 5 days before the general meeting. If not approved, changes to external and statutory auditors should be avoided.

In this research, auditors changed literature, and some concepts from agency theory or other considerations of corporate governance have been used. The added value in this study is the increase of the modified auditor's opinion variable as a mediating variable, which is expected to influence the independent variable through the mediating variable on the dependent variable. This research was done for two reasons. First, centralized and decentralized ownership can influence the auditor's opinion and consequently the auditor's change. The issue of "auditor switching" has implications for the credibility of financial reporting and monitoring costs [24]. Although there have been extensive studies on the auditor's role, due to the conflicting results of previous studies, this study re-identifies the relationship between ownership concentration and auditor change with regard to the mediating role of the auditor's modified opinion to strengthen the results of previous studies. Secondly, the reasons for the auditor's decision are not announced in the annual report, nor are the stakeholders informed, and the facts are hidden by the companies [25]. Based on this phenomenon, it is necessary and exciting to investigate the relationship between the auditor's opinions that mainly leads to discretionary change.

This research expands the theoretical literature related to auditor change and also confirms the role of agency theory in the phenomenon of voluntary auditor change. This study can also provide stakeholders with information about auditor change.

The remainder of this paper is structured as follows. Section 2 will be background, literature review, and hypotheses development, while section 3 will describe the research methods. Section 4 will report the results, and finally, section 5, discussion and conclusions, will discuss the paper.

2. Background, literature review, and hypotheses development

Agency theory expresses the agreed and achieved working relationship between principal and agent [26]. This relationship includes a conflict of interest, and this conflict of interest is mentioned as one of the reasons for changing auditors. On one side of this relationship, there is management. The management observes and is aware of the company's conditions, and most of the time, the interests and goals of the management are not in line with the interests and goals of the shareholders [27]. Therefore, the problems between company owners and managers should be solved by a third party. An independent auditor can mediate representation problems between company owners and company managers. The auditor provides a statement regarding the evaluation of the accuracy of the financial statements, and it is called an audit statement. The main objective of an external audit is to improve the quality of a company's accounting information. If the auditor's opinion is not according to the client's expectations, it often leads to a change of auditor. According to the signal theory, by changing the auditor, companies send a signal about the quality and reliability of their financial statements to the public [4]. Generally, the clients of the company like the audit opinion that shows the absence of deviation in the accounting standards and shows the fairness of the financial statements. Therefore, when the auditor issues an opinion on the financial statements that does not meet the expectations of the client, there is often an incentive for management to change the auditor. It should be noted that usually the stock price and credit of financial reports of a company that receives audit opinions contrary to management's expectations are usually reduced [28]. Ho et al. [9] found that in listed companies where the level of ownership is concentrated, there is a positive and significant relationship between the level of profit management and the willingness to issue modified audit opinions by auditors. However, there is no significant relationship between issuing modified audit opinions and auditor change. In contrast, for listed companies with a dispersed ownership structure, there is no significant relationship between high levels of earnings management and the willingness to issue modified audit opinions. In addition, there is a positive and significant relationship between the issuance of modified audit opinions and the probability of changing the auditor. Meckling and Jensen [26] stated that information asymmetry is usually higher in firms with concentrated ownership, which increases agency problems between managers and owners. Therefore, solving this situation requires an independent and high-quality audit of the company's financial statements, and this is only possible with an independent audit. An external or independent audit assures shareholders that all financial activities are based on fair duties. Lin and Liu [29] found that in companies where the chairman of the board and the CEO are the same person or where the concentration of ownership is high, they tend to change their auditor to a smaller auditor rather than a larger auditor. The findings of many researches confirm that there is a significant relationship between corporate governance (including ownership concentration) and auditor characteristics (including auditor opinion and change) [30–33].

According to the background of the research and in order to achieve the goals of the research and answer the research questions, the following hypothesis is formulated:

Hypothesis 1 (H1): There is an association between ownership concentration and auditor switching.

Hypothesis 2 (H2): There is an association between ownership concentration and modified audit opinion.

Hypothesis 3 (H3): There is an association between modified audit opinion and auditor switching.

Hypothesis 4 (H4): Modified audit opinion mediates the association between ownership concentration and auditor switching.

3. Methodology

3.1. Data and sample selection

This study's initial sample consists of all firms listed on the Tehran stock exchange from 2013 to 2022. After previous checks and to ensure the accuracy of the research data, (1) financial firms were excluded from the sample due to different investment choices, (2) firms with missing accounting data were excluded, and (3) firms whose financial year does not end at the end of March were excluded. After the exclusions and data matching, the final sample consisted of 1410 firm-year observations, representing 141 firms on the Tehran Stock Exchange for the period 2013–2022. In this paper, data on ownership concentration, modified audit opinions, and changes in auditor characteristics were collected manually from annual reports, and financial and other data were collected from the Tehran Stock Exchange database. In order to mitigate the effects of outliers, all continuous variables are winsorized at the 1st and 99th percentiles.

3.2. Variables measurement

Dependent variable—Change of auditor ($\boldsymbol{AuditorSwitch_{i,t}}$): Auditor switch is a dummy variable. If there is a change of auditor in listed companies, the auditor change (switch) is equal to 1; otherwise, the change is 0.

Independent variables—Ownership concentration ($OC_{i,t}$): The Herfindahl-Hirschman Index (HHI) has been used to measure ownership concentration. This index is calculated using the ratio of shares of the largest shareholders. HHI is constructed as a variable by summing the square of the fraction of equity held by each shareholder with at least a 5% ownership stake. In this study, a shareholder who owns at least 5% of the company's shares is considered as a large owner. The following equation shows how to measure HHI:

$$HHI_{i,t}=\sum_{i=1}^{nj}(Share)_{i,j}{}^{2} \tag{1}$$

where, $Share_{i,j}$ indicates the percentage of ownership equal to and greater than 5% is meant. The higher this index is, the greater the concentration in the company's ownership structure.

Mediation Variable—Modified audit opinion ($\boldsymbol{MAO_{i,t}}$): where MAO is a dummy variable. If the auditor issues a modified opinion, MAO equals one; otherwise, MAO equals zero. The auditor's report in the company's annual report specifies the type of audit opinion. This information is collected from the TSE library.

Control variables—based on previous research, corporate governance variables that influence the change of auditors of companies have been used as control variables [34]. The financial and corporate governance features are: Discretionary accruals ($DAcc_{i,t}$), Following the existing earnings management literature [35,36], discretionary accruals (DAcc) are used in this study to measure earnings management. Total accrual items can be divided into discretionary accruals and nondiscretionary accruals items, and it is obtained by deducting cash flows from operating activities (CFO) from net income (NI).

$$Acc_{i,t}=\left(NI_{i,t}-CFO_{i,t}\right)/\overline{TA}_{i,t} \tag{2}$$

By studying previous studies, it can be seen that the modified Jones model has been used by researchers more than other models. In this study, we also use the modified Jones model [37] to break down company-level accruals (whole) into normal accruals and discretionary accruals:

$$Acc_{i,t}/\overline{TA}_{i,t}=\alpha_1/\overline{TA}_{i,t}+\alpha_2\Delta Rev_{i,t}/\overline{TA}_{i,t}+\alpha_3 PPE_{i,t}/\overline{TA}_{i,t}+\varepsilon_{i,t} \tag{3}$$

where, $\Delta Rev_{i,t}$ is the change in sales revenues in year *t* for firm *i*, and $PPE_{i,t}$ is gross property, plant, and equipment in year *t* for firm *i*. The ordinary least square (OLS) method is used to estimate Equation (3) in cross-section for each industry and year combination. We denote the predicted values of the Jones model as normal accruals and the residuals as discretionary accruals (DAcc). In other words, the development of the Jones model started with decomposing total accruals (TA) into current accruals (CA) and noncurrent accruals (NCA). In the second step, they derive a statistical model. In the third step, the statistical model is standardized by beginning total assets (At-1). The final step of the modeling is to select proxy variables for current accruals and noncurrent accruals, respectively. The Jones model uses ΔREV as a proxy for current accruals and PPE as a proxy for noncurrent accruals. The modified Jones model slightly modifies the Jones model by replacing ΔREV with ΔCREV as a proxy for current accruals. Below are the stages in the development of the Jones model:

The first step (Decomposition of total accruals):

$$TA=CA+NCA \tag{4}$$

The second step (Transformation into a statistical model):

$$TA=\beta_0+\beta_1 CA+\beta_2 NCA+\varepsilon \tag{5}$$

The third step (Standardization by At-1 to control for size effect):

$$TA/A_{t-1}=\beta_0(1/A_{t-1})+\beta_1 CA/A_{t-1}+\beta_2 NCA/A_{t-1}+\varepsilon \tag{6}$$

The final step (Selection of proxy variables for current and noncurrent accruals):

$$TA/A_{t-1}=\beta_0(1/A_{t-1})+\beta_1\Delta REV/A_{t-1}+\beta_2 PPE/A_{t-1}+\varepsilon \tag{7}$$

Board size ($BoardSize_{i,t}$) represents the size of the board of directors. Meeting ($Meeting_{i,t}$) is the number of times the board meeting per year. Dual ($Dual_{i,t}$) is a dummy variable. If a member of the board of directors is also the CEO, Dual equals 1; otherwise, Dual equals 0. We also consider industry and year fixed effects to control for the regression results.

3.3. Regression model

The analysis of the logistic regression model was used to examine the relationship between ownership concentrations and auditor switches with modified audit opinions as mediating variables (based on the variables described below). The functional form of the logistic regression model is as follows:

$$AuditorSwitch_{i,t}=\alpha+\beta_1 OC_{i,t}+\sum_{i=1}^{4}\gamma_i control\ variables_{i,t}+\varepsilon_{i,t} \tag{8}$$

$$MAO_{i,t}=\alpha+\beta_1 OC_{i,t}+\sum_{i=1}^{4}\gamma_i control\ variables_{i,t}+\varepsilon_{i,t} \tag{9}$$

$$AuditorSwitch_{i,t}=\alpha+\beta_1 MAO_{i,t}+\sum_{i=1}^{4}\gamma_i control\ variables_{i,t}+\varepsilon_{i,t} \tag{10}$$

$$AuditorSwitch_{i,t}=\alpha+\beta_1 OC_{i,t}+\beta_1 MAO_{i,t}+\sum_{i=1}^{4}\gamma_i control\ variables_{i,t}+\varepsilon_{i,t} \tag{11}$$

where, $AuditorSwitch_{i,t}$ is the change of auditor in year *t* for firm *i*, $MAO_{i,t}$ is the modified audit opinion in year *t* for firm *i*, $OC_{i,t}$ is the ownership concentration in year *t* for firm *i*. $DAcc_{i,t}$ is the discretionary accruals in year *t* for firm *i*, $BoardSize_{i,t}$ is the size of the board of directors in year t for firm *i*. $Meeting_{i,t}$ is the number of times the board meeting in year *t* for firm *i*. $Dual_{i,t}$ is the board of directors is also the CEO, We also consider industry and year fixed effects to control for the regression results.

4. Experimental results

4.1. Descriptive statistics

Table 1 presents the descriptive statistics of all the variables. The results show that 40% of the total sample indicated the presence of a modified audit opinion and 41.2% of the total sample indicated the presence of a change of auditor. In other words, almost 40% of the firm-year (564 units) had modified audit opinions, and also approximately 41.2% (581 units) of the firm-year had change of auditor. Further, the average ownership concentration is 53%, which shows that the ownership concentration in the sample companies is neither high nor low and is within the normal range. The average discretionary accruals are 21.7%.

Table 1. Descriptive statistics.

	Variable	Obs	Mean	Median	Max	Min	S.D
1	Switch	1410	0.412	0.000	1.000	0.000	0.492
2	MAO	1410	0.400	0.000	1.000	0.000	0.490
3	OC	1410	0.530	0.620	0.884	0.140	0.257
4	DAcc	1410	0.217	0.225	0.276	0.164	0.032
5	BoardSize	1410	5.082	5.000	7.000	5.000	0.397
6	Meeting	1410	5.700	5.500	7.000	5.000	0.781
7	Dual	1410	0.800	1.000	1.000	0.000	0.400

4.2. Correlation

With the results of the correlation test, we examined the basic relationship between the variables (univariate analysis), and, according to the results of **Table 2**, we can say that there is a relationship between the variables, and we can investigate these relationships more closely. In order to calculate the correlation coefficient of research variables, the Pearson correlation coefficient is used. There is a negative correlation of 0.266 between the Modified Audit Opinion and Change of Auditor of the company, with a significance of less than 1%, which shows that the modified opinion auditor did not change the auditor. There is a positive correlation of 0.134 between the ownership concentration and change of the auditor of the company, with a significance of less than 1%, which shows that the concentration of ownership at a high level in companies has been effective in changing auditors. There is a negative correlation of 0.380 between the Modified Audit Opinion and the ownership concentration of the company, with a significance of less than 1%, which shows that the ownership concentration at a high level in companies has not been effective in the auditor's modified opinion. The correlation coefficients between all independent variables are small (with a maximum of 0.489), which indicates the absence of a collinearity problem. Also, the variance inflation factor (VIF) of independent and control variables is within the permissible limit (less than 10), and therefore there is no collinearity. All research variables (based on the generalized Dickey-Fuller test) are at the significance level.

Table 2. Pearson correlation matrix.

	1	**2**	**3**	**4**	**5**	**6**	**7**
1	1.000						
2	-0.266^{***}	1.000					
3	0.134^{***}	-0.380^{***}	1.000				
4	-0.123^{***}	-0.325^{***}	0.057^{**}	1.000			
5	0.008	−0.016	−0.029	0.031	1.000		
6	0.292^{***}	0.314^{***}	0.076^{***}	-0.177^{***}	0.002	1.000	
7	-0.093^{***}	-0.102^{***}	0.152^{***}	-0.489^{***}	−0.004	0.448^{***}	1.000

Note: *, **, and*** denote 10%, 5%, and 1% significance levels, respectively.

4.3. Multivariate analysis

In order to test the hypothesis, the estimation results of the model presented in **Table 3** have been used with the panel data approach. The logistic regression method is used to estimate the model. The logistic regression results are present in **Table 3**. The model consists of independent variables (ownership concentration), dependent variables (auditor switching), control variables (DAcc, BoardSize, Meeting, Dual) with modified audit opinion as mediation variables.

Table 3. Results of regression analyses.

Dependent variable	Switch	MOA	Switch	Switch
	(1)	**(2)**	**(3)**	**(4)**
C	-4.90^{***} (−4.42)	4.30^{***} (3.39)	-12.57^{***} (−4.50)	-14.92^{***} (−3.71)
OC	10.40^{***} (11.18)	-4.51^{***} (−10.79)	-	-13.94^{***} (−12.90)
MAO	-	-	-11.02^{***} (−17.66)	-19.56^{***} (−15.38)
DAcc	-99.04^{***} (−11.51)	-50.50^{***} (−15.45)	-120.62^{***} (−13.83)	-99.78^{***} (−8.23)
BoardSize	0.12 (0.71)	−0.13 (−0.62)	0.14 (48)	0.07 (0.20)
Meeting	4.83^{***} (12.28)	2.18^{***} (15.84)	9.46^{***} (16.83)	11.09^{***} (17.37)
Dual	-11.99^{***} (−11.85)	-4.06^{***} (−11.93)	-18.06^{***} (−17.18)	-18.54^{***} (−16.67)
Industry fixed effects	YES	YES	YES	YES
Year fixed effects	YES	YES	YES	YES
McFadden R-squared	0.28	0.43	0.71	0.80
LR statistic	534.02^{***}	818.03^{***}	1354.45^{***}	1537.52^{***}

Note: *, **, and*** denote 10%, 5%, and 1% significance levels, respectively.

Model 1 shows that ownership concentration has a positive significant effect on auditor switching. This means that companies' ownership structures are highly concentrated and tend to do auditor switching before the specified time. Model 2 shows that ownership concentration has a significant negative effect on modified audit opinion. This means that companies' ownership structures are highly concentrated and tend not to receive the auditor's modified opinion. Model 3 shows that modified audit opinion has a significant negative effect on auditor switching. This means that companies have the ability to not replace an auditor before the specified time if they get a modified audit opinion. Model 4 shows that ownership concentration and modified audit opinion have a significant negative effect on auditor switching. This means that the modified auditor opinion variable mediates the effect of ownership concentration on auditor switching.

The second way to test the mediation hypothesis is to use the Sobel test. In statistics, the Sobel test is used to test the mediation hypothesis. This test is based on the work of Sobel [38,39] and the application of the delta method. In the absence of a mediator variable, the relationship between the independent variable and the dependent variable is a direct effect. In the presence of a mediator variable, the relationship between the independent variable and the dependent variable is considered an indirect effect (**Table 4**). The presence of a third variable has an influence. The mediator variable is a mediator. Therefore, when a mediator variable is added to a regression analysis model and placed next to an independent variable, the mediator variable absorbs part of the effect of the independent variable on the dependent variable, thereby reducing the effect of the independent variable and leaving a significant state—effect of the mediator variable. The Sobel test is a special t-test that provides a way to determine whether the reduction in the effect of the independent variable after including the mediator in the model is significant and whether the mediation effect is statistically significant. According to the model (8) and the obtained coefficients, in this section, the absolute value of the number obtained from the Sobel test is compared with the number 1.96, and if the Z value is greater than 1.96, the significance of the effect of the mediator variable is confirmed. In this formula, the Z score is the Sobel test statistic, and a is the effect of the independent variable on the mediator (also called the "a path"), and b is the effect of the mediator on the dependent variable. Where control is the independent variable, S_a is the standard error of a, and S_b is the standard error of b. S_a and S_b are readily available from the statistics output. The Sobel test is calculated as follows:

Sobel test equation:

$$Z - Value = a \times b / SQRT\left(b^2 \times s_a^2 + a^2 \times s_b^2\right) \tag{12}$$

Aroian test equation:

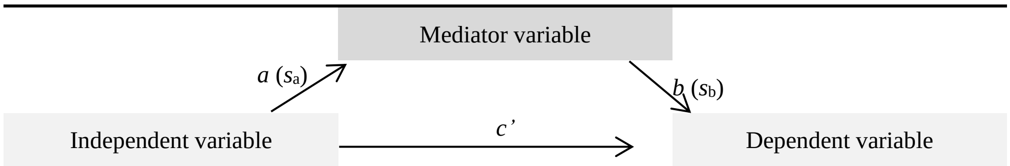
$$Z - Value = a \times b / SQRT\left(b^2 \times s_a^2 + a^2 \times s_b^2 + s_a^2 \times s_b^2\right) \tag{13}$$

Goodman test equation:

$$Z - Value = a \times b / SQRT\left(b^2 \times s_a^2 + a^2 \times s_b^2 - s_a^2 \times s_b^2\right) \tag{14}$$

$$Z - Value = -4.51 \times -19.56 / SQRT(-19.56^2 \times 0.42^2 + -4.51^2 \times 1.27^2) = 8.81 \tag{15}$$

Table 4. The role of the mediator.



Here's a simple version of your text: "A picture showing how mediation works:"

a, *b*, and *c'* are numbers that show how things are related to each other. The numbers in parentheses show the standard errors of the path coefficients.

Description of needed numbers:

A = raw (unstandardized) regression coefficient shows how the independent variable is related to the mediator.

s_a = the usual error of a.

b = the measure of how the mediator and the dependent variable are related when the independent variable also affects the dependent variable.

s_b = standard error of *b* means s_b is a measure of how much b could vary if we took many samples.

To obtain numbers:

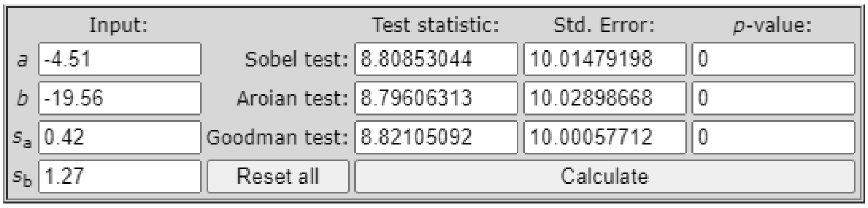
Sure. Please provide the text you would like me to rewrite in simple words. Do a regression analysis where the independent variable (IV) predicts the middle factor (mediator). This will provide a and s_a.

Sure. Please provide the text you would like me to rewrite in simple words. Do a regression analysis using the independent variable (IV) and the mediator to predict the dependent variable (DV).

This will give you *b* and s_b. Remember that s_a and s_b should always be positive.

To carry out the Sobel test:

You can find more information in the works by Baron and Kenny [40], Sobel [38], Goodman [41], and MacKinnon et al. [42]. Put the values for *a*, *b*, s_a, and s_b into the boxes below. This program will compute the critical ratio to check if the indirect effect of the independent variable (IV) on the dependent variable (DV) through the mediator is significantly different from zero. The results of the mediation tests are shown in **Figure 1**.



Input:			Test statistic:	Std. Error:	*p*-value:
a	-4.51	Sobel test:	8.80853044	10.01479198	0
b	-19.56	Aroian test:	8.79606313	10.02898668	0
s_a	0.42	Goodman test:	8.82105092	10.00057712	0
s_b	1.27	Reset all	Calculate		

Figure 1. Mediation test results.

Since the number from the Sobel test (8.81) is greater than 1.96 and less than—1.96, it shows that the effect of the mediating variable is significant.

5. Discussion and conclusion

The research objective of this study is to investigate the impact of ownership concentration on auditor switch decisions with modified audit opinions as a mediating variable. Based on a sample size of 141 manufacturing firms (1410 firm-year observations) listed on the Tehran Stock Exchange from 2013–2022, the results suggest that there is a significant positive relationship between ownership concentration and auditor switching, and this relationship is mediated by the modified audit opinion. Hence, firms with larger controlling owners (high ownership concentration) are more likely to switch to auditors. The findings of this study show the effect of ownership concentration on auditor change decisions through the mediating variable of modified audit opinion, which may provide insights to help shareholders recognize the importance of a balanced ownership structure. Ownership concentration is the deciding factor for auditor change. The finding of this study supports the principal-agent theory that high ownership concentration affects modified audit opinion and auditor switching. Consistent with previous studies [9,29,43], this study concludes that there is a significant relationship between ownership concentration and auditor switching. This study contributes to the audit literature by investigating the relationship between ownership concentration and auditor change decisions with modified audit opinions of privatized companies in emerging markets as a mediating variable. This study provides important additional evidence on the importance of ownership concentration in auditor switch and modified audit opinion, complementing other studies on auditor switch. This extends the research on auditor turnover that has mainly focused on the principal-agent theory issue and complements previous studies on the impact of ownership concentration on auditor change decisions with modified audit opinion as a mediating variable. The study recommends that future studies should consider additional ownership structure variables such as management and family ownership or use a combination of these ownership forms. Finally, they could also cover a longer period to provide a realistic picture of the topic.

Conflict of interest: The author declares no conflict of interest.

References

1. Woo ES, Koh HC. Factors associated with auditor changes: a Singapore study. Accounting and Business Research. 2001; 31(2): 133-144. doi: 10.1080/00014788.2001.9729607
2. Abubakar H. The impact of corporate governance on the productivity of a firm. Nasarawa State University, Keffi; 2010; Unpublished work.
3. Eisenhardt KM. Agency Theory: An Assessment and Review. The Academy of Management Review. 1989; 14(1): 57. doi: 10.2307/258191
4. Wati Y. Auditor Switching: New Evidence from Indonesia. The Indonesian Journal of Accounting Research. 2020; 23(01). doi: 10.33312/ijar.464
5. Lennox C. Do companies successfully engage in opinion-shopping? Evidence from the UK. Journal of accounting and economics. 2000; 29(3): 321-337.
6. Chen F, Francis JR, Hou Y. Opinion shopping through same-firm audit office switches. 2019.
7. Dodgson MK, Agoglia CP, Bennett GB, et al. Managing the Auditor-Client Relationship Through Partner Rotations: The Experiences of Audit Firm Partners. The Accounting Review. 2019; 95(2): 89-111. doi: 10.2308/accr-52556
8. Pereira AN. Corporate Governance and Quality of Accounting Information: The Relation of Content and Logit Betwing Fiscal Board and Independent Auditor Opinion in Brazilian Companies Listed at Bovespa (2006). SSRN Electronic Journal. 2009. doi: 10.2139/ssrn.1415522
9. Hu M, Muhammad A, Yang J. Ownership concentration, modified audit opinion, and auditor switch: New evidence and method. The North American Journal of Economics and Finance. 2022; 61: 101692. doi: 10.1016/j.najef.2022.101692
10. Carey PJ, Geiger MA, O'connell BT. Costs Associated with Going-Concern-Modified Audit Opinions: An Analysis of the Australian Audit Market. Abacus. 2008; 44(1): 61-81. doi: 10.1111/j.1467-6281.2007.00249.x
11. Chow CW, Rice SJ. Qualified audit opinions and auditor switching. Accounting Review. 1982: 326-335.
12. Craswell AT. The Association Between Qualified Opinions and Auditor Switches. Accounting and Business Research. 1988; 19(73): 23-31. doi: 10.1080/00014788.1988.9728832
13. Citron DB, Taffler RJ. The Audit Report under Going Concern Uncertainties: An Empirical Analysis. Accounting and Business Research. 1992; 22(88): 337-345. doi: 10.1080/00014788.1992.9729449
14. Schwartz KB, Menon K. Auditor switches by failing firms. Accounting Review. 1985: 248-261.
15. Haskins ME, Williams DD. A contingent model of intra-Big Eight auditor changes. Auditing: A Journal of Practice & Theory. 1990: 9(3).
16. DeAngelo LE. Mandated successful efforts and auditor choice. Journal of Accounting and Economics. 1982; 4(3): 171-203.
17. Newton NJ, Persellin JS, Wang D, et al. Internal Control Opinion Shopping and Audit Market Competition. The Accounting Review. 2015; 91(2): 603-623. doi: 10.2308/accr-51149
18. Fauziyyah W, Sondakh JJ, Suwetja IG. The Effect of Financial Distress, Company Size, Audit Opinion, and Cap Reputation on Voluntary Auditor Switching in Manufacturing Companies Listed on the Indonesia Stock Exchange (Indonesian). Jurnal EMBA: Jurnal Riset Ekonomi, Manajemen, Bisnis Dan Akuntansi. 2019: 7(3).
19. Faradila Y, Yahya MR. The effect of audit opinion, financial distress, and client company growth on auditor switching (study of Manufacturing companies listed on the Indonesia Stock Exchange in 2010-2014) (Indonesian). Journal Ilmiah Mahasiswa Ekonomi Akuntansi. 2016; 1(1): 81-100.
20. Putra I, Suryanawa IK. The effect of audit opinion and KAP reputation on auditor switching with financial distress as a moderating variable (Indonesian). E-Jurnal Akuntansi Universitas Udayana. 2016; 14(2): 1120-1149.

21. Suryandari D, Dwiyanti F. The Effect of Company Size and Audit Opinion on Auditor Switching with Moderated by the Auditor's Reputation. In: Proceedings of the 3rd International Conference on Economics, Business and Economic Education Science, ICE-BEES 2020; 22-23 July 2020; Semarang, Indonesia. doi: 10.4108/eai.22-7-2020.2307882
22. Winata AS, Anisykurlillah I. Analysis of Factors Affecting Manufacturing Companies in Indonesia Performing a Switching Auditor. Jurnal Dinamika Akuntansi. 2018; 9(1): 82-91. doi: 10.15294/jda.v9i1.11998
23. Pawitri NMP, Yadnyana K. The effect of audit delay, audit opinion, auditor reputation and management turnover on voluntary auditor switching (Indonesian). E-jurnal akuntansi universitas udayana. 2015; 10(1): 214-228.
24. Huson MR, Parrino R, Starks LT. Internal Monitoring Mechanisms and CEO Turnover: A Long-Term Perspective. The Journal of Finance. 2001; 56(6): 2265-2297. doi: 10.1111/0022-1082.00405
25. Nazri SNFSM, Smith M, Ismail Z. Factors influencing auditor change: evidence from Malaysia. Asian Review of Accounting. 2012; 20(3): 222-240. doi: 10.1108/13217341211263274
26. Meckling WH, Jensen MC. Theory of the Firm. Managerial Behavior, Agency Costs and Ownership Structure. Journal of Financial Economics. 1976; 3(4): 305-360
27. Lesmana K, Kurnia R. Analysis of the Effect of Management Change, Previous Year's Audit Opinion, Financial Distress, KAP Size, and Client Company Size on Voluntary Auditor Switching (Indonesian). Ultimaccounting: Jurnal Ilmu Akuntansi. 2016; 8(1): 37-52. doi: 10.31937/akuntansi.v8i1.576
28. Skinner DJ, Srinivasan S. Audit Quality and Auditor Reputation: Evidence from Japan. The Accounting Review. 2012; 87(5): 1737-1765. doi: 10.2308/accr-50198
29. Lin ZJ, Liu M. The Determinants of Auditor Switching from the Perspective of Corporate Governance in China. Corporate Governance: An International Review. 2009; 17(4): 476-491. doi: 10.1111/j.1467-8683.2009.00759.x
30. Zaid Alkilani S, Nordin Wan Hussin W, Salim B. Impact of Ownership Characteristics on Modified Audit Opinion in Jordan. International Journal of Accounting, Finance and Risk Management. 2019; 4(3): 71. doi: 10.11648/j.ijafrm.20190403.11
31. Nehme R, Michael A, Haslam J. Directors' Monitoring Role, Ownership Concentration and Audit fees. Australasian Business, Accounting & Finance Journal. 2020; 14(5): 3-25. doi: 10.14453/aabfj.v14i5.2
32. Sulimany HGH. Does Institutional Ownership Moderate the Relationship Between Audit Committee Composition and Audit Report Lag: Evidence From Saudi. Sage Open. 2024; 14(2). doi: 10.1177/21582440241241171
33. Astami E, Pramono AJ, Rusmin R, et al. Do family ownership and supervisory board characteristics influence audit report lag? A view from a two-tier board context. Journal of International Accounting, Auditing and Taxation. 2024; 56: 100638. doi: 10.1016/j.intaccaudtax.2024.100638
34. Liu Q, Lu Z. Corporate governance and earnings management in the Chinese listed companies: A tunneling perspective. Journal of Corporate Finance. 2007; 13(5): 881-906. doi: 10.1016/j.jcorpfin.2007.07.003
35. Healy PM, Wahlen JM. A Review of the Earnings Management Literature and Its Implications for Standard Setting. Accounting Horizons. 1999; 13(4): 365-383. doi: 10.2308/acch.1999.13.4.365
36. Dechow PM, Skinner DJ. Earnings Management: Reconciling the Views of Accounting Academics, Practitioners, and Regulators. Accounting Horizons. 2000; 14(2): 235-250. doi: 10.2308/acch.2000.14.2.235
37. Jones JJ. Earnings Management During Import Relief Investigations. Journal of Accounting Research. 1991; 29(2): 193. doi: 10.2307/2491047
38. Sobel ME. Asymptotic Confidence Intervals for Indirect Effects in Structural Equation Models. Sociological Methodology. 1982; 13: 290. doi: 10.2307/270723
39. Sobel ME. Some New Results on Indirect Effects and Their Standard Errors in Covariance Structure Models. Sociological Methodology. 1986; 16: 159. doi: 10.2307/270922
40. Baron RM, Kenny DA. The moderator–mediator variable distinction in social psychological research: Conceptual, strategic, and statistical considerations. Journal of Personality and Social Psychology. 1986; 51(6): 1173-1182. doi: 10.1037/0022-3514.51.6.1173
41. Goodman LA. On the Exact Variance of Products. Journal of the American Statistical Association. 1960; 55(292): 708-713. doi: 10.1080/01621459.1960.10483369
42. MacKinnon DP, Warsi G, Dwyer JH. A simulation study of mediated effect measures. Multivariate behavioral research. 1995; 30(1): 41-62.
43. Liu M. Corporate governance, auditor choice and auditor switch: Evidence from China [PhD thesis]. Hong Kong Baptist University; 2007.

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Article

Autoregressive moving average approaches for estimating continuous non-negative time series

Gisele de Oliveira Maia[1,*], Ana Julia Alves Camara[2]

[1] Institute of Pure and Applied Sciences, Federal University of Itajubá, Itajubá 35903-087, Brazil
[2] Department of Statistics, Federal University of Espírito Santo, Vitória 29075-910, Brazil
* **Corresponding author:** Gisele de Oliveira Maia, giseleemaia07@gmail.com

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Abstract: This study performs a comparative analysis of autoregressive moving average models for non-negative time series within a financial context, aiming to identify the model that offers the best fit and forecasting accuracy. The analysis is applied to two financial datasets: the stock trading volume of Banco Bradesco and the insurance volume of Porto Seguro. Four models are fitted in the process: Autoregressive Moving Average (ARMA), Rayleigh Autoregressive Moving Average (RARMA), Generalized Autoregressive Moving Average (GARMA), and Generalized Linear Autoregressive Moving Average (GLARMA). Model performance is evaluated through fit comparison metrics and forecasting accuracy measures to determine the most effective model.

Keywords: non-negative time series; stock volume; GARMA; GLARMA

1. Introduction

Statistical methods have assumed that data follow a normal distribution for many years. However, data often deviate from this pattern, showing asymmetry, kurtosis, heteroscedasticity, and non-linear trends. In finance, variables such as salaries, financial returns, and stock trade volumes often display these asymmetric behaviors, which can challenge traditional analysis methods. Forcing data into a normal distribution can distort its true structure, leading to biased estimates, incorrect conclusions, and unreliable predictions. Most financial and macroeconomic data are time series, where observations are collected over time and are not independent of each other. This lack of independence can further complicate the analysis. Therefore, it is important to evaluate the data's distribution carefully and before applying statistical methods, ensuring that the analyses are accurate and appropriately represent the nature of the data.

Linear models were frequently employed to describe random phenomena, even when the data exhibited autocorrelated observations. However, alternative methods designed for data with time-dependent structures began to emerge over time. Among these, the autoregressive moving average (ARMA) model, introduced by Box and Jenkins [1], is particularly significant. This method assumes that the data follows a normal distribution. This model has been widely used to analyze linear characteristics, such as autocorrelation, in financial time series and is considered a benchmark model. Choudhury and Jones [2] used the ARMA model to assess crop yield estimates for insurance purposes in Ghana, Tang [3] adjusted the model to predict prices of Apple Inc from 2018 to the end of 2019, Ibrahim et al. [4] used the methodology to predict price movement's direction of Bitcoin for the next 5-minute time frame, among others. Despite the extensive use of the ARMA methodology, advances in computational methods have enabled the development of new approaches to explaining various phenomena in practical situations.

However, it is crucial to use models that capture the specific characteristics of the variable of interest, such as asymmetry. New models have been developed to address this need that extend autoregressive moving average (ARMA) models to non-Gaussian time series. In this context, we can mention the Rayleigh autoregressive moving average (RARMA) model, originally proposed by Bayer et al. [5] for signal and image processing, offers an alternative for modeling continuous, asymmetric, and non-negative processes. It models the mean of Rayleigh-distributed discrete-time signals using a dynamic structure incorporating autoregressive (AR) and moving average (MA) terms, a set of regressors, and a link function. De Araújo et al. [6] considered this methodology to model stock trading volumes.

Zeger and Qaqish [7] introduced a quasi-likelihood approach to time series regression, which Benjamin et al. [8] later generalized into the generalized autoregressive moving average (GARMA) models. GARMA extends the ARMA model to non-Gaussian processes where the conditional mean (given past information in a time series setting) belongs to the exponential family. Specifically, GARMA-GAMMA is used for non-negative time series. Recently, Alcoforado et al. [9] applied the GARMA model to predict an index related to cattle spot and future prices, while de Araújo et al. [6] used the GARMA model to analyze and forecast the trading volume of Banco Bradesco S.A. (BBD) stocks. Although is an interesting alternative Zheng et al. [10] argues that unless an identity link function is used, the model's error sequence does not form a martingale difference sequence, complicating the study of the series' probabilistic properties and the asymptotic behavior of its estimators. On the other hand, for non-negative time series, GARMA models with an identity link function do not accommodate negative autocorrelation, which is quite common in real-world applications. Albarracin et al. [11] states that the structure of the GARMA model can lead to multicollinearity issues.

In parallel, Davis et al. [12] proposed the generalized linear autoregressive moving average (GLARMA) model as an extension of the generalized linear model (GLM) to handle time-dependent data, originally designed for count and discrete time series. Maia [13] later introduced the GLARMA for positive continuous processes, named GLARMA-GAMMA and GLARMA-IG, and derived some properties. In this work, the GLARMA-GAMMA and GLARMA-IG models are applied to estimate the trading volume of a Brazilian insurance company and Banco Bradesco. Although theoretical properties of the class of GLARMA models have only been rigorously established for a very limited case, Davis et al. [14] in a recent literature review highlighted that, despite this complexity, this family remains one of the most flexible and easily applicable methods.

This work aims to compare the prediction capability of the ARMA, RARMA, GARMA, and GLARMA models. To the best of our knowledge, comparisons involving the cited methods in the financial context have not yet been explored, and this work aims to fill that gap. The comparison is based on real-world data from two applications. The first focuses on forecasting the trading volume of Banco Bradesco for the period from 14 February 2022 to 10 February 2023. The second application involves forecasting the trading volume of the insurance company Porto Seguro from January 2005 to April 2023, incorporating an explanatory variable related to stock volume.

Section 2 outlines the methodologies under comparison. Section 3 presents and discusses the results of the real data applications for stock volume. Finally, Section 4 concludes the study, summarizing the main findings.

2. Materials and methods

In this section, we detail the procedures adopted to conduct this study, including the tools used and the methodology applied. Initially, we present the models employed. Next, we describe the data processing and analysis techniques, highlighting the parameters used and the criteria for model selection. Finally, we address the tools and software used to implement the analyses and validate the results.

2.1. ARMA models

The autoregressive moving average (ARMA(p, q)) model is a methodology for analyzing stationary time series based on four steps: identification, estimation, validation, and forecasting. The order p represents the autoregressive component, which captures the relationships between a value and its previous values. The order q corresponds to the moving average component, which reflects how forecast errors are incorporated into future predictions. For non-stationary time series, where the mean and variance change over time, the ARIMA(p, d, q) model is used. Here, the "I" denotes the integrated component, accounting for trends or other systematic changes in the data.

Let $\{Y_t\}_{t\in\mathbb{N}}$ be a stationary process, the ARMA model is defined as

$$\tilde{Y}_t = \phi_1\tilde{Y}_{t-1} + ... + \phi_p\tilde{Y}_{t-p} + a_t - \theta_1 a_{t-1} - ... - \theta_q a_{t-q} \tag{1}$$

where $\tilde{Y}_t = Y_t - \mu$ and a_t is an white noise. The orders p and q are identified using the autocorrelation function (ACF) and the partial autocorrelation function (PACF). The PACF measures the correlation between two observations, excluding the influence of any intermediate observations. Once the orders p and q are determined, the parameters $\boldsymbol{\phi} = (\phi_1, \phi_2, ..., \phi_p)^\top$ and $\boldsymbol{\theta} = (\theta_1, \theta_2, ..., \theta_q)^\top$ are estimated. The maximum likelihood method is widely used for parameter estimation in the model. The Akaike Information Criterion (AIC) proposed by Akaike [15]), the Bayesian Information Criterion (BIC), and the Hannan-Quinn information criteria (HQ) are used to determine the appropriate number of parameters to include in the model.

For validation, the adequacy of the estimated models can be assessed through residual analysis. According to Box et al. [16] the residuals in ARMA(p,q) are defined as

$$\hat{a}_t = \tilde{Y}_t - \hat{\phi}_1\tilde{Y}_{t-1} - ... - \hat{\phi}_p\tilde{Y}_{t-p} + \hat{\theta}_1 a_{t-1} + ... + \hat{\theta}_q a_{t-q}$$

The assumption that a_t, $t = 1, 2, .., n$ are independent must be satisfied.

2.2. RARMA models

Let $\{Y_t\}_{t\in\mathbb{N}}$ be a discrete-time signal and $\mathcal{F}_{t-1} = \sigma\{Y_s, s \leq t-1\}$ is the past of the observations up to time t. It is assumed that each Y_t conditioned to $\mathcal{F}_{t-1}$ is a Rayleigh with conditional mean μ_t, and the conditional distribution density of Y_t is given by

$$f(y_t|\mathcal{F}_{t-1}) = \frac{\pi y_t}{2{\mu_t}^2}\exp\left(-\frac{\pi {y_t}^2}{4{\mu_t}^2}\right) \tag{2}$$

where $y_t > 0$ and $\mu_t > 0$

A strictly monotonic and twice differentiable link function $g(\cdot)$ maps μ_t into a linear predictor (η_t). Thus, the structure of the RARMA model is given by

$$g(\mu_t) = \eta_t = \boldsymbol{X}_t^\top\boldsymbol{\beta} + Z_t \tag{3}$$

where $\boldsymbol{X}_t = (1, X_{1,t}, \ldots, X_{k,t})^\top$ is a k-dimensional vector of regressors observed for $t = 1, ..., n$, and $\boldsymbol{\beta} = (\beta_0, \beta_1, \ldots, \beta_k)^\top$ are the regression coefficients. The component $Z_t = \sum_{i=1}^{p}\phi_i g(y_{t-i}) + \sum_{j=1}^{q}\theta_j r_{t-j}$, where $r_{t-j} = g(y_t) - g(\mu_t)$, adds the autoregressive and moving average terms to the linear predictor, where $\boldsymbol{\phi} = (\phi_1, \phi_2, ..., \phi_p)^\top$ and $\boldsymbol{\theta} = (\theta_1, \theta_2, ..., \theta_q)^\top$ are respective parameters to be estimated. The estimators are obtained upon maximizing the conditional log-likelihood function. For more details, see Bayer et al. [5].

2.3. Gamma-GARMA models

Let $\{Y_t\}_{t\in\mathbb{N}}$ be the observations and $\mathcal{F}_{t-1} = \sigma\{Y_s, s \le t-1; X_{i,s}, 1 \le i \le k, s \le t\}$, where Y_s is the past of the observed positive process and $X_{i,s}$ is the past and present of the regressor variables. The density function of $Y_t|\mathcal{F}_{t-1}$ follows a Gamma(μ_t, ν) distribution, which can be written in exponential family as

$$f(y_t|\mathcal{F}_{t-1}) = exp[\nu(-y_t\mu_t) - log\mu_t - log\Gamma(\nu) + \nu log(\nu y_t) - logy_t] \tag{4}$$

where $y_t > 0, \nu > 0, \mu_t = E(Y_t|\mathcal{F}_{t-1})$ is positive and $\Gamma(\nu) = \int_0^\infty t^{\nu-1}e^{-t}dt$ is the gamma function.

The linear prediction expression is given by

$$g(\mu_t) = \eta_t = \boldsymbol{X}_t^\top \boldsymbol{\beta} + Z_t \tag{5}$$

where $\boldsymbol{X}_t = (1, X_{1,t}, \ldots, X_{k,t})^\top$ is a k-dimensional vector of regressors observed for $t = 1, ..., n$, and $\boldsymbol{\beta} = (\beta_0, \beta_1, \ldots, \beta_k)^\top$ are the regression coefficients. The link function used is the logarithm. The term $Z_t = \sum_{i=1}^{p} \phi_j \left\{g(y_{t-j}) - \boldsymbol{X}_t^T \boldsymbol{\beta}\right\} + \sum_{j=1}^{q} \theta_j \left\{g(y_{t-j}) - \eta_{t-j}\right\}$ captures the autoregressive and moving average components, where $\boldsymbol{\phi} = (\phi_1, \phi_2, ..., \phi_p)^\top$ and $\boldsymbol{\theta} = (\theta_1, \theta_2, ..., \theta_q)^\top$ are the corresponding parameters. Equations (4) and (5) define the GARMA-GAMMA(p, q) model. The parameters $\boldsymbol{\beta}, \boldsymbol{\phi}, \boldsymbol{\theta}$ and ν are estimated via maximum likelihood. For further details, refer to Benjamin et al. [8].

2.4. GLARMA-GAMMA and GLARMA-IG models

Let $\{Y_t\}_{t\in\mathbb{N}}$ be the positive continuous time series and $\mathcal{F}_{t-1} = \sigma\{Y_s, s \le t-1; X_{i,s}, 1 \le i \le k, s \le t\}$, denote the past information available on the response series and the past and present information on the regressors. The distribution of Y_t conditioned on $\mathcal{F}_{t-1}$ is assumed to be a Gamma(μ_t, ν) or Inverse Gaussian(μ_t, ν), where $\mu_t = E(Y_t|\mathcal{F}_{t-1})$ is positive and $\nu > 0$ is the shape parameter of the conditional distribution. The conditional density of the GLARMA-GAMMA model is defined in Equation (4), while the conditional density of the GLARMA-IG model is given by

$$f(y_t|\mathcal{F}_{t-1}) = \exp\left\{\nu\left[\frac{y_t}{2\mu_t^2} + \frac{1}{\mu_t}\right] + \frac{1}{2}\log\nu - \frac{1}{2}\log(2\pi y_t^3) + \frac{\nu}{2y_t}\right\}$$

The linear prediction expression is given by

$$g(\mu_t) = \eta_t = \boldsymbol{X}_t^\top \boldsymbol{\beta} + Z_t$$

where the logarithmic function is used as the link function, $\boldsymbol{X}_t = (1, X_{1,t}, \ldots, X_{k,t})^\top$ is a k-dimensional vector of regressors observed for $t = 1, ..., n$, and $\boldsymbol{\beta} = (\beta_0, \beta_1, \ldots, \beta_k)^\top$ denotes the regression coefficients. The term $Z_t = \sum_{i=1}^{\infty} \zeta_i e_{t-i}$ introduces the time-dependent structure into the model, with ζ_i as the parameters and e_t as the error term. After some mathematical manipulations, the term Z_t can be rewritten as

$$Z_t = \phi_1(Z_{t-1} + e_{t-1}) + \cdots + \phi_p(Z_{t-p} + e_{t-p}) + \theta_1 e_{t-1} + \cdots + \theta_q e_{t-q}$$

where $\boldsymbol{\phi} = (\phi_1, \phi_2, ..., \phi_p)^\top$ and $\boldsymbol{\theta} = (\theta_1, \theta_2, ..., \theta_q)^\top$ are the autoregressive and moving average parameters, and the error terms are

$$e_t = \frac{Y_t - \mu_t}{\mu_t} \quad \text{and} \quad e_t = \frac{Y_t - \mu_t}{\sqrt{\mu_t^3}}$$

for the Gamma and Inverse Gaussian distribution, respectively. The parameters $\boldsymbol{\beta}$, $\boldsymbol{\phi}$, $\boldsymbol{\theta}$ and ν are estimated using maximum likelihood. Refer to Davis et al. [12] and Maia [13] for more information.

3. Results and discussion

All analyses were conducted using the R software ([17]), a platform widely used for statistical analysis and time series modeling. R provides a wide range of specialized packages that were employed to fit the models. The adjustments of the RARMA and GARMA models were carried out with the help of the R-package *PTSR* (for more details, see Prass et al. [18]), while for the ARMA model, we used the R-package *stats*. In the case of the GLARMA model, we utilized custom codes developed by the authors, which are available upon request.

The selection and comparison of the models were based on the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Hannan-Quinn Information Criterion (HQ). These three measures are defined as follows respectively,

$$\begin{aligned} AIC &= -2l(\hat{\delta}) + 2r, \\ BIC &= -2l(\hat{\delta}) + r\log(n), \\ HQ &= 2l(\hat{\delta})\left(\frac{n}{n-m}\right) + r\log[\log(n)] \end{aligned} \tag{6}$$

where $l(\cdot)$ is the log-likelihood function, δ is the parameter vector estimate, n is the sample size of the time series and r is the number of parameters in the model. The models were fitted with different combinations of p and q ($\max(p, q) = 4$), with selection based on the AIC, BIC, HQ criteria, combined with the residual analysis. That is, the final selection of p and q values involves choosing the smallest values according to the criteria mentioned, ensuring that the model can capture the characteristics of the series and results in residuals that behave like white noise. Additionally, in the comparison of forecast values between the models, we used the following measures: Mean Square Error (MSE), Mean Absolute Percentage Error (MAPE), and Mean Absolute Scaled Error (MASE). The MSE, MAPE, and MASE measures can be expressed as

$$\begin{aligned} MSE &= \frac{1}{h_0}\sum_{h=1}^{h_0}(y_h - \hat{y}_h)^2, \\ MAPE &= \frac{1}{h_0}\sum_{h=1}^{h_0}\frac{|y_h - \hat{y}_h|}{|y_h|}, \\ MASE &= \frac{1}{h_0}\sum_{h=1}^{h_0}\frac{|y_h - \hat{y}_h|}{\frac{1}{h-1}\sum_{h=2}^{h_0}|y_h - y_{h-1}|} \end{aligned} \tag{7}$$

respectively, where y_h are the observed values, and $\hat{y}_h$ are the predicted values for the forecast horizon ($h = 1, \ldots, h_0$). The forecast uses a one-step approach, where the parameters are re-estimated in each step. This methodology was also adopted in the works of Agosto et al. [19], Maia et al. [20] and Mendes et al. [21].

3.1. Trading volume of Banco Bradesco

The first data set refers to the trading volume of Banco Bradesco (U.S. Dollars (US$)). The time series is available at *Yahoo Finance* website [22]. This positive continuous time series was observed from 14 February 2022 to 10 February 2023, totaling 250 observations. Since trading volumes are usually large-scale, we divided the series by one hundred million. The last thirty observations of the series were removed to obtain accuracy measures for choosing the best model. This dataset has been analyzed by de Araújo et al. [6] under the BXII autoregressive moving average (BXII-ARMA) time series model.

Table 1 presents descriptive statistics of the dataset, while **Figure 1** displays the time series of Banco Bradesco's trading volume, along with its corresponding Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF). The ACF plot indicates a positive correlation, with most observations falling outside the confidence interval (CI). In contrast, the PACF plot shows that nearly all observations remain within the CI.

Table 1. Descriptive statistics for the trading volume of Banco Bradesco.

Minimum	Median	Mean	Maximum	Variance
US$ 0.126	US$ 0.332	US$ 0.356	US$ 1.485	US$ 0.025

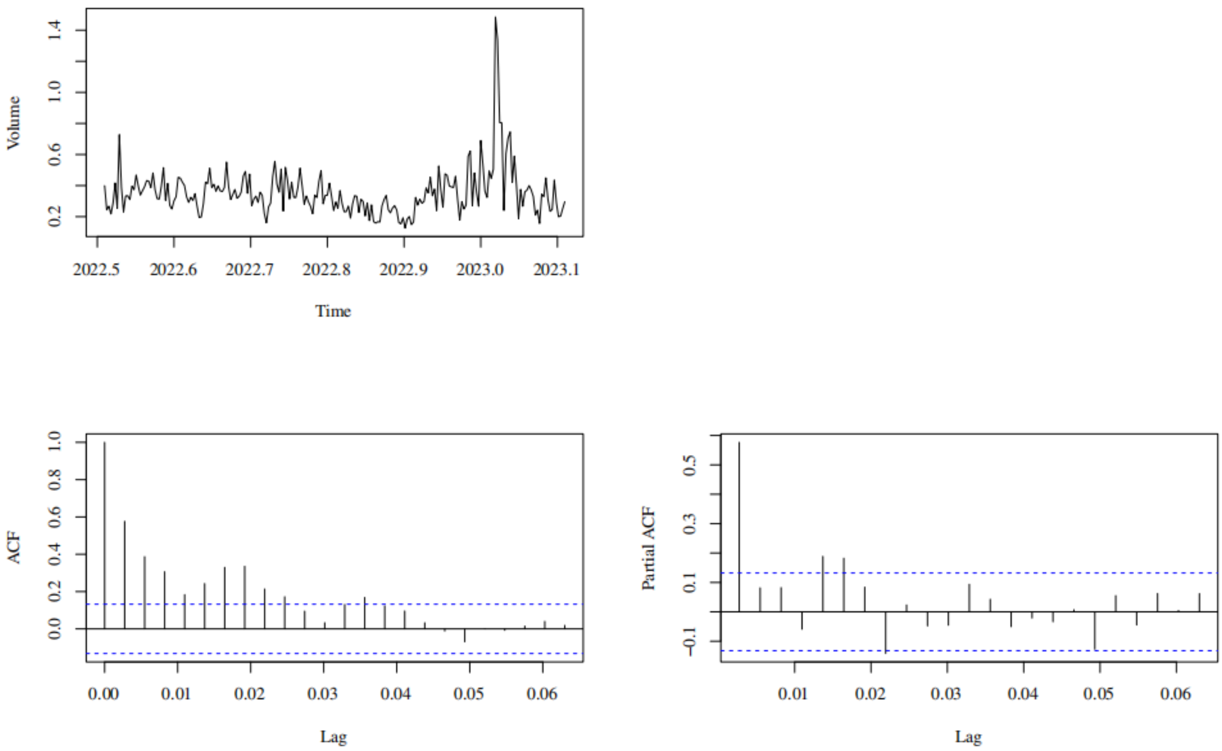

Figure 1. Plots of the trading volume of Banco Bradesco from 14 February 2022 to 10 February 2023 (top) and its associated ACF (bottom to the left) and PACF (bottom to the right).

We performed the Phillips-Perron (PP) test to verify the trading volume series's stationarity. The results confirmed that the series is stationary, with a p-value of 0.01. Subsequently, the ARMA(3,1), RARMA(0,3), Gamma-GARMA(4,4), and GLARMA-GAMMA models, with autoregressive components at lags 1, 5, and 6, were fitted to the time series. In **Table 2**, we present the parameter estimates, the standard errors (SE), and the p-value for each estimate. We observed that only the parameter ϕ_2 is not significant at the 5% level. For the Gamma-GARMA(4,4) model, only the parameters ϕ_4, θ_1, and θ_4 were significant at the 5% level. Meanwhile, for the RARMA and GLARMA models, all parameters were significant at the 5% level. **Table 3** presents the AIC, BIC, and HQ measures used to evaluate the fitted models, highlighting the GLARMA model, which exhibited the lowest values. However, it is observed that the measures for the GARMA model are quite close to those of the GLARMA model.

Table 2. Estimates, standard errors, and p values of the parameters for the ARMA, RARMA, Gamma-GARMA and GLARMA-GAMMA adjustments in the Banco Bradesco's trading volume time series.

Coef.	**Estimate**	**SE**	***p* value**	**Coef.**	**Estimate**	**SE**	***p*-value**
ARMA(3,1)				RARMA(0,3)			
Int.	0.3548	0.0225	0.0000	α	-1.2183	0.0744	0.0000
ϕ_1	-0.4062	0.0868	0.0000	θ_1	1.4985	0.1649	0.0000
ϕ_2	0.5398	0.0777	0.0589	θ_2	1.1429	0.3237	0.0004
ϕ_3	0.1370	0.0725	0.0000	θ_3	1.1465	0.2201	0.0000
θ_1	0.9574	0.0623	0.0000	-	-	-	-
Gamma-GARMA(4,4)				GLARMA-GAMMA(1,5,6)			
α	-0.3834	0.2041	0.0603	Int.	-1.0613	0.0735	0.0000
ϕ_1	0.0103	0.1038	0.9210	ϕ_1	0.4481	0.0567	0.0000
ϕ_2	0.1596	0.1195	0.1816	ϕ_5	0.1297	0.0550	0.0184
ϕ_3	0.1625	0.0874	0.0630	ϕ_6	0.1740	0.0557	0.0018
ϕ_4	0.2823	0.0813	0.0005	ν	11.7766	1.1074	0.0000
θ_1	1.1676	0.2224	0.0000	-	-	-	-
θ_2	0.5008	0.4396	0.2546	-	-	-	-
θ_3	0.4505	0.3387	0.1834	-	-	-	-
θ_4	-0.8959	0.2311	0.0001	-	-	-	-
φ	12.2942	1.1567	0.0000	-	-	-	-

Table 3. Information criteria for the best fits in each Banco Bradesco time series model class.

Model	**AIC**	**BIC**	**HQ**
ARMA(3,1)	-277.1876	-256.8258	-280.7615
RARMA(0,3)	-263.387	-249.8125	-264.6461
Gamma-GARMA(4,4)	-388.0148	-354.0785	-391.1626
GLARMA-GAMMMA(1,5,6)	-388.2893	-371.3211	-389.8632

The last 30 observations of Banco Bradesco's trading volume series were used for forecasting. **Table 4** presents the comparison metrics between the forecast values and the real observations. The results indicate that the RARMA model showed the lowest forecast accuracy measures. However, since the differences in measures between the models are small, a more in-depth analysis is needed to determine whether these differences are statistically significant. For this, first, the nonparametric Friedman test was conducted to assess whether there were significant differences between the predictions of each model, which was confirmed with a p-value of 0.022. Next, we conducted more detailed analyses, still using the Friedman test, all considering a significance level of 5%, obtaining the results: the ARMA model has forecast similar to the RARMA, GARMA, and GLARMA models, as there was no significant difference between them. The RARMA model differs from GARMA and GLARMA, indicating that the predictions of these two models are significantly different from RARMA. The GARMA model is similar to GLARMA, as there was no significant difference between them. Therefore, the main conclusion is that, although ARMA is similar to the other three models in terms of forecast, RARMA stands out with predictions different from those of GARMA and GLARMA, while GARMA and GLARMA are similar to each other. Finally, **Figure 2** compares the actual and fitted values of Banco Bradesco's trading volume, showing the close fit of the models.

Table 4. Forecasting performance comparison among different fitted models in each Banco Bradesco time series class.

Model	MSE	MAPE	MASE
ARMA(3,1)	0.159	1.138	1.882
RARMA(0,3)	0.159	1.101	1.807
Gamma-GARMA(4,4)	0.167	1.153	1.897
GLARMA-GAMMMA(1,5,6)	0.167	1.178	1.957

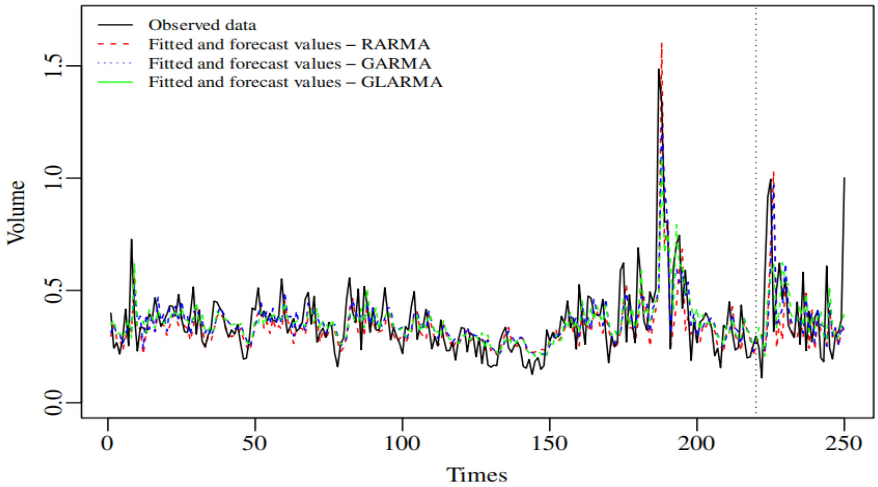

Figure 2. Observed and adjusted values of the RARMA, GLARMA and GARMA models for the Banco Bradesco time series.

3.2. Trading volume of porto seguro

We now analyze the monthly trading volume of Porto Seguro, a Brazilian insurance company, for the period from January 2005 to April 2023, comprising a total of 220 observations. The last 10 observations are reserved for forecasting, leaving $n = 210$ observations for model fitting. To manage the large scale of the data, the series was divided by one hundred million. The time series can be obtained in *Yahoo Finance* website [22]. Furthermore, in this application we use the Dollar Price covariate, which was sourced from *Investing.com* website [23].

Table 5 provides the descriptive statistics of the series, while **Figure 3** presents the time series of monthly trading volume for the analyzed period, along with the corresponding autocorrelation and partial autocorrelation functions. The ACF plot shows a positive correlation with most points outside the CI, while the PACF plot has nearly all points within the CI.

Table 5. Descriptive statistics for the trading volume of Porto Seguro.

Minimum	Median	Mean	Maximum	Variance
US$ 0.826	US$ 2.518	US$ 2.864	US$ 9.537	US$ 2.097

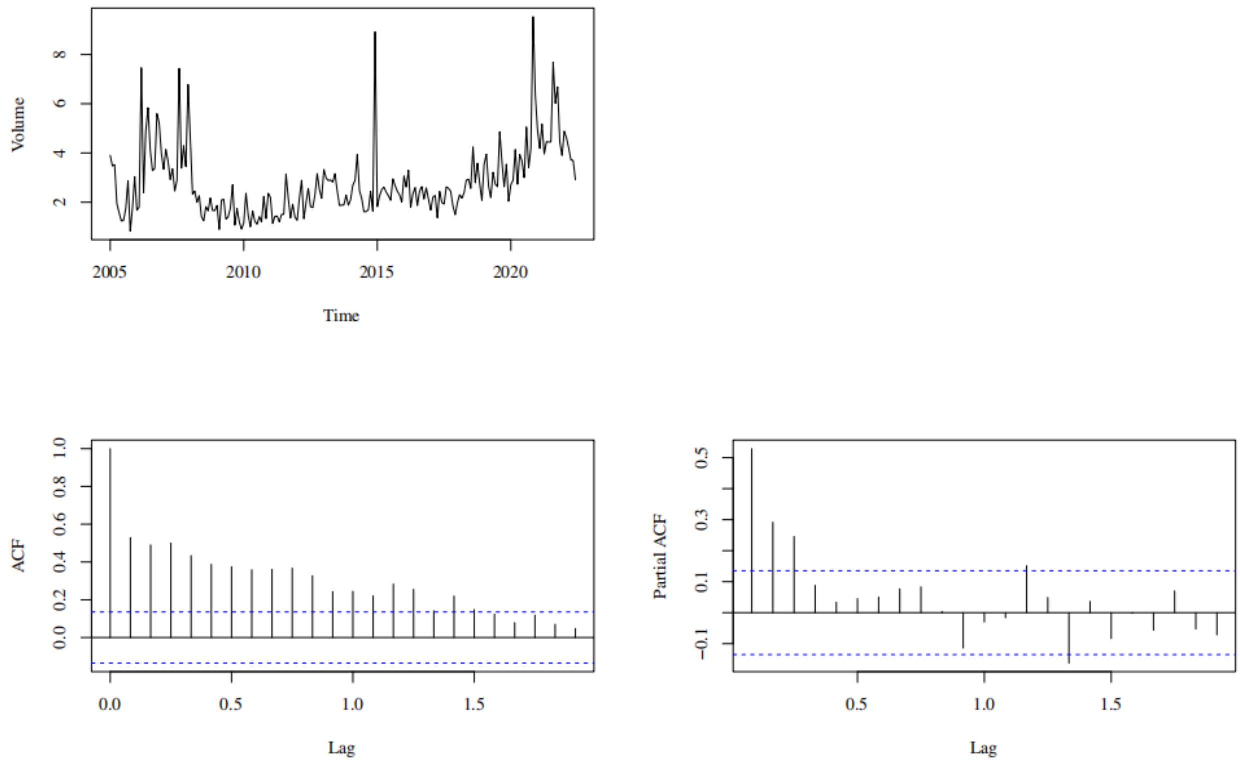

Figure 3. Plots of the monthly trading volume of Porto Alegre from January 2005 to June 2022 (top to the left) and its associated ACF (bottom to the left) and PACF (bottom to the right).

First, the PP test was conducted to check the stationarity of the series, resulting in a p-value of 0.01, indicating that the insurer's trading volume series is stationary. The following covariates were considered: the dollar opening price and a dummy variable representing the peak around August 2014 to February 2015. This peak can be explained by a 32.9% increase in net income reported by the insurer compared to the same period in the previous year. After a preliminary analysis, the following models were fitted to the data: ARMA(1,1,2), RARMA(3,0), Gamma-GARMA(3,0), and GLARMA-IG, with autoregressive components at lags 1 and 3.

Table 6 presents the parameter estimates, with respective standard errors and p-values for each model. Considering a 5% significance level, the parameter β_2 is not significant in the ARMA and Gamma-GARMA model fits, while the parameter β_1 is not significant in the RARMA model fit. In **Table 7**, we compare the fit quality of the ARMA, RARMA, GARMA and GLARMA models, noting that both, GARMA and GLARMA models, show superior performance based on lower AIC, BIC, and HQ criteria.

Finally, forecasts were made for the last 10 observations removed from the series. To evaluate the accuracy of these predictions for each model, we calculated the MSE, MAPE, and MASE measures, with the results presented in **Table 8**. **Figure 4** compares the actual trading volume observations of Porto Seguro with the predicted values. Again, the Friedman test was performed to verify the presence of significant differences between the forecast of all models, which was confirmed with a p-value of 5.283×10^{-5}. We then compared the predictions of each model using the Friedman test, with a significance level of 5%. We obtained: the ARMA model has predictions significantly different from the RARMA and GARMA models, but is similar to GLARMA. The RARMA model differs from both GARMA and GLARMA. The GARMA model is different from GLARMA. Therefore, we conclude that the predictions of the ARMA and GLARMA models show greater accuracy. The performance metrics for the RARMA model indicate the best results; however, when analyzing **Figure 4**, we observe the opposite.

Table 6. Estimates, standard errors, and p values of the parameters for the ARMA, RARMA, Gamma-GARMA and GLARMA-IG adjustments in the Porto Seguro's trading volume time series.

Coef.	**Estimate**	**SE**	***p* value**	**Coef.**	**Estimate**	**SE**	***p*-value**
ARMA(1,1,2)				RARMA(3,0)			
-	-	-	-	α	0.606	0.199	0.002
β_1	0.517	0.202	0.011	β_1	0.062	0.041	0.136
β_2	-0.386	0.596	0.517	β_2	-0.628	0.247	0.011
ϕ_1	0.919	0.047	0.0000	ϕ_1	0.075	0.035	0.031
θ_1	-1.684	0.087	0.000	ϕ_2	0.056	0.031	0.071
θ_2	0.684	0.086	0.000	ϕ_3	0.102	0.035	0.004
Gamma-GARMA(3,0)				GLARMA-IG(1,3)			
α	0.242	0.096	0.012	Int.	1.017	0.252	0.000
β_1	0.148	0.058	0.012	β_1	0.167	0.055	0.002
β_2	-0.131	0.225	0.560	β_2	-0.515	0.186	0.006
ϕ_1	0.261	0.072	0.000	ϕ_1	0.409	0.090	0.000
ϕ_2	0.158	0.070	0.024	-	-	-	-
ϕ_3	0.279	0.072	0.000	ϕ_3	0.331	0.095	0.000
ν	8.726	0.836	0.000	ν	20.595	2.010	0.000

Table 7. Information criteria for the best fits in each model class for the Porto Seguro time series.

Model	**AIC**	**BIC**	**HQ**
ARMA(1,1,2)	651.092	671.146	647.474
RARMA(3,0)	637.910	657.993	635.970
Gamma-GARMA(3,0)	549.150	572.580	546.886
GLARMA-IG(1,3)	551.822	571.905	549.882

Table 8. Forecasting performance comparison among different fitted models in each class for the Porto Seguro time series.

Model	**MSE**	**MAPE**	**MASE**
ARMA(1,1,2)	17.112	1.016	3.937
RARMA(3,0)	8.055	0.696	2.706
Gamma-GARMA(3,0)	18.336	1.052	4.082
GLARMA-IG(1,3)	16.840	1.005	3.907

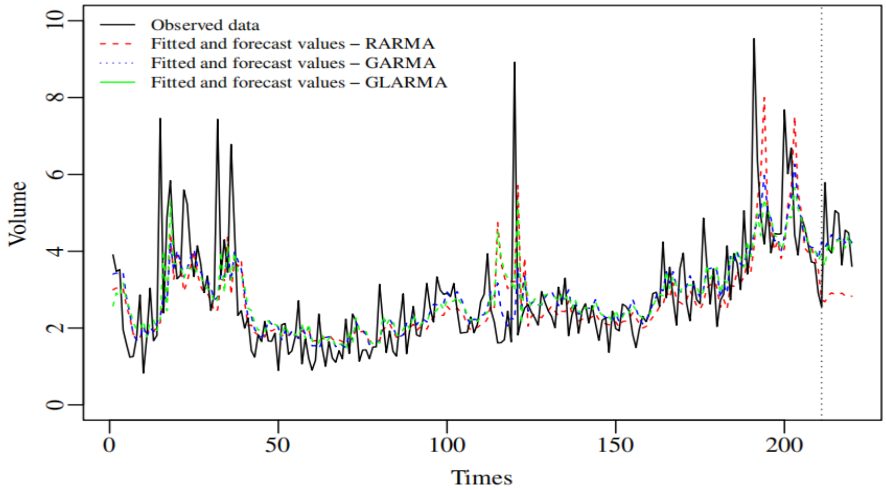

Figure 4. Observed and adjusted values of the RARMA, GARMA, and GLARMA model for the Porto Seguro time series.

4. Conclusion

The present study aimed to perform a comparative analysis of the ARMA, RARMA, GARMA, and GLARMA models applied to financial data to identify the model with the best performance in terms of fit and forecasting. Two financial time series were used, and the models were fitted and evaluated based on model selection criteria such as AIC, BIC, and HQ. Forecast accuracy measures included MSE, MAPE, and MASE.

In the first application, the GARMA and GLARMA models stood out for their goodness of fit compared to the ARMA and RARMA models, exhibiting lower values for the AIC, BIC, and HQ criteria. However, the ARMA and RARMA models presented more accurate results in forecasting future observations. In the second application, GARMA and GLARMA obtained the lowest values for the criteria, with the ARMA and GLARMA models standing out as the most accurate in forecasting future observations.

Although the GARMA and GLARMA models demonstrated a better fit to the data, as indicated by the lower values of the AIC, BIC, and HQ criteria, the ARMA and RARMA models were more effective in forecasting future observations in the trading volume of Banco Bradesco application. However, in the trading volume of Porto Seguro application, the GLARMA model was the most accurate in both fitting and forecasting, but obtaining results similar to the ARMA model in the forecast. This suggests that the choice of the ideal model depends on the objective (fitting or forecasting) and the specific context of the application.

Future research is recommended to apply these models in other financial areas and to consider additional models, such as BXII-ARMA ([6]) and CHARMA ([24]).

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References

1. Box GE, Jenkins GM. Time series analysis: forecasting and control. Holden-Day; 1976.
2. Choudhury A, Jones J. Crop yield prediction using time series models. Journal of Economics and Economic Education Research. 2014; 15(3): 53–67.
3. Tang H. Stock prices prediction based on ARMA model. In: Proceedings of the 2021 International Confer-ence on Computer, Blockchain and Financial Development (CBFD); 23–25 April 2021; Nanjing, China. doi: 10.1109/CBFD52659.2021.00046
4. Ibrahim A, Kashef R, Corrigan L. Predicting market movement direction for bitcoin: A comparison of time series modeling methods. Computers & Electrical Engineering. 2021; 89: 106905. doi: 10.1016/j.compeleceng.2020.106905
5. Bayer FM, Bayer DM, Marinoni A, Gamba P. A novel Rayleigh dynamical model for remote sensing data interpretation. IEEE Transactions on Geoscience and Remote Sensing. 2020; 58(7): 4989–4999. doi: 10.1109/TGRS.2020.2971345
6. de Araújo FJM, Guerra RR, Peña-Ramírez FA. Quantile-based dynamic modeling of asymmetric data: a novel Burr XII approach for positive continuous random variables. International Journal of Data Science and Analytics. 2024; 1–20. doi: 10.1007/s41060-024-00533-w
7. Zeger SL, Qaqish B. Markov regression models for time series: a quasi-likelihood approach. Biometrics. 1988; 1019–1031. doi: 10.2307/2531732
8. Benjamin MA, Rigby RA, Stasinopoulos DM. Generalized autoregressive moving average models. Journal of the American Statistical association. 2003; 98(461): 214–223. doi: 10.1198/016214503388619238
9. Alcoforado RG, Egídio dos Reis AD, Bernardino W, Santos JAC. Modelling Risk for Commodities in Brazil: An Application for Live Cattle Spot and Futures Prices. Commodities. 2023; 2(4): 398–416. doi: 10.3390/commodities2040023
10. Zheng T, Xiao H, Chen R. Generalized ARMA models with martingale difference errors. Journal of Econometrics. 2015; 189(2): 492–506. doi: 10.1016/j.jeconom.2015.03.040
11. Albarracin OYE, Alencar AP, Ho LL. Generalized autoregressive and moving average models: multicollinearity, interpretation and a new modified model. Journal of Statistical Computation and Simulation. 2019; 89(10): 1819–1840. doi: 10.1080/00949655.2019.1599892
12. Davis RA, Dunsmuir WT, Streett SB. Observation-driven models for Poisson counts. Biometrika. 2003; 90(4): 777–790. doi: 10.1093/biomet/90.4.777
13. Maia G. GLARMA model for temporal data analysis: Extensions for positive continuous data and a bootstrap proposal for inference on model parameters [PhD thesis]. Federal University of Minas Gerais; 2024
14. Davis RA, Fokianos K, Holan SH, et al. Count time series: A methodological review. Journal of the American Statistical Association. 2021; 116(535): 1533–1547. doi: 10.1080/01621459.2021.1904957
15. Akaike H. Information theory and an extension of the maximum likelihood principle. In: Petrov B, Csaki F (Eds.) Proceedings of the 2nd International Symposium on Information Theory. Akademiai Kiado; 1973. pp. 267–281
16. Box GE, Jenkins GM, Reinsel GC, Ljung GM. Time series analysis: forecasting and control. John Wiley & Sons; 2015.
17. R Core Team. R: A Language and Environment for Statistical Computing. R Foundation for Statistical Computing, Vienna, Austria, 2024. Available online: https://www.R-project.org/ (accessed on 20 August 2024).
18. Prass TS, Pumi G, Taufemback CG, Carlos JH. Positive time series regression models: theoretical and computational aspects. Computational Statistics. 2024; 1–31. doi: 10.1007/s00180-024-01531-z
19. Agosto A, Cavaliere G, Kristensen D, Rahbek A. Modeling corporate defaults: Poisson autoregressions with exogenous covariates (PARX). Journal of Empirical Finance. 2016; 38: 640–663. doi: 10.1016/j.jempfin.2016.02.007
20. Maia G, Barreto-Souza W, de Souza Bastos F, Ombao H. Semiparametric time series models driven by latent factor. International Journal of Forecasting. 2021; 37(4): 1463–1479. doi: 10.1016/j.ijforecast.2020.12.007
21. Mendes FG, Barreto-Souza W, Ndreca S. Gamma-Driven Markov Processes and Extensions with Application to Realized Volatility. Journal of Business & Economic Statistics. 2024; 1–13. doi: 10.1080/07350015.2024.2321375
22. Finance Y. Stock Market Live, Quotes, Business & Finance News, 2024. Available online: https://finance.yahoo.com/most-active (accessed on 1 February 2024).
23. Investing.com. Site de investimentos, Bolsas de Valores, Finanças e Economia, 2023. Available online: https://br.investing.com/currencies/usd-brl-historical-data (accessed on 1 February 2024).
24. Stone RF, Loose LH, Melo MS, Bayer FM. The Chen Autoregressive Moving Average Model for Modeling Asymmetric Positive Continuous Time Series. Symmetry. 2023; 15(9): 1675. doi: 10.3390/sym15091675

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Article

Multi-target linear shrinkage estimation of large precision matrix

Yuang Xue, Lihong Wang*

School of Mathematics, Nanjing University, Nanjing 210093, China
*** Corresponding author:** Lihong Wang, lhwang@nju.edu.cn

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Abstract: In this paper we propose a multi-target linear shrinkage estimator of the precision matrix by shrinking the inverse of the sample covariance matrix directly, which is a generalization of the single-target linear shrinkage estimator. The explicit expression of the weights of multi-target linear shrinkage estimator is derived when the ratio of the dimension p and the sample size n tends to a positive constant $c \in (0, 1)$. The numerical simulation and an empirical analysis of financial market data are provided to compare the multi-target linear shrinkage estimator with some other estimators of the precision matrix proposed in the literature. The computation results show the improvement of the multi-target linear shrinkage estimator.

Keywords: frobenius norm; linear shrinkage estimator; multiple target matrices; precision matrix

1. Introduction

The estimation of a large covariance matrix and its inverse matrix, known as the precision matrix, is central to statistical learning theory and econometrics and has been receiving growing attention from both researchers and practitioners. For example, principal component analysis and factor analysis involve covariance matrix estimation, while Fisher linear discriminant analysis needs precision matrix estimation. Moreover, to implement the mean-variance portfolio in practice, the accuracy in estimating the covariance structure of returns and its precision matrix is vital. As pointed out in Elton et al. [1] and Markowitz [2], a suitable precision matrix estimator leads to a good estimation for different types of optimal portfolios.

Various methods have been proposed to estimate the covariance matrix and the precision matrix in high-dimensional setting from different directions. Bodnar et al. [3] reviewed the recent advances in the shrinkage-based estimation for high-dimensional covariance and precision matrices. See also, e.g., Bodnar et al. [4]; Fan et al. [5]; Fan et al. [6]; Friedman et al. [7]; Ikeda et al. [8]; Kuismin et al. [9]; Ledoit and Wolf [10, 11]; Liu and Tang [12]; van Wieringen and Peeters [13]; Wang et al. [14]; and Wang et al. [15].

For the estimation of precision matrix, one direction is to construct linear shrinkage estimate of the covariance matrix, and then use its inverse as the estimator of the precision matrix. The shrinkage estimator shrinks the eigenvalues of the sample covariance matrix and forms a linear combination of the sample covariance matrix and a pre-chosen target matrix, see, e.g., Dey and Srinivasan [16], Ledoit and Wolf [17, 18]. Another approach is to shrink directly the inverse of the sample covariance matrix itself, instead of shrinking the sample covariance matrix and then inverting it. When both the sample size n and the dimension p tend to infinity but their ratio tends to a positive constant, Bodnar et al. [19] proposed a direct single target shrinkage estimator for the precision matrix.

Most of the linear shrinkage estimates in the literature use single target matrix. However, the performance of the estimator strongly depends on the choice of the target matrix (cf. Engel et al. [20] and Gray et al. [21]). In order to incorporate uncertainty about the target choice, Gray et al. [21] proposed a multi-target linear shrinkage estimator for the covariance matrix that allows for shrinkage of the sample covariance matrix towards multiple targets simultaneously. The multi-target estimator is less sensitive to target misspecification and leads to equal or improved estimates compared to single-target linear shrinkage estimators, see also Bartz et al. [22] and Lancewicki and Aladjem [23]. However, to the best of our knowledge, we have not found any multi-target shrinkage estimation methods for the precision matrix in the literature. The explicit expression of the shrinkage weights and the theoretical properties of the multi-target linear shrinkage estimation of precision matrix, when both the sample size n and the dimension p tend to infinity, are still unavailable.

Inspired by the works of Bodnar et al. [19] and Gray et al. [21], this paper considers the linear shrinkage estimation of the precision matrix based on multiple shrinkage target matrices. This multi-target shrinkage estimator forms a linear combination of the inverse of the sample covariance matrix and multiple general shrinkage target matrices. In Section 2, we derive the explicit expression of the weights of the multi-target linear shrinkage estimator in the case of $p/n \to c \in (0,1)$. In Section 3, numerical simulation and an empirical analysis of portfolio optimization are provided to compare the multi-target linear shrinkage estimator with some other estimators for the precision matrix proposed in the literature. The simulation results show the improvement of the multi-target linear shrinkage estimator. The proofs of the theorems are placed in the appendix.

2. Multi-target linear shrinkage estimator

The following notations are used throughout the paper. Let n be the sample size, $p = p(n)$ is the number of the variables. Σ_n^{-1} stands for the true precision matrix, and $\widehat{\Sigma}_n^{-1}$ denotes the estimator of Σ_n^{-1}. Since the dimension p is a function of the sample size n, we use the subscript n for the covariance matrix Σ_n which depends on n via $p(n)$. Let $H_n(t)$ denote the empirical distribution function of the eigenvalues of Σ_n.

Let Y_n be a $p \times n$ matrix which consists of independent and identically distributed (i.i.d.) random variables with zero mean and unit variance. The observation matrix is defined as $X_n = \Sigma_n^{\frac{1}{2}} Y_n$. S_n denotes the sample covariance matrix, i.e. $S_n = \frac{1}{n} X_n X_n' = \frac{1}{n} \Sigma_n^{\frac{1}{2}} Y_n Y_n' \Sigma_n^{\frac{1}{2}}$.

$\|A\|_F^2 = tr(AA')$ denotes the Frobenius norm of a matrix A, and $\frac{1}{p}\|A\|_F^2$ denotes the normalized Frobenius norm of $p \times p$ matrix A, while $\|A\|_{tr} = tr\left[(AA')^{\frac{1}{2}}\right]$ stands for the trace norm, and $\frac{1}{p}\|A\|_{tr}$ is the normalized trace norm. Define the Frobenius norm loss function as $L_F^2(\widehat{\Sigma}_n^{-1}, \Sigma_n^{-1}) = \|\widehat{\Sigma}_n^{-1} - \Sigma_n^{-1}\|_F^2$.

Bodnar et al. [19] proposed a single-target linear shrinkage estimator for the precision matrix which is a convex combination of the inverse of the sample covariance matrix and a target matrix and minimizes the Frobenius norm loss. The linear shrinkage estimators are motivated from Bayes methods which are developed to nonparametric linear estimation by estimating the regularization parameter ω nonparametrically through the optimal weight. For more details on the justification, motivation and application of such linear shrinkage estimation, we refer to Bodnar et al. [19]; Ikeda et al. [8]; Ledoit and Wolf [17]; and Robbins [24].

One of the disadvantage of the single-target linear shrinkage estimation is that the performance of the shrinkage estimator depends to a great extent on the selection of target matrix. The choice of target should be guided by the presumed structure of the population covariance matrix. There is often no single ideal target, and it is difficult to identify a sensible target. The single-target estimator may be misspecified, because every target has a different bias-variance trade-off with respect to the unknown population covariance matrix. In order to reduce the estimation error caused by the improper selection of the target matrix, we consider using a set of target matrices.

Let $\Pi_1, \cdots, \Pi_k$ be the set of target matrices, where Π_i is symmetric, positive definite and has uniformly bounded normalized Frobenius norm for $i = 1, \cdots, k$. We propose the multi-target linear shrinkage estimator $\widehat{\Sigma}_n^{-1}$ of the precision matrix

$$\widehat{\Sigma}_n^{-1} = (1 - \omega_1 - \cdots - \omega_k) S_n^{-1} + \omega_1 \Pi_1 + \cdots + \omega_k \Pi_k \tag{1}$$

which minimizes the loss function

$$L_F^2(\widehat{\Sigma}_n^{-1}, \Sigma_n^{-1}) = \|(S_n^{-1} - \Sigma_n^{-1}) - \omega_1(S_n^{-1} - \Pi_1) - \cdots - \omega_k(S_n^{-1} - \Pi_k)\|_F^2$$

Here $\Omega = (\omega_1, \cdots, \omega_k)'$ is restricted by $\omega_i \geq 0$ for $i = 1, \cdots, k$ and $\omega_1 + \cdots + \omega_k \leq 1$.

Taking the derivatives of L_F^2 with respect to ω_i, $i = 1, \cdots, k$, and setting them equal to zero,

$$\begin{aligned}
&\frac{\partial L_F^2(\widehat{\Sigma}_n^{-1}, \Sigma_n^{-1})}{\partial \omega_i} \\
&= -2\, tr\Big[\big((S_n^{-1} - \Sigma_n^{-1}) - \omega_1(S_n^{-1} - \Pi_1) - \cdots - \omega_k(S_n^{-1} - \Pi_k)\big)\big(S_n^{-1} - \Pi_i\big) \Big] \\
&= 0
\end{aligned}$$

we obtain the optimal weight Ω^* satisfying

$$A_n \Omega^* = B_n \tag{2}$$

where

$$A_n = \frac{1}{p}\begin{pmatrix} tr[(S_n^{-1} - \Pi_1)(S_n^{-1} - \Pi_1)] & \cdots & tr[(S_n^{-1} - \Pi_k)(S_n^{-1} - \Pi_1)] \\ \vdots & \ddots & \vdots \\ tr[(S_n^{-1} - \Pi_1)(S_n^{-1} - \Pi_k)] & \cdots & tr[(S_n^{-1} - \Pi_k)(S_n^{-1} - \Pi_k)] \end{pmatrix}$$

and

$$B_n = \frac{1}{p}\begin{pmatrix} tr[(S_n^{-1} - \Sigma_n^{-1})(S_n^{-1} - \Pi_1)] \\ \vdots \\ tr[(S_n^{-1} - \Sigma_n^{-1})(S_n^{-1} - \Pi_k)] \end{pmatrix}$$

Since S_n^{-1} is a biased estimator of Σ_n^{-1} when $p/n \to c \in (0,1)$, the shrinkage estimator (1) actually balances the trade-off between bias and variance in an effective way. Thus $\widehat{\Sigma}_n^{-1}$ can be regarded as a maximum likelihood estimator with special ridge penalty terms, where ω_i, $i = 1, \cdots, k$, are the regularization parameters.

To estimate the optimal weight Ω^*, we suggest a method for estimating A_n and B_n consistently. The following lemma provides the limits of $\|S_n^{-1}\|_F^2$ and $tr(S_n^{-1}\Theta)$ for some symmetric positive definite matrix Θ. The proof can be found in Theorem 3.2 of Bodnar et al. [19]. Using this result one can easily obtain the consistent estimator of the weight Ω^*.

Lemma 1. *Let $p/n \to c \in (0,1)$. Assume that the elements of the matrix Y_n have uniformly bounded $4+\epsilon$ moments, where $\epsilon > 0$, $H_n(t)$ converges to a limit $H(t)$ at all points of continuity of H, and for n large enough there exists the compact interval $[h_0, h_1] \subset (0, +\infty)$ which contains the support of H_n. Then as $n \to \infty$,*

$$\frac{1}{p}\left| \|S_n^{-1}\|_F^2 - \Big(\frac{1}{(1-c)^2}\|\Sigma_n^{-1}\|_F^2 + \frac{c}{p(1-c)^3}\|\Sigma_n^{-1}\|_{tr}^2 \Big) \right| \xrightarrow{a.s.} 0 \tag{3}$$

and for symmetric positive definite matrix Θ which has uniformly bounded trace norm,

$$\left| tr(S_n^{-1}\Theta) - \frac{1}{1-c} tr(\Sigma_n^{-1}\Theta) \right| \stackrel{a.s.}{\to} 0 \tag{4}$$

Throughout the paper we assume that the conditions of Lemma 1 are satisfied. These conditions also ensure that Σ_n^{-1} has uniformly bounded normalized Frobenius norm and normalized trace norm. Note that we need the condition $c < 1$ to keep the sample covariance matrix S_n invertible. The case of $c > 1$ is very difficult to handle because of the loss of information as the dimension p is greater than the sample size n (cf. Bodnar et al. [19]). Although the estimator S_n^{-1} can be replaced by the generalized inverse matrix of the sample covariance matrix, it is not clear how to estimate the optimal weight consistently. Since the theory is yet to be developed fully for the $c > 1$ case, we leave it for future research.

Let $A_0 = (a_{0ij})_{k\times k}$ and $B_0 = (b_{0i})_k$, where

$$\begin{aligned} a_{0ij} = &\frac{1}{p(1-c)^2}\|\Sigma_n^{-1}\|_F^2 + \frac{c}{p^2(1-c)^3}\|\Sigma_n^{-1}\|_{tr}^2 - \frac{1}{p(1-c)} tr[\Sigma_n^{-1}(\Pi_i + \Pi_j)] \\ &+ \frac{1}{p} tr(\Pi_i \Pi_j) \end{aligned}$$

and

$$b_{0i} = \frac{c}{p(1-c)^2}\|\Sigma_n^{-1}\|_F^2 + \frac{c}{p^2(1-c)^3}\|\Sigma_n^{-1}\|_{tr}^2 - \frac{c}{p(1-c)} tr(\Sigma_n^{-1}\Pi_i)$$

Theorem 1 shows the non-random limit of A_n and B_n.

Theorem 1. *Under the conditions of Lemma 1, we have as $n \to \infty$,*

$$A_n - A_0 \stackrel{a.s.}{\to} 0, \;\; B_n - B_0 \stackrel{a.s.}{\to} 0$$

in the sense that each element converges almost surely.

Theorem 1 implies that the asymptotic optimal weight vector $\Omega_0 = (\omega_{01}, \cdots, \omega_{0k})'$ satisfies

$$A_0\Omega_0 = B_0 \tag{5}$$

From Equation (5), we have $\Omega_0 = 0$ and then Ω^* tends to 0 in the case of $c = 0$, which means that the inverse of the sample covariance matrix is an asymptotically best estimator for the precision matrix in terms of minimizing the Frobenius norm loss. In contrast if p increases, the sample covariance matrix becomes ill-conditioned and hence the linear shrinkage estimator (1) improves the performance of the sample estimator. The impact of this improvement becomes larger as p approaches n, i.e. as c approaches 1. In this case, each element of Ω_0 tends to $1/n$ and hence Ω^* approaches $(n^{-1}, \cdots, n^{-1})'$.

By Equation (5), to derive the a.s. consistent estimator of Ω_0, it suffices to construct $\widehat{A}_0$ and $\widehat{B}_0$, the a.s. consistent estimators of A_0 and B_0, which are provided in the following Theorem 2.

Theorem 2. *Under the conditions of Lemma 1, the a.s. consistent estimators of A_0 and B_0 are given by*

$$\widehat{A}_0 = A_n, \;\; \widehat{B}_0 = \frac{1}{p}\begin{pmatrix} \frac{p}{n}\|S_n^{-1}\|_F^2 + \frac{1}{n}\|S_n^{-1}\|_{tr}^2 - \frac{p}{n}\, tr(S_n^{-1}\Pi_1) \\ \vdots \\ \frac{p}{n}\|S_n^{-1}\|_F^2 + \frac{1}{n}\|S_n^{-1}\|_{tr}^2 - \frac{p}{n}\, tr(S_n^{-1}\Pi_k) \end{pmatrix}$$

Assume that $\widehat{A}_0$ is invertible. Then we get the estimator $\widetilde{\Omega}$ of the asymptotic optimal weight Ω_0

$$\widetilde{\Omega} = (\tilde{\omega}_1, \cdots, \tilde{\omega}_k)' = \widehat{A}_0^{-1}\widehat{B}_0 \tag{6}$$

Since Ω is restricted on $\omega_i \geq 0$ for $i = 1, \cdots, k$ and $\omega_1 + \cdots + \omega_k \leq 1$, it is reasonable to define the a.s. consistent estimators of the shrinkage intensities as follows.

Rewrite the multi-target linear shrinkage estimator (1) as

$$\widehat{\Sigma}_n^{-1} = (1-\beta)S_n^{-1} + \beta\big\{(1-\alpha_1-\cdots-\alpha_{k-1})\Pi_1 + \alpha_1\Pi_2 + \cdots + \alpha_{k-1}\Pi_k\big\}$$

where β and α_i, $i = 1, \cdots, k-1$ satisfy that $0 \leq \alpha_i,\ \beta \leq 1$, and

$$\omega_1 + \cdots + \omega_k = \beta,\ \ \omega_1 = (1-\alpha_1-\cdots-\alpha_{k-1})\beta,\ \ \omega_i = \alpha_{i-1}\beta \tag{7}$$

for $i = 2, \cdots, k$. Then we construct the estimators of $(\alpha_1, \cdots, \alpha_{k-1}, \beta)$ using Equation (6) and the relation Equation (7), namely

$$\tilde{\beta} = \tilde{\omega}_1 + \cdots + \tilde{\omega}_k,\ \ \tilde{\alpha}_i = \tilde{\omega}_{i+1}/\tilde{\beta},\ \ i = 1, \cdots, k-1$$

Note that $0 \leq \alpha_i,\ \beta \leq 1$, we estimate β and α_i, $i = 1, \cdots, k-1$, by $\hat{\beta} = \max\big(0, \min(1, \tilde{\beta})\big)$ and $\hat{\alpha}_i = \max\big(0, \min(1, \tilde{\alpha}_i)\big)$, $i = 1, \cdots, k-1$. Using Equation (7) again, we obtain the estimators

$$\hat{\omega}_1 = (1-\hat{\alpha}_1-\cdots-\hat{\alpha}_{k-1})\hat{\beta},\ \ \hat{\omega}_i = \hat{\alpha}_{i-1}\hat{\beta},\ \ i = 2, \cdots, k$$

Now the genuine multi-target linear shrinkage estimator of the precision matrix is given by

$$\widehat{\Sigma}_n^{-1} = (1-\hat{\omega}_1-\cdots-\hat{\omega}_k)S_n^{-1} + \hat{\omega}_1\Pi_1 + \cdots + \hat{\omega}_k\Pi_k \tag{8}$$

Theorems 1 and 2 immediately imply the estimator $\widehat{\Omega} = (\hat{\omega}_1, \cdots, \hat{\omega}_k)'$ converges almost surely to the asymptotic weight vector Ω_0 in Equation (5), and thus converges almost surely to its oracle optimal intensity Ω^* in Equation (2) as $n \to \infty$. This result is presented in the following Theorem 3, which implies that the multi-target linear shrinkage estimator in Equation (8) performs as well as its oracle one.

Theorem 3. *Assume that $\hat{A}_0$ is invertible. Then, under the conditions of Lemma 1, as $n \to \infty$,*

$$\widehat{\Omega} - \Omega^* \overset{a.s.}{\to} 0$$

Let

$$A_n^0 = \frac{1}{p}\begin{pmatrix} tr[(\Sigma_n^{-1}-\Pi_1)(\Sigma_n^{-1}-\Pi_1)] & \cdots & tr[(\Sigma_n^{-1}-\Pi_k)(\Sigma_n^{-1}-\Pi_1)] \\ \vdots & \ddots & \vdots \\ tr[(\Sigma_n^{-1}-\Pi_1)(\Sigma_n^{-1}-\Pi_k)] & \cdots & tr[(\Sigma_n^{-1}-\Pi_k)(\Sigma_n^{-1}-\Pi_k)] \end{pmatrix}$$

The following theorem shows that the shrinkage intensities tend almost surely to zero when $p/n \to c = 0$ as $n \to \infty$. This implies that the multi-target linear shrinkage estimator is asymptotically equivalent to the sample estimator. Bai and Shi [25] showed that the sample precision matrix is consistent in the case when $p/n \to 0$. This implies that the multi-target linear shrinkage estimator $\widehat{\Sigma}_n^{-1}$ is a consistent estimator for the precision matrix in this case.

Theorem 4. *Assume that A_n^0 is invertible and $p = o(n)$. Then, as $n \to \infty$,*

$$\widehat{\Omega} \overset{a.s.}{\to} 0$$

Regarding the choice of the target matrices, selecting suitable targets require some diligence. It is worth mentioning that the theoretical results derived in Theorems 1–4 are based on the assumption that the target matrices are non-random. However, the data driven target matrices may result in more accurate estimators, as these target matrices provide more information about the structure of the precision matrix. Therefore, in practice, one can use the estimators $\hat{\Pi}_1, \cdots, \hat{\Pi}_k$ of the targets to construct the multi-target linear shrinkage estimator of the precision matrix. The theoretical investigation on the properties of the estimator when using the estimated targets is not yet available. This topic will be pursued in the future research.

In the absence of prior information, the nine target matrices described in **Table 1** in next section can be included due to their popularity in the literature (e.g. Gray et al. [21]). This is illustrated in Section 3 using simulations and a real data example. It is also possible to further enrich the target set with any covariance structures not listed in **Table 1**. Examples include Toeplitz, higher-order autoregressive, or latent factor structures, see, e.g., Chen [26] and Ledoit and Wolf [27].

An overview was given in Schäfer and Strimmer [28], where six types of commonly used targets were proposed. These targets are included in the following **Table 1**, denoted as $\hat{\Pi}_1$ (diagonal, unit variance), $\hat{\Pi}_2$ (diagonal, common variance), $\hat{\Pi}_3$ (diagonal, unequal variance), $\hat{\Pi}_4$ and $\hat{\Pi}_5$ (common (co)variance), $\hat{\Pi}_6$ (unequal variance, constant correlation), and $\hat{\Pi}_7$, $\hat{\Pi}_8$ and $\hat{\Pi}_9$ (decaying correlation). In the literature it is easy to find examples where one of the above targets is employed, see, e.g., Dobra et al. [29]; Friedman [30]; Hastie et al. [31]; and Ledoit and Wolf [27,32].

3. Numerical and empirical studies

The purpose of this section is to compare the performance of the proposed approach with existing ones. We also apply the proposed multi-target linear shrinkage estimator for an empirical analysis of financial market data.

3.1. Simulation study

In this subsection we investigate the numerical performance of the proposed estimator through simulation. We generate data as follows. Let $d_i = 0.1 + 10 \times U_i$, U_i is generated from the uniform distribution on the interval (0, 1). Let σ_{ij} be the (i, j)th element of the true covariance matrix Σ. We consider six types of covariance structures:

(Model 1) $\sigma_{ij} = d_i d_j \rho^{|i-j|}$,

(Model 2) $\sigma_{ij} = \rho^{|i-j|}$,

(Model 3) $\sigma_{ij} = d_i d_j \{|i-j+1|^{2h} - 2|i-j|^{2h} + |i-j-1|^{2h}\}/2$,

(Model 4) $\sigma_{ij} = \{|i-j+1|^{2h} - 2|i-j|^{2h} + |i-j-1|^{2h}\}/2$,

(Model 5) $\Sigma = \begin{pmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{pmatrix}$ is a block diagonal matrix, where the elements of Σ_1 and Σ_2 are from (Model 1) and (Model 3), respectively,

(Model 6) Σ is a block diagonal matrix similar to (Model 5), where the elements of the two blocks are from (Model 2) and (Model 4), respectively.

(Model 2) and (Model 4) correspond to the covariance structures of an autoregressive process and a fractional Brownian Motion, respectively. (Model 5) and (Model 6) represent more complex covariance structures. The data $x_1, ..., x_n$ are generated by $x_i = \Sigma^{\frac{1}{2}} y_i$, where $y_i = (y_{1i}, ..., y_{pi})'$. $y_{1i}, ..., y_{pi}$ are mutually independently distributed as

(Case 1) $y_{ji} \sim N(0,1)$,
(Case 2) $y_{ji} \sim \sqrt{(m-2)/m} z_{ji},\ z_{ji} \sim t_m$,
(Case 3) $y_{ji} = (z_{ji} - \tau)/\sqrt{2\tau},\ z_{ji} \sim \chi^2_\tau$,
where t_m is a t-distribution with m degrees of freedom and χ^2_τ is a chi-square distribution with τ degrees of freedom. (Case 2) and (Case 3) include heavy-tailed and skewed distributions, respectively.

Gray et al. [21] listed nine popular target matrices in the literature. We choose them as the target matrix set of our multi-target linear shrinkage estimator for the precision matrix, see **Table 1** for the set of target matrices.

Table 1. Target matrices.

correlation	zero($r_{ij} = 0$)	constant ($r_{ij} = \bar{r}$)	decaying($r_{ij} = \bar{r}^{\lvert i-j \rvert}$)
unit variance ($\nu_i = 1$)	$\hat{\Pi}_1$	$\hat{\Pi}_4$	$\hat{\Pi}_7$
common variance ($\nu_i = \bar{s}$)	$\hat{\Pi}_2$	$\hat{\Pi}_5$	$\hat{\Pi}_8$
unequal variances ($\nu_i = s_{ii}$)	$\hat{\Pi}_3$	$\hat{\Pi}_6$	$\hat{\Pi}_9$

Here the target matrix $\hat{\Pi} = V^{\frac{1}{2}} R V^{\frac{1}{2}}$, with $V = diag(\nu_1, ..., \nu_p)$ a diagonal matrix, $R = (r_{ij})_{p \times p}$ a correlation matrix. s_{ij} denotes the (i,j)th element of the sample covariance matrix S_n, while $\bar{s},\ \bar{r}$ are the averages of the sample variances and correlations, respectively.

To assess the performance of the estimators, we calculate the PRIAL (Percentage Relative Improvement in Average Loss) presented in Ledoit and Wolf [18]. The PRIAL indicates the extent to which the MSE of the estimator $\widehat{\Sigma}_n^{-1}$ outperforms in percentage terms the MSE of the sample precision matrix. Let $\widehat{\Sigma}_n^{-1}$ be an estimator of the precision matrix, the PRIAL is defined by

$$\text{PRIAL}(\widehat{\Sigma}_n^{-1}) = \left(1 - \frac{\text{E}\|\widehat{\Sigma}_n^{-1} - \Sigma_n^{-1}\|_F^2}{\text{E}\|S_n^{-1} - \Sigma_n^{-1}\|_F^2}\right) \times 100\%$$

which actually measures the relative improvement of an estimator over the sample precision matrix. Thus, the improvement of the shrinkage estimator over the sample precision matrix will be measured by how closely this estimator approximates Σ_n^{-1} relative to the sample precision matrix. The PRIAL being closer to 100% indicates the stronger improvement of an estimator.

In order to investigate the performance of the suggested multi-target linear shrinkage (MULTI) estimator for the precision matrix, we introduce three benchmark estimators. Since there exist extensive comparisons of the single-target linear shrinkage estimators and other estimators given in the statistical literature, see, e.g. Bodnar et al. [19] and Ikeda et al. [8], here we only compare the multi-target estimator with the single-target estimators.

The first estimator considered is the nonlinear shrinkage estimator "EV" proposed and studied in Ledoit and Wolf [10,18,33], which is defined as

$$\widehat{\Sigma}_{EV}^{-1} = U A^* U'$$

where U is an orthogonal matrix whose columns are the eigenvectors of the sample covariance matrix S_n, A^* is a diagonal matrix whose elements are real univariate functions which depend on S_n. The exact formula of A^* can be found in (4.3) of Ledoit and Wolf [18].

The second one is based on a single-target linear shrinkage estimator (OLSE) of the covariance matrix provided by Bodnar et al. [34], then the OLSE estimator of the precision matrix is given by

$$\widehat{\Sigma}_{OLSE}^{-1} = \left(\widehat{\Sigma}_{OLSE}\right)^{-1},\ \ \widehat{\Sigma}_{OLSE} = \alpha S_n + \beta \Pi$$

where $\alpha = 1 - \frac{\frac{1}{n}\|S_n\|_{tr}^2 \|\Pi\|_F^2}{\|S_n\|_F^2 \|\Pi\|_F^2 - (tr(S_n \Pi))^2}$, $\beta = \frac{tr(S_n \Pi)}{\|\Pi\|_F^2}(1-\alpha)$. The third estimator is the single-target linear shrinkage (ONE) estimator with $k = 1$ in Equation (1).

For both linear shrinkage estimators, we choose the target matrix from **Table 1**. In simulations we tried each of the shrinkage targets in **Table 1**. Unsurprisingly, the performance of both estimators varies across the different scenarios and depends on the choice of shrinkage target. Overall, the single-target estimators with the best choice of target matrix does not outperform the suggested multi-target estimator which does not need to choose the suitable shrinkage target. It appears that the proposed multi-target estimator is less sensitive to misspecification of the targets.

Now we report the comparison of the performance of the estimators EV, OLSE, ONE and MULTI. Since the comparisons show no significant difference among the nine shrinkage targets in **Table 1**, we only present the results using the target matrix $\hat{\Pi}_1$ for OLSE and ONE estimators.

Table 2 summarises the simulation results with $m = 5$, $\tau = 3$, $p = 100$, $\rho = 0.2, 0.8$, $h = 0.8$, and $n = 120, 200, 300, 500$, respectively. The PRIALs are approximated by an average over 1000 simulation runs for each scenario.

It is observed that the multi-target shrinkage estimator performs better than the other three methods for all six models in almost all scenarios, except for the case of $n = 120$, where MULTI and EV have similar performance. Moreover, we conclude that, for all estimators in each scenario, the relative improvement measure PRIAL decreases when the sample size n increases. This shows that, when the dimension p is fixed, the performance of the sample estimator improves as the sample size increases. It is also seen that even if the sample size n becomes larger, the PRIAL of MULTI estimator is still greater than 95.5%, better than EV estimator and much better than the other two estimators. In general, EV estimator outperforms OLSE and ONE estimators, while the performance of MULTI estimator is the best and most stable one.

On the other hand, the computation results show that the performance of OLSE and ONE estimators differs greatly for different covariance matrix structures. This reveals that the correct selection of the shrinkage target is crucial to OLSE and ONE methods. In contrast, the MULTI estimator achieves a similar performance for all models without having to choose the correct shrinkage target, especially for (Model 5) and (Model 6) with complex covariance structures. This demonstrates the robustness of the proposed multi-target method in estimating the complicated precision matrices compared to single-target shrinkage.

Next, we consider the case of fixing $c = p/n = 0.2, 0.4, 0.6, 0.8$, where n takes values in $\{50 + 20m, m \in \mathbb{N}\} \cap [50, 500]$. **Figures 1** and **2** depict the PRIALs of EV, OLSE, ONE and MULTI estimators for different c values under two covariance matrix structures, respectively.

It can be seen from **Figure 1** that the multi-target linear shrinkage estimator has the best performance for sample size $n \geq 90$. When the sample size is less than 90, MULTI still outperforms other estimators except for the case of $c = 0.8$. This highlights the multi-target estimator's relative benefit in small sample scenarios and provides insights for its applications with severe data constraints. Moreover, with the increase of sample size n, the PRIAL of MULTI estimator increases continuously. The PRIAL of OLSE estimator is unstable and having sudden drops. For (Model 2), the performance of ONE estimator is the worst, which may be caused by improper selection of shrinkage target. This again shows the importance of using multiple target matrices in the estimation procedure. Compared with the EV, OLSE and ONE estimators, the performance of MULTI estimator is the best and the PRIALs are the most stable. The PRIALs of the EV, OLSE and ONE estimators have relatively large volatility when $c = 0.2$.

Figure 2 further shows the superiority of multi-target linear shrinkage estimator for sample size $n \geq 90$. The simulation results indicate that three direct shrinkage procedures outperform the OLSE method for (Model 4). It is noted that the PRIAL of the OLSE estimator becomes smaller as the sample size n increases. The reason may be that the OLSE estimator needs to first estimate the covariance matrix and then invert the estimator of the covariance matrix to obtain the estimator of the precision matrix, while the MULTI, EV and ONE estimators directly shrink the sample precision matrix.

Table 2. PRIALs of EV, OLSE, ONE and MULTI estimators for six models based on 1000 replications.

		Model 1				**Model 2**			
	$\boldsymbol{n}$	**EV**	**OLSE**	**ONE**	**MULTI**	**EV**	**OLSE**	**ONE**	**MULTI**
$N(0,1)$ $\rho=0.2$	120	100	100	99.7	99.9	100	100	100	100
	200	98.9	98.7	98.7	99.8	98.9	98.8	98.7	99.7
	300	95.8	95.4	95.4	99.7	95.7	95.4	95.2	99.6
	500	89	87.9	87.8	99.2	88	87.1	86.7	99
t_5 $\rho=0.2$	120	100	100	100	100	100	100	99.9	100
	200	99.1	99.1	99.1	99.1	98.9	98.7	98.7	99.3
	300	95.9	95.5	95.4	98.8	95.9	95.6	95.5	98.6
	500	89.3	88.1	87.7	97.1	89.4	88.6	88.1	97.8
$N(0,1)$ $\rho=0.8$	120	99.9	99.8	99.5	99.5	99.9	99.7	99.2	99.9
	200	96.9	94.8	91.1	99.7	96.8	94.7	90.5	99.9
	300	92.3	88.1	79.8	99.5	91.3	86.8	78.2	99.7
	500	81.8	73.6	61.3	99.3	80.9	72.3	60.1	99.2
t_5 $\rho=0.8$	120	99.9	99.9	99.6	100	99.9	99.8	99.6	100
	200	97.3	95.6	92	99.6	97.7	96.5	92.7	99.7
	300	92.5	88.7	79.6	98.7	93.3	89.7	81	98.9
	500	81.3	72.5	59.5	98.6	82.4	75.1	63.7	99
		Model 3				Model 4			
	n	EV	OLSE	ONE	MULTI	EV	OLSE	ONE	MULTI
$N(0,1)$	120	100	99.8	99.9	100	100	99.9	99.7	99.8
	200	98.1	92.5	95.7	99.8	97.8	92.3	95.6	99.9
	300	94	77.9	89.2	99.4	93.3	77.6	87.8	99.5
	500	83.7	48.8	71.5	98.2	83.8	50.9	74.1	98.6
χ_3^2	120	99.9	99.7	99.8	100	100	99.8	99.9	100
	200	98.1	90.7	96	99.5	98.1	92.1	95.9	99.5
	300	93.7	73.5	88.4	98.7	94.2	76.1	88.6	98.7
	500	84.7	53.9	73.2	97.8	83.7	53.8	74.1	98
		Model 5				Model 6			
	n	EV	OLSE	ONE	MULTI	EV	OLSE	ONE	MULTI
$N(0,1)$ $\rho=0.2$	120	97.5	89.8	89.2	97.8	97.2	87.3	88.9	97.3
	200	95.1	83.5	84.6	97.5	95.8	83.4	85.2	97.1
	300	90.1	69.7	78.2	97	92.6	70.5	75.9	96.7
	500	81.7	44.5	60.7	96.9	80.8	46	63.1	96.2
t_5 $\rho=0.2$	120	97.2	88.8	89	96.8	97	86.8	87.6	97
	200	94.9	81.7	84.9	96.5	94.6	82.5	82.4	96.6
	300	90.8	65.1	73.9	96.2	91.9	70.1	72.5	96.5
	500	81.2	50.1	55.3	95.5	77.3	44.9	55.4	95.8

Generally speaking, the above simulation evidence reveals that the proposed multi-target linear shrinkage estimator has better performance than the single-target linear shrinkage estimators. It is a great alternative to the existing estimation methods.

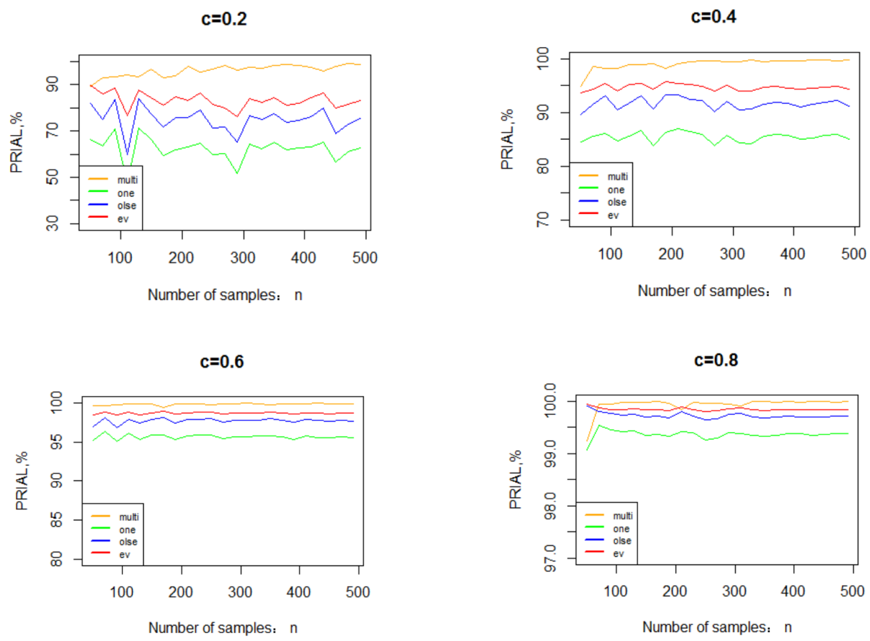


Figure 1. The PRIALs for (Model 2) with ρ=0.8 and $y \sim t_5$.

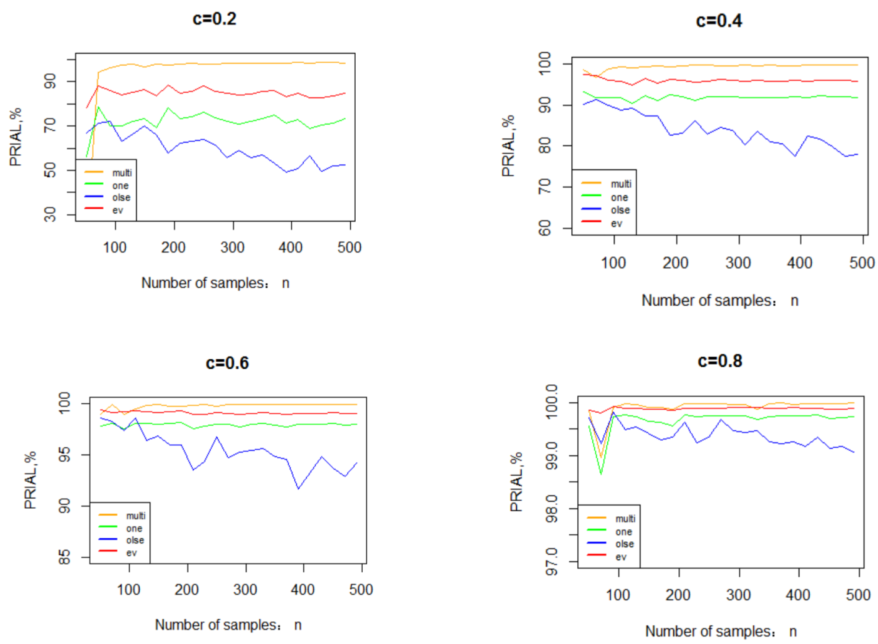


Figure 2. The PRIALs for (Model 4) with $h = 0.8$ and $y \sim N(0, 1)$.

3.2. Empirical study

The ground breaking work of Markowitz [2]—the mean-variance efficient portfolio theory is one of the key tools for portfolio management. However, one needs to know the unobservable covariance matrix and precision matrix to implement this framework. Therefore, it is of vital importance to construct a high-performance precision matrix estimator.

In this subsection we apply the estimator of precision matrix to portfolio optimization problems. Namely, we use the proposed multi-target linear shrinkage method to estimate the weights of a portfolio. We consider a portfolio with p stocks. Denote the expected return of p stocks as $\mu = (\mu_1, ..., \mu_p)'$, the covariance matrix as Σ. Let $\gamma = (\gamma_1, ..., \gamma_p)'$ be the weight of the portfolio, then the return of the portfolio is $\mathrm{E}(R_p) = \gamma'\mu$, the risk (variance) is $\mathrm{Var}(R_p) = \gamma'\Sigma\gamma$. Let $\mathbf{1} = (1, \cdots, 1)'_p$.

The following two popular models are applied to find the efficient frontier of the portfolio (see, e.g., Amenc and Sourd [35]; Cai et al. [36]; Ding et al. [37]; and Joo and Park [38]):

(I). Global minimum variance portfolio

$$\min_{\gamma} \mathrm{Var}(R_p), \text{ subject to } \gamma'\mathbf{1} = 1$$

(II). Maximum expected return portfolio with fixed risk σ_p^2

$$\max_{\gamma} \mathrm{E}(R_p), \text{ subject to } \mathrm{Var}(R_p) = \sigma_p^2, \;\; \gamma'\mathbf{1} = 1$$

The solution to Model (I) is

$$\gamma_1^* = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}$$

From Amenc and Sourd [35], the solution of Model (II) is $\gamma_2^* = (\gamma_{21}^*, ..., \gamma_{2p}^*)'$, where

$$\gamma_{2i}^* = \frac{\mathrm{E}(R_p)\sum_{j=1}^{p} \nu_{ij}(C\mu_j - A) + \sum_{j=1}^{p} \nu_{ij}(B - A\mu_j)}{BC - A^2}, \quad i = 1, ..., p \tag{9}$$

ν_{ij} is the (i, j)th element of Σ^{-1}, and

$$A = \sum_{i=1}^{p}\sum_{j=1}^{p} \nu_{ij}\mu_j, \;\; B = \sum_{i=1}^{p}\sum_{j=1}^{p} \nu_{ij}\mu_i\mu_j, \;\; C = \sum_{i=1}^{p}\sum_{j=1}^{p} \nu_{ij},$$

$$\mathrm{E}(R_p) = \frac{A}{C} + \frac{1}{C}\sqrt{(BC - A^2)C\big(\sigma_p^2 - \frac{1}{C}\big)}$$

We now consider the portfolio management for the stocks in the China CSI Smallcap 500 index which is a prominent benchmark that measures the performance of 500 mid and small-cap A-share stocks listed on the Shanghai and Shenzhen Stock Exchanges. In order to build an efficient portfolio, we use $p = 120$ randomly selected stocks from the components of CSI 500 index with a relatively large total market value and a high turnover ratio. Specifically, the data set contains the daily closing price of the selected 120 stocks from 21 June 2017 to 20 December 2019 with $n = 613$ observations.

The data are divided into three sets: the training set (the first 240 observations), the validation set (the next 240 observations) and the test set (the last 133 observations). The training set is used to estimate the precision matrix, the validation set is utilized to determine the optimal portfolio weight γ^*, while the test set is applied to evaluate the return and risk of the portfolios under different methods.

The parameter μ is calculated from the average return of each stock, Σ^{-1} is estimated by five methods: the sample precision matrix (S^{-1}), EV estimator, OLSE estimator, single-target linear shrinkage (ONE) estimator and the multi-target linear shrinkage (MULTI) estimator, respectively. Among the 120 stocks, the unequal variances and correlations are dominant structures. Based on the data characteristics, we compute two MULTI estimators, one with the total nine targets and the other one (MULTI$_6$) obtained when using only six targets $\hat{\Pi}_1$, $\hat{\Pi}_3$, $\hat{\Pi}_6$, $\hat{\Pi}_7 - \hat{\Pi}_9$.

Using the estimators $\hat{\mu}$ and $\hat{\Sigma}^{-1}$, the portfolio weight vectors for both models can be calculated by $\hat{\gamma}_1^* = \frac{\hat{\Sigma}^{-1}\mathbf{1}}{\mathbf{1}'\hat{\Sigma}^{-1}\mathbf{1}}$, and $\hat{\gamma}_2^* = (\hat{\gamma}_{21}^*, ..., \hat{\gamma}_{2p}^*)'$, where $\hat{\gamma}_{2i}^*$, $i = 1, ..., p$, are obtained by plugging in these estimators in Equation (9). Now one can compute the expected return and the risk of a portfolio as well as the CV, where CV is the coefficient of variation, which reflects the ratio of the square root of the risk to the expected return. Obviously, a smaller value of CV means the better risk-return trade-off of the portfolio.

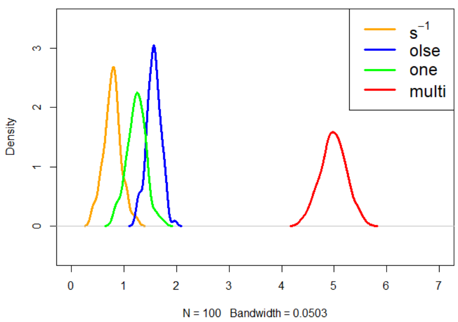
The results of the portfolio obtained by Model (I) and Model (II) are shown in **Table 3**. **Table 3** indicates that in Model (I), the performance of MULTI estimator is the best with the maximum return, minimum risk and lowest CV. MULTI_6 achieves a similar result. EV and ONE estimators have similar performance. The return obtained by S^{-1} is the smallest, while the risk and CV of OLSE estimator are larger than S^{-1}, EV and ONE methods. Meanwhile, in Model (II), when the investment risk is fixed to be equal to 0.1, the MULTI estimators still result in the highest expected return and smallest CV. Moreover, the S^{-1}, EV and ONE estimators have similar results, but the OLSE estimator is the worst.

In both cases, MULTI and MULTI_6 have very similar performance. This highlights the key strength of the proposed multi-target estimator that it is less sensitive to misspecification of the targets.

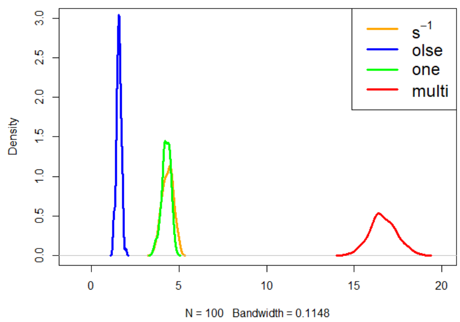
Table 3. The results of the portfolio obtained by Models (I) and (II).

		S^{-1}	EV	OLSE	ONE	MULTI_6	MULTI
Model (I)	return	0.9	1.4	1.8	1.3	4.9	5
	risk	2.3×10^{-4}	5.5×10^{-4}	7.4×10^{-3}	5.2×10^{-4}	1.7×10^{-4}	1.7×10^{-4}
	CV	1.7×10^{-2}	1.7×10^{-2}	4.8×10^{-2}	1.8×10^{-2}	2.7×10^{-3}	2.6×10^{-3}
Model (II)	return	42.4	55	17.5	48.2	133.9	135.1
(with risk $= 0.1$)	CV	7.5×10^{-3}	5.8×10^{-3}	1.8×10^{-2}	6.6×10^{-3}	2.4×10^{-3}	2.3×10^{-3}

In order to further evaluate the performance of the estimators, we randomly divide 613 observations into the training set, validation set and test set with sample size of 240, 240 and 133, respectively. Since the performance of EV and ONE are quite similar, and we are more interested in comparing the single- and multi-target linear shrinkage methods, we use only OLSE, ONE and MULTI estimators together with the sample precision matrix to calculate the return and risk of the portfolios. The procedure is repeated 100 times and the simulation results are recorded by **Figure 3** which shows the density functions of portfolio returns in 100 replications using S^{-1}, OLSE, ONE and MULTI estimators under Model (I) and Model (II) with the investment risk changing from 0.001 to 0.2, respectively. In **Figure 3a** represents the density for Model (I), **Figure 3b–f** represent the densities for Model (II) with risk = 0.001, 0.005, 0.05, 0.1 and 0.2, respectively.



(a)



(b)

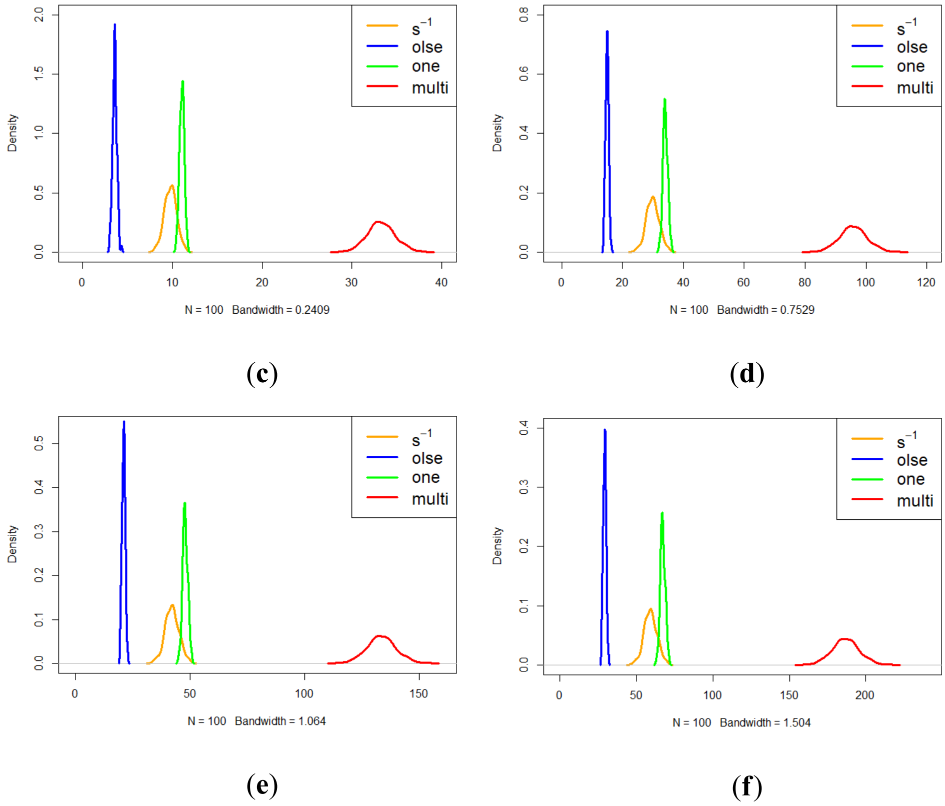


Figure 3. The density functions of portfolio returns with 100 realizations.

The simulation evidence in **Figure 3** illustrates that the greater the investment risk, the larger the return, and for both models, the return of the portfolio obtained by using MULTI estimator is significantly higher than the other estimators. For model (I), the OLSE estimator is superior to S^{-1} and ONE estimator. For model (II), the performance of the OLSE estimator is poor. As the risk increases, the ONE estimator performs better than the sample precision matrix. Overall, the proposed multi-target linear shrinkage estimator has superior performance in all cases. Our findings show that the multi-target shrinkage approach is quite useful for reducing the estimation errors of the precision matrix and increasing the performances of the portfolios. The proposed method yields more accurate portfolio weights than those of other methods, resulting in higher returns and lower risks.

4. Conclusions

A new estimation method for the precision matrix is considered in this paper. The multi-target linear shrinkage estimator by shrinking the inverse of the sample covariance matrix directly is proposed. This approach generalizes single-target shrinkage methods by allowing the estimator to incorporate multiple targets. The estimator is applied to the simulated data and a financial market dataset, and compared with several existing estimators. The computation results show the improvement of the multi-target linear shrinkage estimator particularly for high-dimensional problems where choosing a single target matrix might limit performance.

It is clear that a careful analysis on determining the optimal number and the optimal choice of the target matrices can greatly help in improving the performance of the estimator. Designing the adaptive methods that automatically select optimal targets based on empirical data characteristics would be an interesting research topic to explore in the future. Another open question worth pursuing further is to investigate the multi-target OLSE method which should also have some nice properties. The multi-target OLSE procedure first creates the multi-target linear shrinkage for the sample covariance matrix and thereafter inverts it to obtain the estimator of the precision matrix. The research of this topic is ongoing.

Author contributions: Conceptualization, YX and LW; methodology, YX and LW ; software, YX; validation, YX and LW; formal analysis, YX and LW; investigation, YX and LW; resources, YX and LW; data curation, YX; writing—original draft preparation, YX and LW; writing—review and editing, LW; visualization, YX; supervision, LW; project administration, LW; funding acquisition, LW. All authors have read and agreed to the published version of the manuscript.

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References

1. Elton EJ, Gruber MJ, Brown SJ, Goetzmann WN. Modern Portfolio Theory and Investment Analysis. Wiley Publishing; 2009.
2. Markowitz H. Portfolio selection. J. Finance. 1952; 7: 77–91.
3. Bodnar O, Bodnar T, Parolya N. Recent advances in shrinkage-based high-dimensional inference. J. Multivar. Ana. 2022; 188: 104826.
4. Bodnar T, Okhrin Y, Parolya N. Optimal shrinkage based portfolio selection in high dimensions. J. Bus. Econ. Stat. 2023; 41(1): 140–156.
5. Fan J, Liao Y, Liu H. An overview of the estimation of large covariance and precision matrices. Econom. J. 2016; 19: C1–C32.
6. Fan J, Liao Y, Mincheva M. High-dimensional covariance matrix estimation in approximate factor models. Ann. Stat. 2017; 39: 3320–3356.
7. Friedman JH, Hastie T, Tibshirani R. Sparse inverse covariance estimation with the graphical lasso. Biostatistics. 2008; 9: 432–441.
8. Ikeda Y, Kubokawa T, Srivastava MS. Comparison of linear shrinkage estimators of a large covariance matrix in normal and non-normal distributions. Comput. Statist. Data Anal. 2016; 95: 95–108.
9. Kuismin MO, Kemppainen JT, Sillanpää MJ. Precision matrix estimation with ROPE. J. Comput. Graph. Statist. 2017; 26: 682–694.
10. Ledoit O, Wolf M. Nonlinear shrinkage of the covariance matrix for portfolio selection: Markowitz meets goldilocks. Rev. Financ. Stud. 2017; 30: 4349–4388.
11. Ledoit O, Wolf M. Shrinkage estimation of large covariance matrices: Keep it simple, statistician? J. Multivar. Anal. 2021; 186: 104796.
12. Liu C, Tang CY. A quasi-maximum likelihood approach for integrated covariance matrix estimation with high frequency data. J. Econom. 2014; 180: 217–232.
13. van Wieringen WN, Peeters CFW. Ridge estimation of inverse covariance matrices from high-dimensional data. Comput. Statist. Data Anal. 2016; 103: 284–303.
14. Wang L, Chen Z, Wang CD, Li R. Ultrahigh dimensional precision matrix estimation via refitted cross validation. J. Econom. 2020; 215: 118–130.
15. Wang X, Zhang B. Target selection in shrinkage estimation of covariance matrix: A structural similarity approach. Stat. Prob. Lett. 2024; 208: 110048.
16. Dey DK, Srinivasan C. Estimation of a covariance matrix under Stein's loss. Ann. Stat. 1985; 13: 1581–1591.
17. Ledoit O, Wolf M. A well-conditioned estimator for large-dimensional covariance matrices. J. Multivar. Anal. 2004a; 88: 365–411.
18. Ledoit O, Wolf M. Nonlinear shrinkage estimation of large-dimensional covariance matrices. Ann. Statist. 2012b; 40: 1024–1060.
19. Bodnar T, Gupta AK, Parolya N. Direct shrinkage estimation of large dimensional precision matrix. J. Multivar Anal,

2016; 146: 223–236.
20. Engel J, Buydens L, Blanchet L. An overview of large-dimensional covariance and precision matrix estimators with applications in chemometrics. J. Chemometrics. 2017; 31: e2880.
21. Gray H, Leday GGR, Vallejos CA, Richardson S. Shrinkage estimation of large covariance matrices using multiple shrinkage targets. arXiv preprint. 2018; 1809: 08024.
22. Bartz D, Höhne J, Müller KR. Multi-target shrinkage. arXiv preprint. 2014; 1412: 2041.
23. Lancewicki T, Aladjem M. Multi-target shrinkage estimation for covariance matrices. IEEE Trans. Signal Process. 2014; 62: 6380–6390.
24. Robbins H. Some thoughts on empirical Bayes estimation. Ann. Stat. 1983; 11: 713–723.
25. Bai J, Shi S. Estimating high dimensional covariance matrices and its applications. Ann. Econom. Finance. 2011; 12: 199–215.
26. Chen CF. Bayesian inference for a normal dispersion matrix and its application to stochastic multiple regression analysis. J. Royal Stat. Soc. Ser. B Stat. Methodol. 1979; 41: 235–248.
27. Ledoit O, Wolf M. Improved estimation of the covariance matrix of stock returns with an application to portfolio selection. J. Empir. Finance. 2003; 10: 603–621.
28. Schäfer J, Strimmer K. A shrinkage approach to large-scale covariance matrix estimation and implications for functional genomics. Stat. Appl. Genet. Mol. Biol. 2005; 4: 32.
29. Dobra A, Hans C, Jones B, et al. Sparse graphical models for exploring gene expression data. J. Multivar. Anal. 2014; 90: 196–212.
30. Friedman JH. Regularized discriminant analysis. J. Amer. Statist. Assoc. 1989; 84: 165–175.
31. Hastie T, Tibshirani R, Friedman J. The Elements of Statistical Learning. Springer Publishing; 2001.
32. Ledoit O, Wolf M. Honey, I shrunk the sample covariance matrix. J. Portfolio. Management. 2024c; 30: 110–119.
33. Ledoit O, Wolf M. Analytical nonlinear shrinkage of large-dimensional covariance matrices. Ann. Stat. 2020d; 48: 3043–3065.
34. Bodnar T, Gupta AK, Parolya N. On the strong convergence of the optimal linear shrinkage estimator for large dimensional covariance matrix. J. Multivar. Anal. 2014; 132: 215–228.
35. Amenc N, Sourd VL. Portfolio Theory and Performance Analysis. John Wiley & Sons Publishing; 2005.
36. Cai T, Hu J, Li Y, Zheng X. High-dimensional minimum variance portfolio estimation based on high-frequency data. J. Econom. 2019; 214: 482–494.
37. Ding Y, Li Y, Zheng X. High dimensional minimum variance portfolio estimation under statistical factor models. J. Econom. 2021; 222: 502–515.
38. Joo YC, Park SY. Optimal portfolio selection using a simple double-shrinkage selection rule. Finance Res. Lett. 2021; 43: 102019.

Appendix

Proof of Theorem 1. Let $a_n \overset{a.s.}{\sim} b_n$ denotes $a_n - b_n \overset{a.s.}{\to} 0$. By Lemma 1, we obtain

$$\begin{aligned}
&\frac{1}{p}\|S_n^{-1}\|_F^2 \overset{a.s.}{\sim} \frac{1}{p(1-c)^2}\|\Sigma_n^{-1}\|_F^2 + \frac{c}{p^2(1-c)^3}\|\Sigma_n^{-1}\|_{tr}^2,\\
&\frac{1}{p}tr(S_n^{-1}\Pi) \overset{a.s.}{\sim} \frac{1}{p(1-c)}tr(\Sigma_n^{-1}\Pi),\\
&\frac{1}{p}tr(S_n^{-1}\Sigma_n^{-1}) \overset{a.s.}{\sim} \frac{1}{p(1-c)}\|\Sigma_n^{-1}\|_F^2
\end{aligned}$$

These yield that, as $n \to \infty$,

$$A_n - A_0 \overset{a.s.}{\to} 0, \ \ B_n - B_0 \overset{a.s.}{\to} 0$$

in the sense that each element converges almost surely.□

Proof of Theorem 2. By Lemma 1, the a.s. consistent estimators of $\frac{1}{p}\|\Sigma_n^{-1}\|_{tr}$, $\frac{1}{p}tr(\Sigma_n^{-1}\Pi)$, and $\frac{1}{p}\|\Sigma_n^{-1}\|_F^2$ can be given by

$$\frac{1}{p}\widehat{\|\Sigma_n^{-1}\|}_{tr} = \frac{(1-p/n)}{p}\|S_n^{-1}\|_{tr} \tag{A1}$$

$$\frac{1}{p}tr\widehat{(\Sigma_n^{-1}\Pi)} = \frac{(1-p/n)}{p}tr(S_n^{-1}\Pi) \tag{A2}$$

and

$$\begin{aligned}
\tfrac{1}{p}\widehat{\|\Sigma_n^{-1}\|}_F^2 &= \tfrac{(1-p/n)^2}{p}\|S_n^{-1}\|_F^2 - \tfrac{1}{pn(1-p/n)}\widehat{\|\Sigma_n^{-1}\|}_{tr}^2\\
&= \tfrac{(1-p/n)^2}{p}\|S_n^{-1}\|_F^2 - \tfrac{(1-p/n)}{pn}\|S_n^{-1}\|_{tr}^2
\end{aligned} \tag{A3}$$

Hence the a.s. consistent estimator of a_{0ij} is given by

$$\begin{aligned}
\hat{a}_{0ij} &= \frac{1}{p}\|S_n^{-1}\|_F^2 - \frac{1}{pn(1-p/n)}\|S_n^{-1}\|_{tr}^2 + \frac{1}{pn(1-p/n)^3}(1-p/n)^2\|S_n^{-1}\|_{tr}^2\\
&\quad -\frac{1}{p(1-p/n)}(1-p/n)\ tr[S_n^{-1}(\Pi_i+\Pi_j)] + \frac{1}{p}tr(\Pi_i\Pi_j)\\
&= \frac{1}{p}\Big(\|S_n^{-1}\|_F^2 - tr(S_n^{-1}\Pi_i) - tr(S_n^{-1}\Pi_j) + tr(\Pi_i\Pi_j)\Big)
\end{aligned}$$

Therefore $\widehat{A}_0 = A_n$.

Similarly, by Equations (A1)–(A3), the a.s. consistent estimator of b_{0i} is given by

$$\begin{aligned}
\hat{b}_{0i} &= \frac{1}{n(1-p/n)^2}\left((1-p/n)^2\|S_n^{-1}\|_F^2 - \frac{(1-p/n)}{n}\|S_n^{-1}\|_{tr}^2\right)\\
&\quad + \frac{1}{pn(1-p/n)^3}(1-p/n)^2\|S_n^{-1}\|_{tr}^2 - \frac{1}{n(1-p/n)}(1-p/n)tr(S_n^{-1}\Pi_i)\\
&= \frac{1}{n}\|S_n^{-1}\|_F^2 + \frac{1}{pn}\|S_n^{-1}\|_{tr}^2 - \frac{1}{n}\ tr(S_n^{-1}\Pi_i)
\end{aligned}$$

Thus

$$\widehat{B}_0 = \frac{1}{p}\begin{pmatrix} \frac{p}{n}\|S_n^{-1}\|_F^2 + \frac{1}{n}\|S_n^{-1}\|_{tr}^2 - \frac{p}{n}\ tr(S_n^{-1}\Pi_1)\\ \vdots \\ \frac{p}{n}\|S_n^{-1}\|_F^2 + \frac{1}{n}\|S_n^{-1}\|_{tr}^2 - \frac{p}{n}\ tr(S_n^{-1}\Pi_k)\end{pmatrix}$$

□

Proof of Theorem 4. Note that, in this case, $p/n \to c = 0$ as $n \to \infty$. Then, for $i = 1, \cdots, k$, $\hat{b}_{0i} \overset{a.s.}{\sim} b_{0i} = o(1)$. Hence, $\widehat{B}_0 \overset{a.s.}{\to} 0$ as $n \to \infty$.

On the other hand, for $i, j = 1, \cdots, k$, the (i, j)th element of A_n,

$$\begin{aligned}\frac{1}{p}tr[(S_n^{-1} - \Pi_i)(S_n^{-1} - \Pi_j)] \overset{a.s.}{\sim} a_{0ij} \\ &\overset{a.s.}{\sim} \frac{1}{p}\Big(\|\Sigma_n^{-1}\|_F^2 - tr[\Sigma_n^{-1}(\Pi_i + \Pi_j)] + tr(\Pi_i\Pi_j)\Big) \\ &= \frac{1}{p}tr[(\Sigma_n^{-1} - \Pi_i)(\Sigma_n^{-1} - \Pi_j)]\end{aligned}$$

Thus $\widehat{A}_0 \overset{a.s.}{\sim} A_n^0$.

Notice that

$$\begin{aligned}&\frac{1}{p}tr[(\Sigma_n^{-1} - \Pi_i)(\Sigma_n^{-1} - \Pi_j)] \\ &\leq \frac{1}{p}\|\Sigma_n^{-1}\|_F^2 + \Big(\frac{1}{p}\|\Sigma_n^{-1}\|_F^2\Big)^{1/2}\Big\{\Big(\frac{1}{p}\|\Pi_i\|_F^2\Big)^{1/2} + \Big(\frac{1}{p}\|\Pi_j\|_F^2\Big)^{1/2}\Big\} \\ &\quad + \Big(\frac{1}{p}\|\Pi_i\|_F^2\frac{1}{p}\|\Pi_j\|_F^2\Big)^{1/2} \\ &= O(1)\end{aligned}$$

Then we obtain $\widehat{A}_0^{-1} = O(1)$ almost surely. That is, $\widetilde{\Omega} = \widehat{A}_0^{-1}\widehat{B}_0 \overset{a.s.}{\to} 0$ as $n \to \infty$, This yields $\widehat{\Omega} \to 0$ almost surely.□

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Article

A novel circular dynamics in financial networks with cross-correlated volatility and asset movements

Subhrajit Saha, Debashis Chatterjee*

Department of Statistics, Siksha Bhavana (Institution of Science), Visva Bharati, Bolpur, Santiniketan 731235, India
*** Corresponding author:** Debashis Chatterjee, debashis.chatterjee@visva-bharati.ac.in

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Abstract: In this paper, we propose a novel application of classical directional statistics to model the cross-correlation of asset volatility in financial networks. The proposed novel Circular Volatility Model (CVM) provides a framework for studying the interdependencies of financial assets whose returns exhibit periodic behaviors. By extending traditional volatility models into a circular framework, we establish new pathways for understanding the cyclicity inherent in market dynamics. Our model is rigorously grounded in classical \& directional statistics, utilizing von Mises distributions for parameter estimation and novel circular covariance structures. We offer formal derivations, maximum likelihood estimates, and a novel goodness-of-fit testing framework for this circular model. We establish our methodologies using simulation studies.

Keywords: circular volatility model; von mises distribution; angular data; financial networks; volatility modeling; Maximum Likelihood Estimation (MLE); confidence intervals

1. Introduction

In financial markets, asset returns frequently exhibit cyclic behaviors influenced by market sentiment, external shocks, and cyclical economic conditions. Traditional linear models fail to account for the periodic nature of such phenomena. In contrast, circular statistics allow us to model returns and volatility within a framework that respects their angular nature. This paper introduces the Circular Volatility Model (CVM), which extends the GARCH volatility models into the circular domain. The CVM utilizes von Mises distributions for angular data and incorporates circular cross-correlations between assets in a financial network.

2. Related works

The dynamics of financial networks, particularly in the context of cross-correlated volatility and asset movements, have garnered significant attention in recent years. A growing body of literature has explored the interconnectedness of financial markets and the implications of this interconnectedness for systemic risk and volatility spillovers.

Addresses modeling and analyzing time series of unit two-dimensional vectors, evaluating multiple model classes for feasibility, and recommends a dual-class approach using standard time series algorithms, applied to wind direction data analysis [1].

Explores modeling and analyzing time series of unit two-dimensional vectors, recommends a dual-class approach using standard time series algorithms, and demonstrates its application to wind direction data [2].

Develops the MF-APCCA method to analyze asymmetric risk transmission between cryptocurrencies and global stock markets, showing stronger cross-correlations between Bitcoin and G7 markets than with E7 markets, with gold providing risk-buffering effects [3].

Examines dynamic risk resonance across Chinese market sectors, finding time-varying effects with transportation and utilities as net transmitters, low-frequency events exerting prolonged impact, and crises amplifying resonance [4].

Uses time and frequency connectedness methods to analyze volatility links between 27 emerging markets and seven major cryptocurrencies, finding increased global risk spillovers post-COVID-19, with key risk transmitters and limited diversification benefits for emerging market portfolios [5].

Uses asymmetric multifractal cross-correlation analysis to reveal that gold is the most efficient asset, with green stocks as net transmitters of shocks and halal tourism stocks and oil as net receivers, while a minimum connectedness portfolio offers strong hedging benefits during major economic events [6].

Uses spillover index and wavelet approaches to analyze multiscale relationships among major cryptocurrencies, finding Monero as a key risk transmitter and Ethereum as the main receiver, with enhanced diversification benefits at lower scales in mixed portfolios [7].

Applies a DCC-GARCH model to analyze volatility connectedness among major cryptocurrencies, revealing that low investor sentiment correlates with heightened market connectedness and volatility, while high sentiment supports greater diversification [8].

Examines cryptocurrency integration and contagion during the COVID-19 pandemic, finding mixed integration levels and no contagion, suggesting cryptocurrencies could serve as a good investment during global shocks [9].

Applies artificial neural networks to predict Bitcoin price trends using symmetric volatility attributes, finding high accuracy and identifying the low price as the primary driver for price predictions [10].

Explores volatility spillovers between crude oil prices and cryptocurrencies, finding bidirectional spillovers between oil and Bitcoin, and unidirectional spillovers from oil to Bitcoin Cash and from cryptocurrencies like Ethereum to oil [11].

Suggests inefficiency in the Brazilian stock market, evidenced by strong long-term cross-correlations with foreign markets, fat-tailed log-returns, and successful predictions of IBOVESPA futures using a neural network model [12].

One of the foundational works in this area is by [13], who developed an analytical model to understand contagion in financial networks. They demonstrated that the network's structure significantly influences the probability and impact of contagion, highlighting a robust yet fragile characteristic of financial systems where low probabilities of contagion can lead to widespread effects during crises.

This notion is echoed in the work of [14], who emphasize the importance of understanding financial interdependencies to mitigate systemic risks. Their model illustrates how organizations' values are interlinked through various financial instruments, which can exacerbate the effects of shocks across the network. This approach complements the work of [14], who drew parallels between financial networks and ecological systems, suggesting that complexity can lead to systemic vulnerabilities.

The COVID-19 pandemic has further illuminated the dynamics of financial networks [15] analyzed the connectedness among stock markets during this period, providing a visualization of pandemic risk that parallels financial risk. Their findings suggest that the degree of connectedness can serve as a critical indicator of systemic risk, reinforcing the need for robust network analysis in understanding financial contagion.

Similarly, [16] employed a GARCH model to assess risk spillovers between crude oil and stock markets during the pandemic, revealing complex interdependencies that traditional methods may overlook.

Research on volatility spillovers has also gained traction, particularly in commodity markets. [17] identified a significant shift in the correlation structure between commodities and stock markets post-2008 financial crisis, indicating that financialization has led to increased risk spillovers.

Supports this [18] who found that stock market shocks have a pronounced impact on agricultural commodity price volatility, particularly following major financial disruptions. Their work underscores the growing integration between financial and commodity markets, which complicates the dynamics of volatility transmission.

The role of uncertainty in financial markets has been another focal point of research. [19] highlighted that uncertainty is a central node in the volatility transmission network, influencing spillovers among various financial markets.

This aligns with the findings of [20], who explored the interdependence of financial markets in South and East Asia, revealing that economic shocks can propagate across borders through volatility transmission.

Moreover, the methodological advancements in analyzing financial networks have contributed to a deeper understanding of these dynamics. [21] introduced a metapopulation network model to study the spreading of financial risk, emphasizing the importance of network structure in understanding risk dynamics. The literature on financial networks reveals a complex interplay of interconnectedness, volatility spillovers, and systemic risk. The insights gained from these studies underscore the necessity for comprehensive models that account for the dynamic nature of financial interdependencies. As financial markets continue to evolve, ongoing research will be essential to navigate the challenges posed by these interconnected systems.

The Circular Volatility Model (CVM) proposed in this paper builds upon classical and circular statistical frameworks, particularly the von Mises distribution, which is pivotal in modeling circular data. The von Mises distribution has been extensively utilized in various fields, including bioinformatics and directional data analysis, due to its ability to capture the cyclic nature of phenomena [22] and [23].

For instance, [23] highlighted the fundamental properties of multivariate von Mises distributions, establishing a foundation for analyzing directional data directly applicable to financial asset returns that exhibit periodic behaviors.

In financial markets, herd behavior has been a focal point in understanding volatility interdependencies. [24] provided empirical evidence of herd behavior across global stock markets, indicating that asset returns often move in tandem during periods of high volatility. This phenomenon aligns with the cyclicity that we aim to model in this paper, as it reflects the collective behavior of investors, which can lead to synchronized fluctuations in asset prices.

Similarly, [25] explored the implications of foreign institutional herding in the Taiwanese stock market, further supporting the notion that market dynamics are influenced by collective investor behavior.

The theoretical underpinnings of the CVM are further supported by advancements in statistical methodologies for directional data. For instance, [26] emphasized the importance of the von Mises distribution in antenna design, showcasing its versatility in modeling directional phenomena. This adaptability is mirrored in financial applications, where asset returns' cyclic nature necessitates a similar modeling approach.

This research offers a novel methodology for analyzing cyclical behaviors in asset returns and volatilities, with rigorous mathematical foundations and wide applicability in finance.

3. Objective and novelty of the research

The primary objective of this research is to develop a rigorous and mathematically sophisticated model that captures the directional dependencies between asset returns and volatilities in financial markets. Unlike traditional multivariate models that operate in a linear space, our model addresses the periodicity of financial data. The novelty of this research lies in:

- Extending the classical GARCH model into the circular domain.
- Introducing a new circular cross-correlation measure that captures the angular dependencies between assets.
- Proposing a novel goodness-of-fit test for circular data.
- Establishing formal parameter estimation techniques using von Mises distributions.

The Circular Volatility Model (CVM) extends the traditional GARCH framework into the circular domain, effectively capturing the periodic and directional dependencies inherent in financial asset returns. Through rigorous mathematical formulation and establishing key statistical properties, the CVM provides a robust tool for modeling and analyzing volatility in complex financial networks. This novel application of directional statistics to financial markets opens new avenues for understanding asset dependencies in complex financial networks.

4. Preliminaries

4.1. Circular statistics

Circular statistics is a branch of statistics that deals with data measured in angles or directions. Let $\theta \in [0,2\pi)$ represent a circular variable, such as the direction of a return in a financial asset. In contrast to linear data, the distance between two angles is not Euclidean. For example, the distance between $350°$ and $10°$ is $20°$, not $340°$. Circular data must be analyzed using specialized techniques, such as the von Mises distribution, which is the circular analog of the normal distribution.

4.2. Von mises distribution

The von Mises distribution is given by:

$$f(\theta; \mu, \kappa) = \frac{e^{\kappa \cos(\theta-\mu)}}{2\pi I_0(\kappa)}$$

where μ is the mean direction, κ is the concentration parameter, and $I_0(\kappa)$ is the modified Bessel function of the first kind. This distribution is used extensively in the modeling of angular data because it captures the periodicity inherent in circular variables.

4.3. GARCH model for volatility

The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model, introduced by Bollerslev [27], is a cornerstone in modeling financial time series volatility. The GARCH (1,1) model, in its simplest form, is defined as:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{1}$$

where:

- σ_t^2 is the conditional variance (volatility) at time t.
- $\alpha_0 > 0$ is the constant term.
- $\alpha_1 \geq 0$ measures the reaction of volatility to past shocks (ARCH term).
- $\beta_1 \geq 0$ captures the persistence of volatility (GARCH term).
- $\epsilon_t = r_t - \mu_t$ is the residual or shock at time r_t where μ_t is the return and μt is the mean return.

The GARCH (1,1) model effectively captures the volatility clustering observed in financial time series, where high-volatility periods tend to cluster together, as do low-volatility periods.

5. Circular volatility model (CVM)

5.1. Motivation for circular volatility modeling

Traditional GARCH models operate within a linear framework, assuming that returns and volatilities evolve over a linear time scale. However, financial markets exhibit periodic and cyclical behaviors influenced by various factors such as trading hours, economic cycles, and seasonal effects. To capture these cyclical dynamics, it is advantageous to extend the GARCH framework into the circular domain, where angular dependencies can be explicitly modeled.

Circular data, characterized by angles or directions, naturally encapsulate periodicity and cyclicity. For instance, intra-day trading patterns may exhibit regular cycles corresponding to market opening and closing times. By incorporating circular distance into the GARCH model, we can better model and understand the directional dependencies and cyclical behaviors inherent in financial data.

5.2. Model definition

The Circular Volatility Model (CVM) is designed to extend the traditional GARCH (1,1) model into the circular domain, where asset returns are treated as angular data. This is crucial when dealing with financial systems where periodicity and cyclic behaviors are present, such as intra-day or quarterly trading cycles. The CVM introduces angular cross-correlations between asset volatilities to capture these cyclical patterns more accurately. Let $\{\theta_i(t)\}_{t\in Z}$ denote the sequence of angular returns for asset i, where $\theta_i(t) \in [0,2\pi)$. The CVM for asset i is defined by the following recursive equation:

$$\sigma_i^2(t) = \alpha_0 + \alpha_1 \sum_{j=1}^{N} w_{ij} \cos\left(\theta_i(t) - \theta_j(t)\right) + \beta_1 \sigma_i^2(t-1) \tag{2}$$

where:

- $\sigma_i^2(t)$ is the conditional variance (volatility) of asset i at time t.
- $\alpha_0 > 0$ is the intercept term, representing the baseline level of volatility.
- $\alpha_1 \geq 0$ captures the influence of the angular cross-correlations between asset i and all other assets j = 1, 2, ..., N.
- $\beta_1 \geq 0$ is the autoregressive parameter, reflecting the persistence of volatility over time.
- $w_{ij} \geq 0$ are weights representing the influence of asset j on asset i, satisfying the
- normalization condition $\sum_{j=1}^{N} w_{ij} = 1$
- $cos\left(\theta_i(t) - \theta_j(t)\right)$ captures the circular (angular) correlation between the returns of assets i and j.
- $\theta_i(t) \sim$ von Mises (μ_i, κ_i), indicating that the angular returns follow a von Mises distribution with mean direction μ_i and concentration parameter κ_i.

The incorporation of circular distance $\Delta_{ij}(t)$ into the GARCH framework allows the model to account for the directional alignment between different assets. Unlike the traditional GARCH model, which considers only the magnitude of past shocks, the CGARCH model integrates directional information, thereby enriching the volatility dynamics with cyclical dependencies.

5.3. Interpretation of model components

The term $\cos\left(\theta_i(t) - \theta_j(t)\right)$ measures the alignment between the directional returns of assets i and j. A value close to 1 implies that the assets are moving in the same direction, thus contributing positively to the volatility of asset i. Conversely, a value near -1 indicates opposing directions, potentially reducing the volatility contribution.

The weights w_{ij} allow for differential influences of various assets on asset i, enabling the model to capture complex interdependencies within the financial network.

Consider two assets, A and B, with angular returns $\theta_A(t)$ and $\theta_B(t)$. If their angular difference $\theta_A(t) - \theta_B(t)$ is close to zero, it implies that these assets are moving in a synchronized manner, increasing the volatility of asset A due to a positive correlation.

If the angular difference is close to π, the assets move in opposite directions, contributing less to the overall volatility. The periodic cosine function handles these variations effectively.

The parameters α_0, α_1, β_1, μ_i, and κ_i provide valuable insights into the behavior of asset volatilities:

1) α0 reflects the baseline volatility level, independent of cross-asset interactions.
2) α1 captures the impact of angular cross-correlations, revealing how much the angular returns of other assets influence the volatility of an asset.
3) β1 indicates the persistence of volatility over time, similar to the autoregressive term in the traditional GARCH model.
4) μ_i and κ_i describe the distribution of angular returns, where μ_i represents the average direction and κ_i represents the concentration of returns around this direction.

These insights are essential for portfolio optimization and risk management, especially when assets exhibit periodic or cyclical behaviors.

6. Parameter estimation using maximum likelihood estimation (MLE)

To estimate the parameters of the Circular Volatility Model (CVM), we employ the Maximum Likelihood Estimation (MLE) method. The MLE process enables us to derive parameter estimates that maximize the likelihood of the observed data under the proposed model.

6.1. Model specification

In the Circular Volatility Model, the observed angular returns for asset *i* at time *t*, denoted by $\theta_i(t) \in [0,2\pi)$, follow a Von Mises distribution with mean direction μ_i and concentration parameter κ_i. The conditional variance $\sigma_i^2(t)$ for each asset *i* is governed by the GARCH (1,1) structure extended into the circular domain:

$$\sigma_i^2(t) = \alpha_0 + \alpha_1 \sum_{j=1}^{N} w_{ij} \cos\left(\theta_i(t) - \theta_j(t)\right) + \beta_1 \sigma_i^2(t-1) \tag{3}$$

where:

- $\alpha_0 > 0$ is the baseline level of volatility.
- $\alpha_1 \geq 0$ quantifies the effect of angular cross-correlations with other assets.
- $\beta_1 \geq 0$ represents the autoregressive effect of past volatility.
- $w_{ij} \geq 0$ are weights for the influence of asset *j* on asset *i*.

6.2. Likelihood function construction

The probability density function (PDF) of the von Mises distribution for each angular return $\theta_i(t)$ is:

$$f(\theta_i(t); \mu_i, \kappa_i) = \frac{e^{\kappa_i \cos(\theta_i(t) - \mu_i)}}{2\pi I_0(\kappa_i)} \tag{4}$$

where:

1) μ_i is the mean direction for asset *i*.
2) κ_i is the concentration parameter, analogous to the precision of the distribution.

3) $I_0(\kappa_i)$ is the modified Bessel function of the first kind, serving as a normalization factor.

Given observed data $\theta_i(t)\}_{t=1}^{T}$for each asset *i* and time period *T*, the likelihood function for the CVM is constructed as the product of the individual probabilities for each $\theta_i(t)$:

$$L(\alpha_0, \alpha_1, \beta_1, \{\mu_i\}, \{\kappa_i\}) = \prod_{i=1}^{N} \prod_{t=1}^{T} f(\theta_i(t); \mu_i, \kappa_i) \tag{5}$$

6.3. Log-likelihood function

To facilitate maximization, we take the natural logarithm of the likelihood function, yielding the log-likelihood function:

$$\mathcal{L}(\alpha_0, \alpha_1, \beta_1, \{\mu_i\}, \{\kappa_i\}) = \sum_{i=1}^{N} \sum_{t=1}^{T} \left(\kappa_i \cos(\theta_i(t) - \mu_i) - \log\left(2\pi I_0(\kappa_i)\right)\right) \tag{6}$$

Maximizing this log-likelihood with respect to α_0, α_1, β_1, μ_i, and κ_i yields the MLE estimates for these parameters.

6.4. Step-by-step MLE process

To obtain the MLEs, we differentiate the log-likelihood function with respect to each parameter, set the derivatives equal to zero, and solve the resulting equations.

Differentiation with Respect to μ_i

The partial derivative of $\mathcal{L}$ with respect to μ_i for each asset *i* is:

$$\frac{\partial \mathcal{L}}{\partial \mu_i} = \sum_{t=1}^{T} \kappa_i \sin(\theta_i(t) - \mu_i) \tag{7}$$

Setting $\frac{\partial \mathcal{L}}{\partial \mu_i} = 0$ gives:

$$\sum_{t=1}^{T} \sin(\theta_i(t) - \mu_i) = 0 \tag{8}$$

This equation can be solved numerically to obtain the MLE $\widehat{\mu_i}$ for each asset *i*.

Differentiation with Respect to κ_i

The partial derivative of $\mathcal{L}$ with respect to κ_i is

$$\frac{\partial \mathcal{L}}{\partial \kappa_i} = \sum_{t=1}^{T} \left(\cos(\theta_i(t) - \mu_i) - \frac{I_1(\kappa_i)}{I_0(\kappa_i)}\right) \tag{9}$$

where $I_1(\kappa_i)$ is the modified Bessel function of the first kind of order 1. Setting $\frac{\partial \mathcal{L}}{\partial \kappa_i} =$ 0 gives:

$$\sum_{t=1}^{T} \cos(\theta_i(t) - \mu_i) = T \frac{I_1(\kappa_i)}{I_0(\kappa_i)} \tag{10}$$

Solving this equation numerically provides the MLE κ_i.

Differentiation with Respect to α_0, α_1 and β_1

For the parameters α_0, α_1 and β_1 in the volatility equation, we use iterative methods to find the estimates. Given the recursive structure of $\sigma_i^2(t)$, we calculate the partial derivatives of $\mathcal{L}$ with respect to these parameters, which involve the chain rule and the autoregressive terms.

- Differentiation with respect to α_0:

$$\frac{\partial \mathcal{L}}{\partial \alpha_0} = \sum_{i=1}^{N} \sum_{t=1}^{T} \frac{\partial \sigma_i^2(t)}{\partial \alpha_0} \tag{11}$$

- Differentiation with respect to α_1:

$$\frac{\partial \mathcal{L}}{\partial \alpha_1} = \sum_{i=1}^{N} \sum_{t=1}^{T} \sum_{j=1}^{N} w_{ij} \frac{\partial \, cos\left(\theta_i(t) - \theta_j(t)\right)}{\partial \alpha_1} \tag{12}$$

- Differentiation with respect to β_1:

$$\frac{\partial \mathcal{L}}{\partial \beta_1} = \sum_{i=1}^{N} \sum_{t=1}^{T} \frac{\partial \sigma_i^2(t-1)}{\partial \beta_1} \tag{13}$$

These derivatives do not have closed-form solutions, so we use numerical optimization techniques, such as the Newton-Raphson or Expectation-Maximization (EM) algorithm, to obtain the MLE estimates $\widehat{\alpha_0}, \widehat{\alpha_1}, and\ \widehat{\beta_1}$.

6.5. Numerical optimization

Given the recursive nature of $\sigma_i^2(t)$ and the nonlinear relationship between parameters, numerical optimization methods are essential for solving the MLE equations. We employ an iterative algorithm that alternates between updating $\{\mu_i\}$, $\{\kappa_i\}$, α_0, α_1, and β_1, with the following steps:

(1) Initialize α_0, α_1, β_1, $\{\mu_i\}$ and $\{\kappa_i\}$ with reasonable starting values.

(2) Calculate the conditional variances $\sigma_i^2(t)$ using the current parameter estimates.

(3) Update each parameter by maximizing the log-likelihood with respect to that parameter while holding the others fixed.

(4) Repeat steps 2–3 until convergence, i.e., until changes in parameter estimates fall below a predefined threshold.

This approach yields the final MLE estimates $\widehat{\alpha_0}, \widehat{\alpha_1}, \widehat{\beta_1}, \{\widehat{\mu_i}\}, and\ \{\widehat{\kappa_i}\}$.

6.6. Interpretation of estimated parameters

Once estimated, these parameters provide insights into the behavior of volatility and directional dependencies in the financial network:

- $\widehat{\alpha_0}$: Baseline level of volatility.
- $\widehat{\alpha_1}$: Influence of angular cross-correlations on volatility.
- $\widehat{\beta_1}$: Persistence of volatility over time.
- $\widehat{\mu_i}$: Average directional return of asset *i*.

- $\widehat{\kappa_i}$: Concentration of angular returns around $\widehat{\mu_i}$, indicating the variability in directional movement.

Together, these estimates enable a comprehensive understanding of the dynamics within circular financial data.

7. Stationarity conditions for CVM

This section provides a rigorous mathematical answer for the stationarity condition of the Circular Volatility Model (CVM). The CVM extends the traditional GARCH (1,1) model into the circular domain, accommodating the periodic nature of financial asset returns. This formal definition ensures clarity and lays the foundation for subsequent theoretical properties and estimations. For details of circular time series, we refer to [2] and [28].

Theorem 1. *(Stationarity of CVM). If the Circular Volatility Model (CVM) defined in Equation (3) is strictly stationary, then* $\beta_1 < 1$ *and* $\alpha_1 \cdot \lambda_{max}(W) + \beta_1 < 1$ *here* $\lambda_{max}(W)$ *is the maximum eigenvalue of the weight matrix* $W = [w_{ij}]$.

Proof of Theorem 1. To establish the stationarity of the CVM, we examine the conditions under which the model's conditional variance $\sigma_i^2(t)$ remains stable over time.

Expectation of the CVM Equation:

Taking expectations on both sides of Equation (3), we have:

$$E[\sigma_i^2(t)] = \alpha_0 + \alpha_1 \sum_{j=1}^{N} w_{ij} E\left[\cos\left(\theta_i(t) - \theta_j(t)\right)\right] + \beta_1 E[\sigma_i^2(t-1)]$$

Stationarity Assumption:

Assume that the process is strictly stationary, implying

$$E[\sigma_i^2(t)] = E[\sigma_i^2(t-1)] = \sigma_i^2$$

Substituting into the above equation:

$$\sigma_i^2 = \alpha_0 + \alpha_1 \sum_{j=1}^{N} w_{ij} E\left[\cos\left(\theta_i(t) - \theta_j(t)\right)\right] + \beta_1 \sigma_i^2$$

Solving for σ_i^2:

Rearranging terms:

$$\sigma_i^2(1-\beta_1) = \alpha_0 + \alpha_1 \sum_{j=1}^{N} w_{ij} E\left[\cos\left(\theta_i(t) - \theta_j(t)\right)\right]$$

Therefore:

$$\sigma_i^2 = \frac{\alpha_0 + \alpha_1 \sum_{j=1}^{N} w_{ij} E\left[\cos\left(\theta_i(t) - \theta_j(t)\right)\right]}{(1-\beta_1)}$$

Ensuring Positive Variance:

For σ_i^2 to be positive and finite, the denominator must satisfy $1 - \beta_1 > 0, i.e., \beta_1 < 1$.

Boundedness of the Cosine Expectation:

The expectation $E\left[\cos\left(\theta_i(t) - \theta_j(t)\right)\right]$ is bounded between -1 and 1. Therefore:

$$-\alpha_1 \times \sum_{j=1}^{N} w_{ij} \leq \alpha_1 \sum_{j=1}^{N} w_{ij} E\left[\cos\left(\theta_i(t) - \theta_j(t)\right)\right] \leq \alpha_1 \times \sum_{j=1}^{N} w_{ij}$$

Given that $\sum_{j=1}^{N} w_{ij} = 1$:

$$-\alpha_1 \leq \alpha_1 \sum_{j=1}^{N} w_{ij} E\left[\cos\left(\theta_i(t) - \theta_j(t)\right)\right] \leq \alpha_1$$

Eigenvalue Condition:

To ensure that the impact of the cross-correlations does not destabilize the volatility process, we impose:

$$\alpha_1 \times \lambda_{\max}(W) + \beta_1 < 1$$

where $\lambda_{max}(W)$ is the largest eigenvalue of the weight matrix W. This condition guarantees that the combined influence of cross-correlations and volatility persistence is controlled, ensuring stationarity.

Under the condition $\alpha_1 \times \lambda_{max}(W) + \beta_1 < 1$, the CVM maintains a stable conditional variance, thereby satisfying the criteria for strict stationarity. □

8. Circular covariance structure

In this section, we introduce the novel concept of a circular covariance matrix to model the interdependencies between angular returns of financial assets. In traditional linear statistics, covariance measures how two variables change together. However, when dealing with circular data, such as angles or periodic variables, the standard definition of covariance must be adapted to account for the wrapping nature of the data, where angles $0°$ and $360°$ represent the same point on a circle. Hence, the circular covariance must capture both the angular relationships and the periodicity inherent in such data.

Let $\theta_i(t)$ and $\theta_j(t)$ represent the angular returns of assets i and j at time t, where $\theta_i, \theta_j \in [0,2\pi)$. The covariance between two angular returns can be expressed as a function of their directional differences. For assets i and j, the circular covariance structure is defined as:

$$\Sigma_{\theta_{ij}} = \kappa_i \kappa_j \cos(\mu_i - \mu_j)$$

where:

- κ_i and κ_j are the concentration parameters (analogous to precision) of the von Mises distributions fitted to $\theta_i(t)$ and $\theta_j(t)$, respectively.
- μ_i and μ_j are the mean directions of the angular returns for assets i and j.

The circular covariance matrix Σ_θ for N assets is thus given by:

$$\Sigma_\theta = \begin{pmatrix} \kappa_1^2 & \kappa_1\kappa_2\cos(\mu_1-\mu_2) & \cdots & \kappa_1\kappa_N\cos(\mu_1-\mu_N) \\ \kappa_2\kappa_1\cos(\mu_2-\mu_1) & \kappa_2^2 & \cdots & \kappa_2\kappa_N\cos(\mu_2-\mu_N) \\ \vdots & \vdots & \ddots & \vdots \\ \kappa_N\kappa_1\cos(\mu_N-\mu_1) & \kappa_N\kappa_2\cos(\mu_N-\mu_2) & \cdots & \kappa_N^2 \end{pmatrix}$$

Consider a simple example of $\mathrm{N} = 3$ assets. Let the mean directions of the angular returns for assets 1, 2, and 3 be $\mu_1 = \frac{\pi}{4}$, $\mu_2 = \frac{\pi}{2}$ and $\mu_3 = \frac{3\pi}{4}$ respectively. Furthermore, assume the concentration parameters are $\kappa_1 = 2$, $\kappa_2 = 3$, and $\kappa_3 = 4$. The circular covariance matrix Σ_θ would be:

$$\Sigma_\theta = \begin{pmatrix} 4 & 6\cos\left(\frac{\pi}{4}-\frac{\pi}{2}\right) & 8\cos\left(\frac{\pi}{4}-\frac{3\pi}{4}\right) \\ 6\cos\left(\frac{\pi}{2}-\frac{\pi}{4}\right) & 9 & 12\cos\left(\frac{\pi}{2}-\frac{3\pi}{4}\right) \\ 8\cos\left(\frac{3\pi}{4}-\frac{\pi}{4}\right) & 12\cos\left(\frac{3\pi}{4}-\frac{\pi}{2}\right) & 16 \end{pmatrix}$$

Simplifying the trigonometric terms, we get:

$$\Sigma_\theta = \begin{pmatrix} 4 & 6\cos\left(-\frac{\pi}{4}\right) & 8\cos\left(-\frac{\pi}{2}\right) \\ 6\cos\left(\frac{\pi}{4}\right) & 9 & 12\cos\left(-\frac{\pi}{4}\right) \\ 8\cos\left(\frac{\pi}{2}\right) & 12\cos\left(\frac{\pi}{4}\right) & 16 \end{pmatrix}$$

Given that $\cos(-\theta) = \cos(\theta)$, we can substitute the cosine values to obtain:

$$\Sigma_\theta = \begin{pmatrix} 4 & 6\times\frac{\sqrt{2}}{2} & 8\cdot 0 \\ 6\times\frac{\sqrt{2}}{2} & 9 & 12\times\frac{\sqrt{2}}{2} \\ 8\times 0 & 12\times\frac{\sqrt{2}}{2} & 16 \end{pmatrix} = \begin{pmatrix} 4 & 3\sqrt{2} & 0 \\ 3\sqrt{2} & 9 & 6\sqrt{2} \\ 0 & 6\sqrt{2} & 16 \end{pmatrix}$$

This matrix fully represents the circular dependencies between the three assets. It is evident that the correlation between assets 1 and 3, for example, is zero because the angular separation between them is $\frac{\pi}{2}$, which corresponds to orthogonal directions.

8.1. Properties of the circular covariance matrix

The circular covariance matrix Σ_θ is a fundamental structure for modeling the dependencies between angular variables in a financial network. Unlike traditional covariance matrices, Σ_θ captures the specific properties of circular data, making it essential for accurate modeling in systems where periodicity is inherent, such as in asset returns that exhibit cyclical behavior. In this section, we elaborate on the key properties of Σ_θ and provide context-specific insights into its relevance in financial applications.

8.1.1. Symmetry

The circular covariance matrix Σ_θ retains the symmetry property, a characteristic shared with traditional covariance matrices. For assets i and j, the covariance between their angular returns is determined by:

$$\Sigma_{\theta_{ij}} = \kappa_i \kappa_j \cos(\mu_i - \mu_j)$$

where κ_i and κ_j represent the concentration parameters of the von Mises distribution fitted to the angular returns θ_i and θ_j, and $\mu_i$and μ_j are their mean directions.

Since the cosine function is symmetric, meaning that:

$$\cos(\theta_i - \theta_j) = \cos(\theta_j - \theta_i)$$

it follows that:

$$\Sigma_{\theta_{ij}} = \Sigma_{\theta_{ji}}$$

Thus, Σ_θ is a symmetric matrix. This symmetry ensures that the relationships between pairs of assets are consistently captured, and this is crucial in financial systems where reciprocal dependencies between asset volatilities are common.

8.1.2. Periodicity

The periodic nature of circular data is one of its defining features, and this is reflected in the covariance structure. The cosine function, $\cos(\theta_i - \theta_j)$, is inherently periodic with a period of 2π.

This implies that any circular difference $(\theta_i - \theta_j)$ is equivalent modulo 2π, meaning:

$$\cos(\theta_i - \theta_j) = \cos\left((\theta_i - \theta_j) + 2k\pi\right) \quad \forall k \in Z$$

This property ensures that the covariance between any two assets is invariant under rotations of 2π, which is essential for modeling the cyclic behaviors present in financial markets. For instance, asset prices may exhibit periodic fluctuations tied to daily or quarterly events, and the circular covariance model captures these cycles naturally, even if the events repeat at different times.

8.1.3. Positive semi-definiteness

An important property of any covariance matrix is positive semi-definiteness, which ensures that the variance of any linear combination of variables is non-negative. Formally, for any non-zero vector $v \in R^N$, the quadratic form:

$$v^T \Sigma_{\theta v} \geq 0$$

Must hold. For the circular covariance matrix Σ_θ this property is preserved due to the nature of the cosine function and the structure of the matrix. The positive semi-definiteness can be understood by examining the fact that for any combination of assets i and j, the covariance $\Sigma_{\theta_{ij}} = \kappa_i \kappa_j \cos(\mu_i - \mu_j)$ is bounded by:

$$-\kappa_i \kappa_j \leq \Sigma_{\theta_{ij}} \leq \kappa_i \kappa_j$$

Thus, the matrix is guaranteed to be positive semi-definite. In the context of financial systems, this ensures that the volatility of any portfolio, modeled as a linear combination of asset volatilities, remains non-negative a critical property for risk management and portfolio optimization.

8.2. Estimation of the circular covariance matrix

To estimate the circular covariance matrix Σ_θ from observed data, we rely on the maximum likelihood estimates (MLE) of the von Mises distribution parameters for each asset's angular returns. Let the observed angular returns of asset i at times $(t_1, t_2, \ldots, t_T)$ be denoted by $\theta_i(t_1), \theta_i(t_2), \ldots, \theta_i(t_T)$. The likelihood function for the von Mises distribution of these angular returns is given by:

$$L(\mu_i, \kappa_i) = \prod_{t=1}^{T} \frac{e^{\kappa_i \cos(\theta_i(t) - \mu_i)}}{2\pi I_0(\kappa_i)}$$

where $I_0(\kappa_i)$ is the modified Bessel function of the first kind. The log-likelihood is:

$$\log L\,(\mu_i, \kappa_i) = \sum_{t=1}^{T} \left(\kappa_i \cos(\theta_i(t) - \mu_i) - \log\left(2\pi I_0(\kappa_i)\right)\right)$$

To find the MLE of the parameters μ_i and κ_i, we take the partial derivatives of the log-likelihood function with respect to μ_i and κ_i and solve the resulting equations. This gives us the estimates $\widehat{\mu_\iota} and \widehat{\kappa_\iota}$, which represent the empirical mean direction and concentration parameter for the angular returns of asset i.

The circular covariance between assets i and j is then computed as:

$$\widehat{\Sigma_{\theta_{\iota J}}} = \widehat{\kappa_\iota \kappa_J}\, cos\left(\widehat{\mu_\iota} - \widehat{\mu_J}\right)$$

where $\widehat{\mu_\iota}$ and $\widehat{\mu_J}$ are the estimated mean directions, and $\widehat{\mu_\iota}$ and $\widehat{\mu_J}$ are the estimated concentration parameters for the angular returns of assets i and j.

Consider two assets, A and B, with observed angular returns over $T = 10$ periods. After fitting the von Mises distributions, suppose we obtain the following estimates:

$$\widehat{\mu_A} = \frac{\pi}{4}, \quad \widehat{\kappa_A} = 2, \quad \widehat{\mu_B} = \frac{\pi}{3}, \quad \widehat{\kappa_B} = 3$$

The estimated circular covariance between assets A and B is then:

$$\widehat{\Sigma_{\theta_{AB}}} = 2 \times 3 \cdot \cos\left(\frac{\pi}{4} - \frac{\pi}{3}\right) = 6 \times cos\left(-\frac{\pi}{12}\right) = 6 \times \cos\left(\frac{\pi}{12}\right)$$

Using the known value $\cos\left(\frac{\pi}{12}\right) \approx 0.9659$, we compute:

$$\widehat{\Sigma_{\theta_{AB}}} \approx 6 \times 0.9659 = 5.795$$

This result shows a strong positive correlation between the angular returns of the two assets, indicating that they tend to move in similar directions during the observed periods.

8.2.1. Implications for financial networks

The ability to estimate and interpret circular covariances is critical for understanding the dependencies in a financial network. Assets with high positive circular covariances tend to exhibit synchronized movements in their angular returns, which may indicate shared underlying factors, such as market sentiment or exposure to the same economic events. Conversely, assets with near-zero or negative circular covariances may behave independently or in opposition, providing insights for diversification strategies.

By estimating the full circular covariance matrix for a portfolio of assets, financial analysts can identify clusters of assets with similar directional movements, enabling better portfolio construction and risk management.

8.3. Applications and illustrations

The circular covariance structure has crucial applications in financial systems, particularly in capturing the interdependencies of asset returns and volatilities within cyclic or periodic environments. Below are a few specific applications within the domain of financial networks:

8.3.1. High-frequency trading and periodic market effects

In high-frequency trading (HFT), price movements occur over very short time intervals, often displaying cyclical patterns due to market microstructures, such as the opening and closing of stock exchanges. These periodic behaviors in asset returns can be effectively modeled using circular statistics. Specifically, the covariance between two assets' angular returns can highlight how closely their price fluctuations align during specific periods of the trading day.

For example, consider two assets, i and j, traded on a market with clearly defined opening and closing times. The price changes of these assets, measured at intervals $(t_1, t_2, \dots, t_T)$, exhibit directional behavior due to the predictability of certain times of the day when large trades or high volatility occurs (e.g., opening, lunchtime, and closing). Using the circular covariance:

$$\Sigma_{\theta_{ij}} = \kappa_i \kappa_j \cos(\mu_i - \mu_j)$$

We can capture the inter-asset dependency based on these recurring periods. For example, asset i might consistently spike at market opening due to high liquidity, while asset j follows similar behavior but is more volatile during the closing hours. By calculating the circular covariance, we measure how synchronized these price movements are over time, which linear methods might miss due to the circular nature of time in financial markets.

In practical terms, this insight could be used by high-frequency traders to optimize their algorithms based on the alignment of assets during key times of the day. Circular covariance allows traders to better understand interdependencies in magnitude, timing, and directional flow.

8.3.2. Volatility clustering in international markets

Global financial markets exhibit strong evidence of volatility clustering, where periods of high volatility are followed by further high volatility, and similarly for low volatility. This clustering often coincides with cyclical events such as national elections, central bank meetings, or earnings reports, which happen periodically across different time zones and economies.

In an international context, the circular covariance matrix can provide a more nuanced view of these volatility clusters across different markets. Let's consider two markets, the U.S. stock market and the European stock market. Their volatilities may peak at different times during a 24-hour cycle. For example, the U.S. market opens after the European market has already been trading for several hours. The circular covariance between their volatilities could be calculated as:

$$\Sigma_{\theta_{ij}} = \kappa_{\mathrm{US}}\kappa_{\mathrm{EU}} \cos(\mu_{\mathrm{US}} - \mu_{\mathrm{EU}})$$

where μ_{US} and μ_{EU} are the mean directions representing the typical times of high volatility for the U.S. and European markets, respectively, and κ_{US} and κ_{EU} represent the concentration of this volatility around these periods.

For instance, if the U.S. market tends to experience a volatility spike around 9:30 AM Eastern Time, and the European market around 3:00 PM Central European Time, the circular covariance will reflect the phase difference between these events. This allows portfolio managers to strategically align their investment strategies based on the synchronized behavior of these markets, using circular covariance to measure cross-market volatility dependencies.

8.3.3. Asset correlations during periodic macroeconomic events

Financial markets are frequently affected by periodic macroeconomic events such as central bank interest rate announcements, corporate earnings releases, and even seasonal patterns as year-end market closures. These events tend to create periodicity in asset returns and volatilities. Traditional linear covariance metrics might fail to capture the cyclic nature of how different assets respond to these events.

For instance, consider the behavior of technology stocks during quarterly earnings season. Suppose asset i (a large technology firm) tends to have significant price movement during earnings announcements in the second week of each quarter. Similarly, asset j (another firm in the same sector) experiences directional movement in the same period, but the peak movement might occur a few days earlier due to different release schedules. The circular covariance:

$$\Sigma_{\theta_{ij}} = \kappa_i\kappa_j \cos(\mu_i - \mu_j)$$

Measures the phase difference between these assets' responses. The circular covariance structure allows investors to understand whether these assets' volatilities tend to synchronize over these periodic events, offering critical insights into how correlated movements evolve cyclically over time.

By applying this method across multiple assets and event windows, fund managers can build a portfolio strategy that accounts for the periodic dependencies between assets, allowing for better risk management and return optimization.

8.3.4. Illustration: Synchronization in financial cycles

To further illustrate the circular covariance structure, consider the following hypothetical scenario. Two large financial institutions, A and B, trade two major currency pairs, EUR/USD and GBP/USD. Each institution has a trading strategy that tends to execute large trades at similar times daily based on global market opening hours.

Let $\mu_A = \frac{\pi}{3}$ represent the typical mean direction of institution A's trading window, and $\mu_B = \frac{\pi}{2}$ represent that of institution B. Let the concentration parameters, $\kappa_A = 5$

and $\kappa_B = 6$, reflect how focused these institutions are in trading at those specific times. The circular covariance between their trading times can be calculated as:

$$\Sigma_{\theta_{AB}} = 5 \times 6 \times \cos\left(\frac{\pi}{3} - \frac{\pi}{2}\right) = 30 \times \cos\left(-\frac{\pi}{6}\right) = 30 \times \frac{\sqrt{3}}{2} = 15\sqrt{3}$$

This positive covariance indicates a strong synchronization between the two institutions' trading windows, suggesting that their behaviors are highly correlated during these periods. Such information could be crucial for other market participants anticipating liquidity surges or price impacts due to synchronized institutional trades.

9. Goodness-of-fit test for circular data

To test the fit of the Circular Volatility Model, we propose a goodness-of-fit test based on a circular chi-squared statistic. The null hypothesis is that the observed angular returns follow a von Mises distribution. The test statistic is:

$$\chi^2 = \sum_{i=1}^{N} \frac{\left(\widehat{\theta_i} - \mu_i\right)^2}{\sigma_i^2}$$

where $\widehat{\theta_i}$ is the observed angular return, μ_i is the mean direction, and σ_i^2 is the volatility. This test provides a rigorous mechanism for assessing the model's goodness of fit.

10. Simulation study: Circular volatility model and parameter estimation

10.1. Simulation of angular data

We simulate angular returns for three assets, denoted as $\theta_1(t), \theta_2(t)$ and $\theta_3(t)$, at time $t = 1,2,\ldots..100$, from a von Mises distribution. The true mean directions μ and concentration parameters κ for each asset are specified as:

$$\mu_1 = \frac{\pi}{4}, \quad \mu_2 = \frac{\pi}{2}, \quad \mu_3 = \frac{3\pi}{4}$$

$$\kappa_1 = 2, \quad \kappa_2 = 3, \quad \kappa_3 = 4$$

10.2. Parameter estimation

To estimate the mean directions μ and concentration parameters κ from the simulated angular data, we use the maximum likelihood estimation (MLE) method. The estimated parameters are denoted by $\hat{\mu}$ and $\hat{\kappa}$.

The formulae used for parameter estimation are as follows:

$$\hat{\mu} = \text{atan2}\left(\sum_{t=1}^{T} \sin(\theta_t), \sum_{t=1}^{T} \cos(\theta_t)\right)$$

$$\hat{\kappa} = \text{function of the mean resultant length } R = \sqrt{C^2 + S^2}$$

where C and S are the averages of $\cos(\theta_t)$ and $\sin(\theta_t)$, respectively.

The results are shown in **Table 1**.

Table 1. True vs. estimated parameters of the von mises distribution.

Asset	True μ	Estimated μ	True κ	Estimated κ
1	0.7854	0.6716	2	1.8597
2	1.5708	1.5683	3	3.6196
3	2.3562	2.3778	4	3.6504

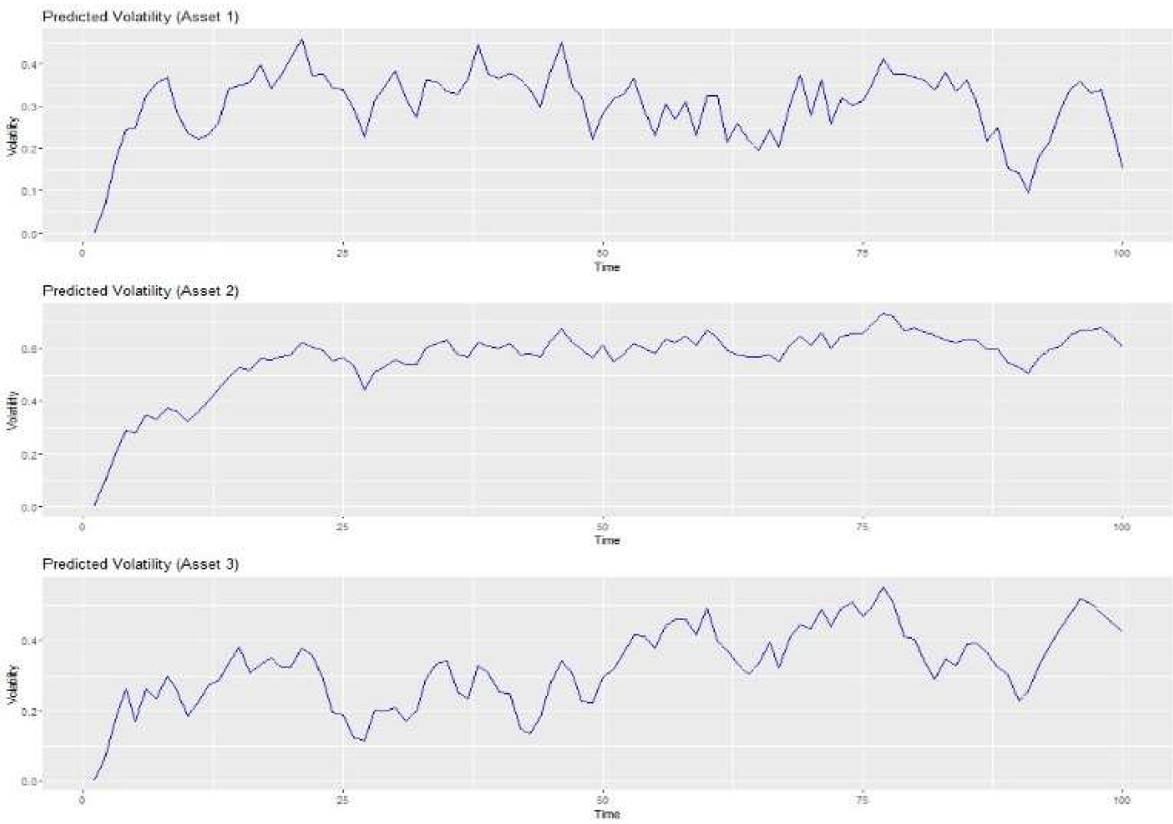
10.3. Circular volatility model

The volatility of each asset is modeled using a Circular Volatility Model (CVM) that extends the GARCH (1,1) model to angular data. The volatility at time t for asset i is defined as:

$$\sigma_i^2(t) = \alpha_0 + \alpha_1 \sum_{j=1}^{N} \cos\left(\theta_i(t) - \theta_j(t)\right) + \beta_1 \sigma_i^2(t-1)$$

where $\alpha_0 = 0.02, \alpha_1 = 0.05$ *and* $\beta_1 = 0.9$ are the model parameters, and $N = 3$ is the number of assets. This model captures both the autoregressive nature of volatility and the angular correlation between the returns of different assets.

Figure 1 shows the predicted volatility for each asset over time.



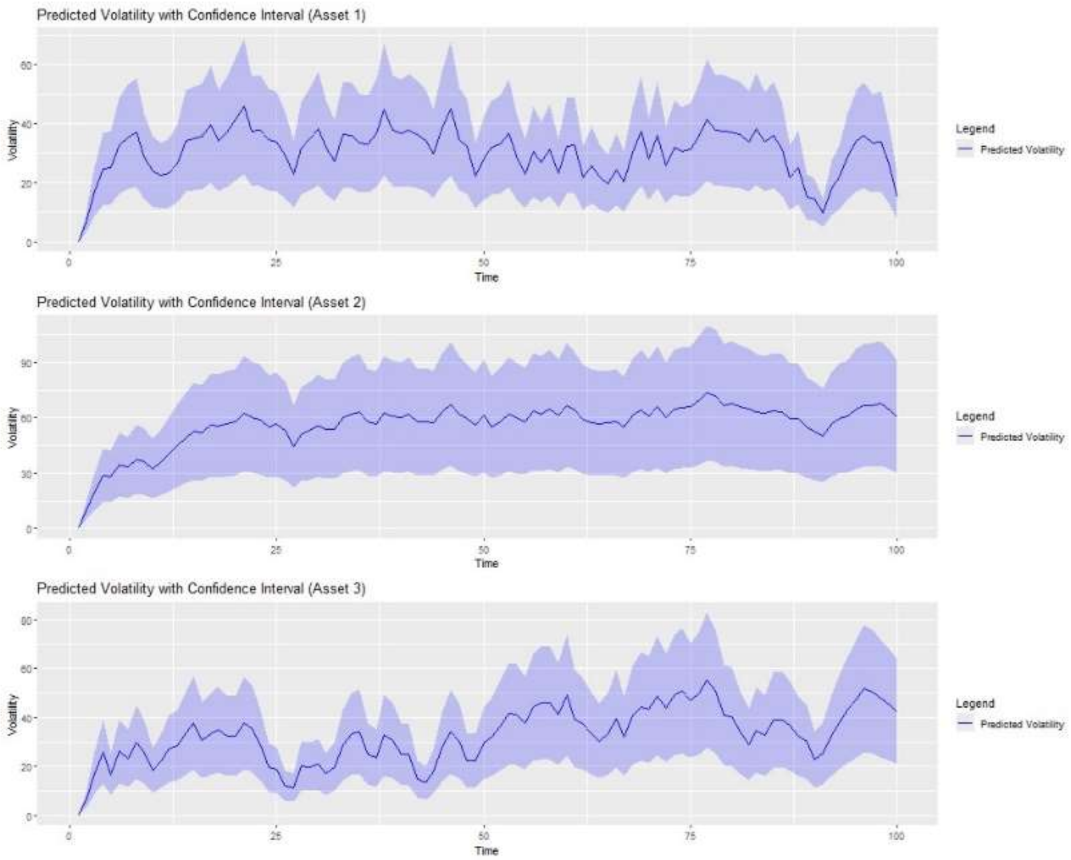


Figure 1. Predicted volatility for Asset 1, Asset 2, and Asset 3 over time, with confidence intervals (scaled with 100 for better visual.

10.4. Circular histograms (rose plots)

The circular histograms (rose plots) for the angular returns of each asset provide a visual representation of the distribution of angular data. The histograms are shown in **Figure 2**. Each plot illustrates how the respective assets' angular returns are distributed across different angles.

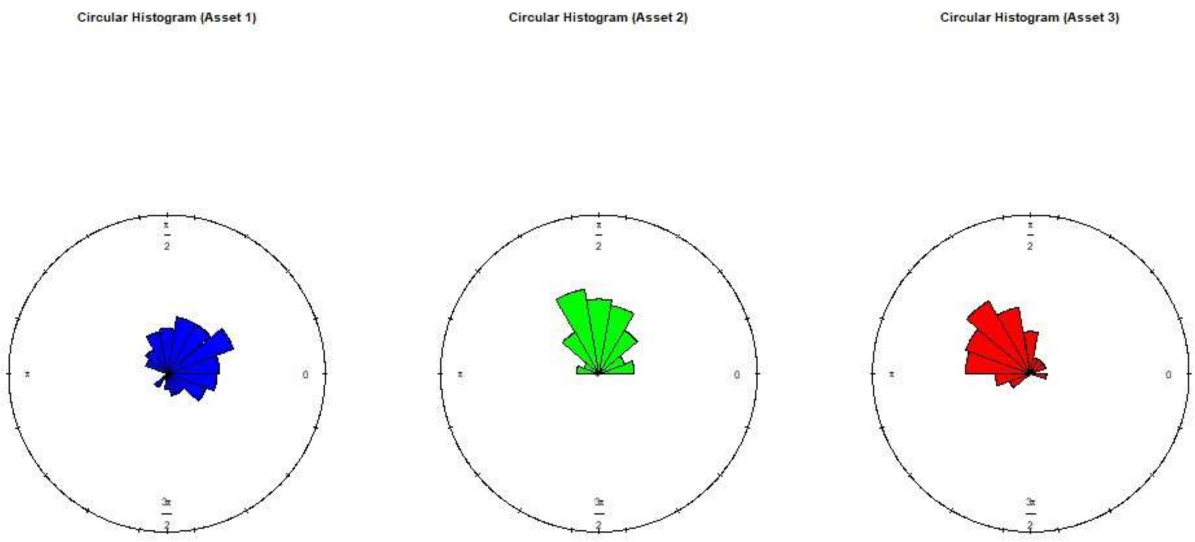


Figure 2. Circular histograms (rose plots) for the angular returns of Asset 1, Asset 2 and Asset 3.

10.5. Comparison of true vs. estimated mean directions

To visually compare the true and estimated mean directions μ for each asset, we plot the directions as arrows on a circle originating from the center. The true mean directions are shown in red, while the estimated ones are in blue. This visualization helps assess how close the estimated μ values are to the true values.

Figure 3 shows the circular plot for the true and estimated mean directions.

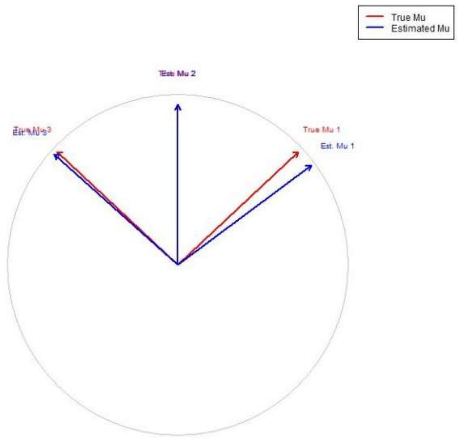


Figure 3. Comparison of true and estimated mean directions (μ) for each asset.

Red arrows represent true μ, and blue arrows represent estimated μ.

10.6. Goodness-of-fit test

he goodness-of-fit of the von Mises distribution for each asset is assessed using the Rayleigh test for uniformity. The null hypothesis of the Rayleigh test is that the data is uniformly distributed on the circle, and a low $p-value$ indicates a good fit to the von Mises distribution.

Table 2 shows each asset's $p-values$ from the Rayleigh test.

Table 2. Goodness-of-fit results (rayleigh test).

Asset *p*-value	**Asset *p*-value**
1	1.70×10^{-20}
2	5.32×10^{-32}
3	4.11×10^{-32}

The extremely low p-values indicate that the angular data for all assets fits the von Mises distribution well.

10.7. Overlay of simulated data, predicted volatility, and confidence intervals

In this subsection, we analyze the overlay of simulated angular returns with the predicted volatility generated by the Circular Volatility Model (CVM). Confidence intervals around the predicted volatility are also plotted to visually assess the variability in the volatility estimates. The confidence intervals are constructed by applying a 15% margin above and below the predicted volatility, giving a visual range for uncertainty around the model predictions.

We simulate the data for three assets and compute the predicted volatility based on the CVM model. The confidence intervals are defined as:

$$\text{Lower Bound} = \sigma^2_{\text{predicted}} \times 0.85$$

$$\text{Upper Bound} = \sigma^2_{\text{predicted}} \times 1.15$$

This provides a range of possible volatility values that capture the uncertainty in the model's predictions.

Each plot shows:

Simulated Data: Represented by a dashed green line corresponding to each asset's actual angular returns.

Predicted Volatility: Represented by a solid blue line, corresponding to the volatility predicted by the CVM model for each asset.

Confidence Interval: A blue-shaded region displays the uncertainty range of the predicted volatility for each asset.

Figure 4 illustrates each asset's overlay.

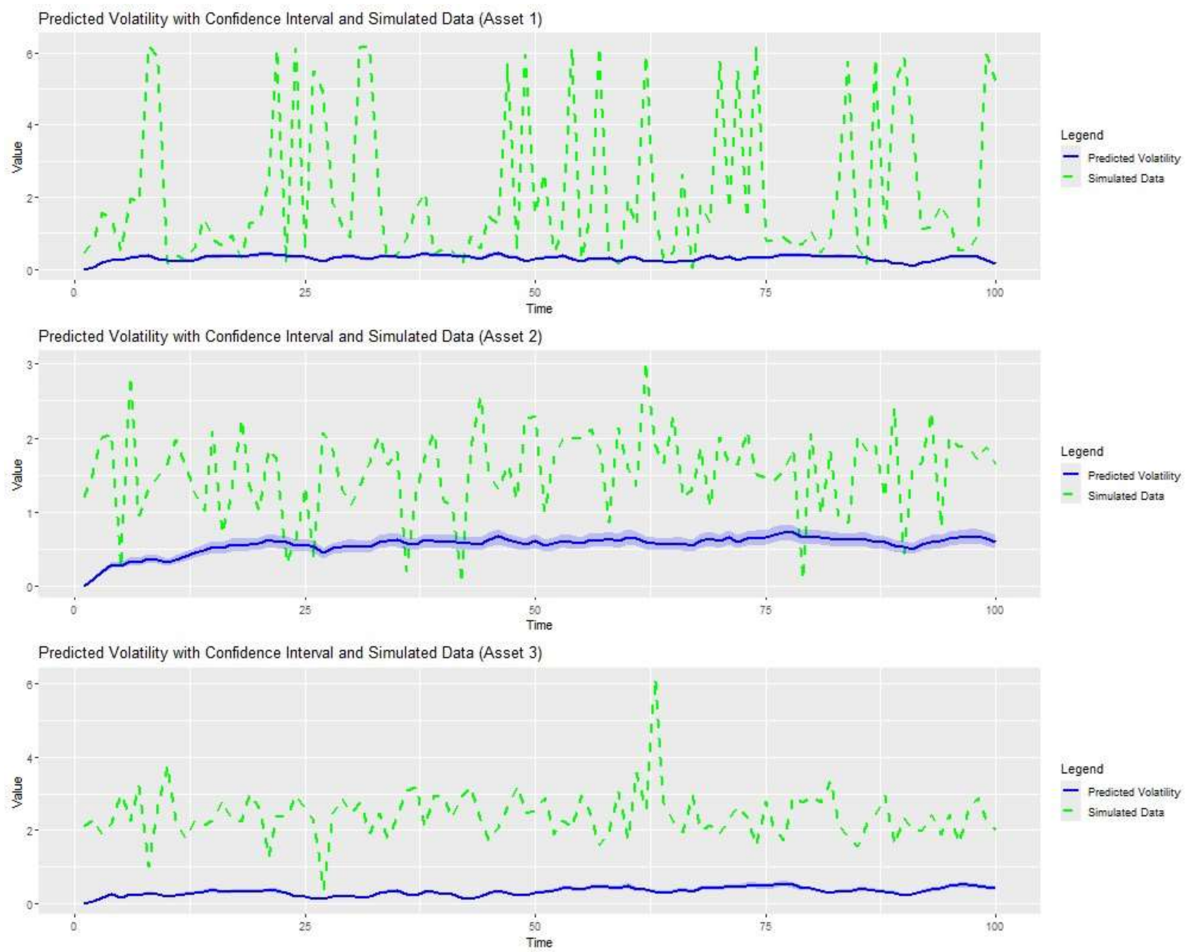


Figure 4. Overlay of simulated data, predicted volatility, and confidence intervals for Asset 1, Asset 2, and Asset 3.

The dashed green line represents the simulated angular returns, the solid blue line represents the predicted volatility, and the shaded blue area represents the confidence interval around the predicted volatility.

The overlay of simulated data predicted volatility and confidence intervals offers valuable insights into the performance of the Circular Volatility Model. For all three assets, the predicted volatility closely tracks the underlying angular returns, with the confidence intervals providing a meaningful range of uncertainty around the predictions.

10.8. Estimation errors

To quantify the difference between the true and estimated parameters, we compute the error for both μ and κ as:

$$\text{Error in } \mu = \hat{\mu} - \mu, \quad \text{Error in } \kappa = \hat{\kappa} - \kappa$$

The errors are shown in **Table 3**.

Figure 5 shows bar plots of the errors in μ and κ for each asset.

Table 3. Errors in Estimated parameters.

Asset	Error in μ	Error in κ
1	−0.1138	−0.1403
2	−0.0025	0.6196
3	0.0216	−0.3496

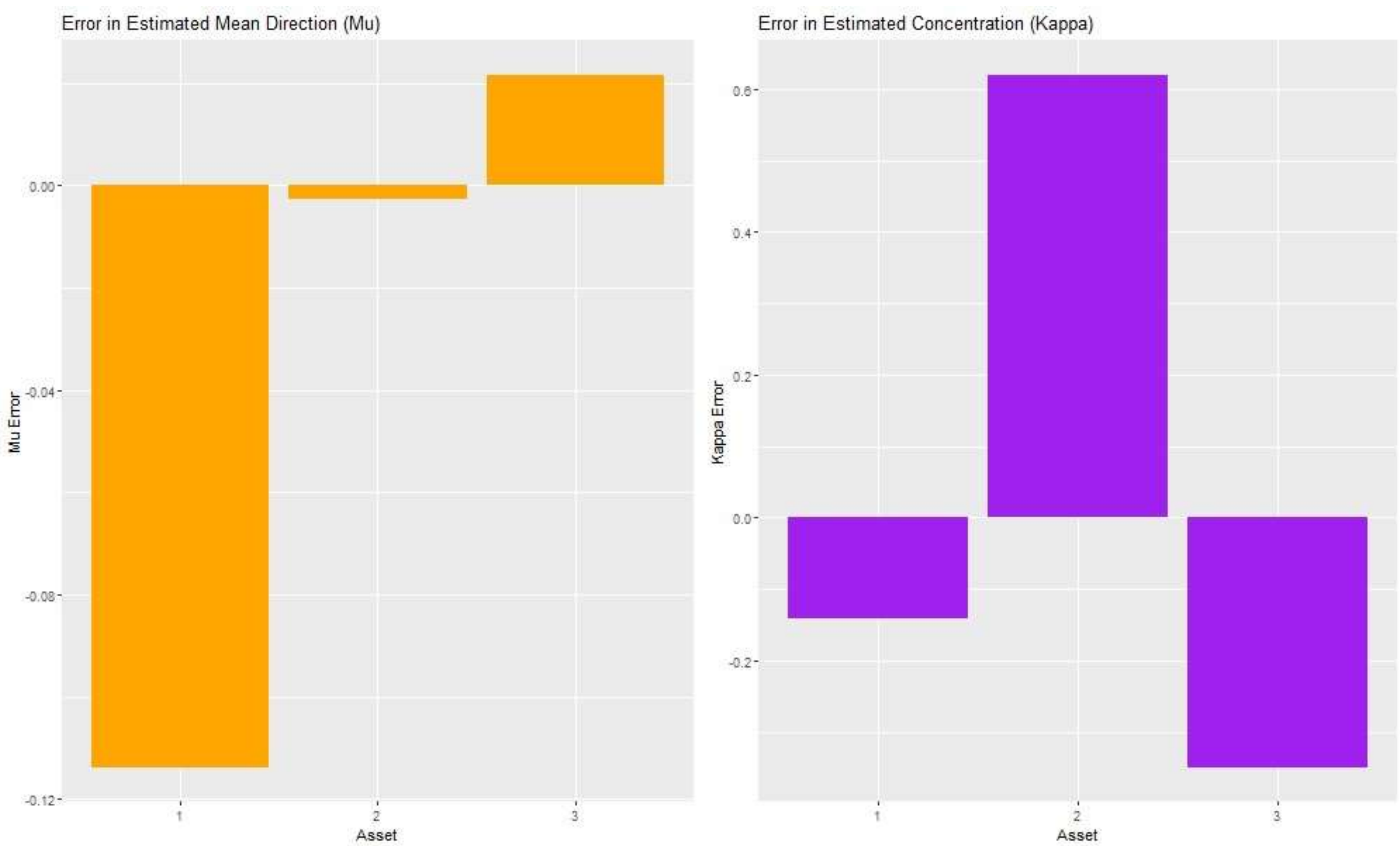


Figure 5. Errors in estimated mean direction (μ) and concentration (κ) for each asset.

10.9. Residual diagnostics using QQ-plots

In this subsection, we present the QQ-plots for the residuals of the predicted volatility for each asset. These plots assess whether the Circular Volatility Model (CVM) residuals follow a normal distribution, a key assumption for the model's validity.

Residuals are the differences between the simulated data and the predicted volatility. For this model to be well-calibrated, we expect the residuals to approximately follow a normal distribution. If the QQ-plot shows a linear pattern, it indicates that the residuals are normally distributed. Deviations from linearity would suggest potential problems with the model's assumptions.

The QQ-plots for residuals of the three assets are shown in **Figure 6**. Each plot compares the quantiles of the residuals with the theoretical quantiles of a normal distribution.

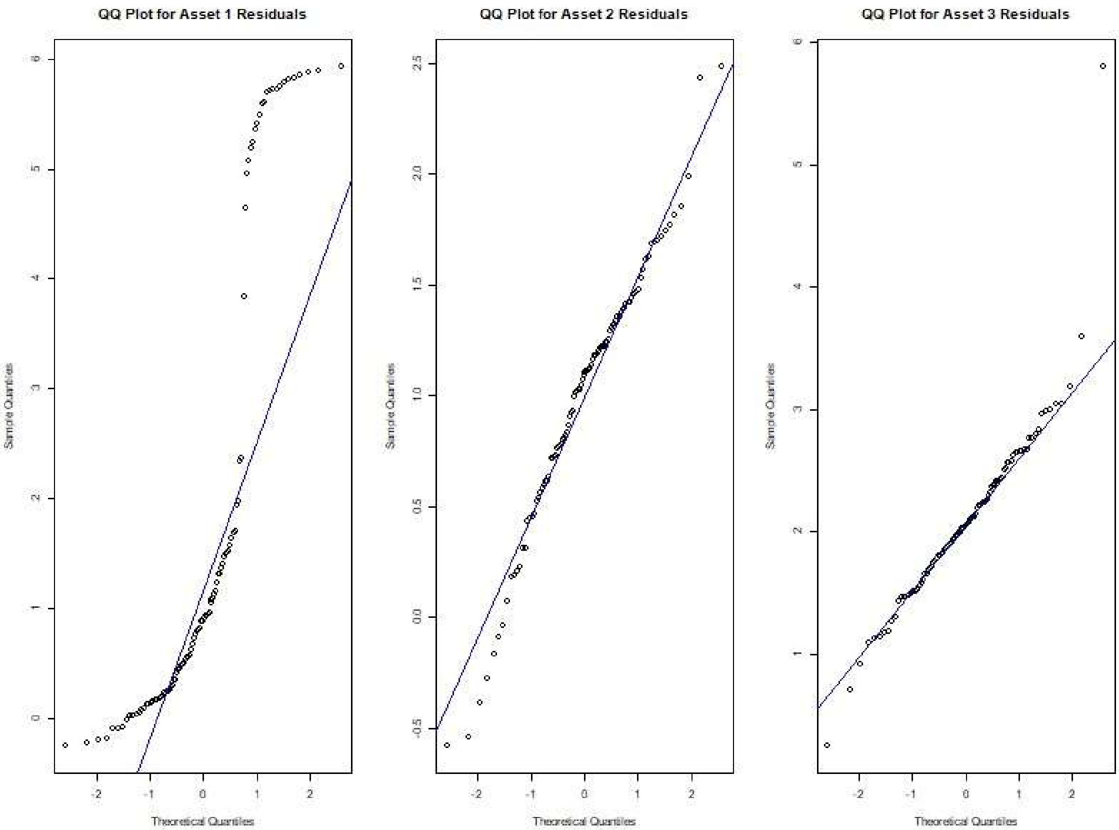


Figure 6. QQ-plots for residuals of the predicted volatility for the three assets.

The residuals are compared against the theoretical quantiles of a normal distribution. Deviations from the diagonal line indicate departures from normality.

Asset 1 Residual: The QQ-plot for Asset 1 shows that the residuals generally follow a normal distribution, as most points lie along the reference line. Some deviations can be observed at the tails. This suggests that the model may have over- or under-predicted volatility for extreme asset returns values.

Asset 2 Residuals: The QQ-plot for Asset 2 shows almost no deviations from normality and is within an acceptable range.

Asset 3 Residuals: Similar to Asset 2, the residuals for Asset 3 ALSO show NEGLIGIBLE deviations from normality, except very few in the extreme quantiles. These deviations might suggest potential issues with the model for higher volatility levels or extreme returns.

The QQ plots indicate that Assets 2 and 3 residuals are approximately normally distributed, supporting the model's validity for this asset. However, Asset 1 shows some deviations from normality, especially in the tails, indicating that the model might not fully capture the volatility dynamics for these assets. Further model refinements or alternative models could be considered to better capture the behavior of the residuals for these assets.

10.10. Summary of results

The parameter estimation and volatility modeling results are summarized in **Table 4**, which includes the true and estimated parameters, the estimation errors, and the $p - values$ from the Rayleigh test.

Table 4. Summary of results.

Asset	True μ	Estimated μ	True κ	Estimated κ	μ Error	κ Error
1	0.7854	0.6716	2	1.8597	−0.1138	−0.1403
2	1.5708	1.5683	3	3.6196	−0.0025	0.6196
3	2.3562	2.3778	4	3.6504	0.0216	−0.3496

10.11. Interpretation of the results

The results in **Table 4** demonstrate the accuracy of the estimated parameters for the von Mises distribution. The estimated mean directions μ are close to the true values, with small errors for all three assets. The concentration parameters κ are reasonably well estimated, although Asset 2 has a slightly larger error for κ. This is likely due to the lower concentration of the data in Asset 2 compared to Asset 1 and Asset 3.

The goodness-of-fit tests, with extremely low $p - values$, confirm that the simulated angular data fits the von Mises distribution well for all three assets. This is consistent with the data being generated using a von Mises distribution.

10.12. Visual assessment

To further assess the quality of the parameter estimation, we visualize the true vs. estimated mean directions using arrows on a circular plot, as shown in **Figure 3**. This plot demonstrates that the estimated mean directions are very close to the true directions for all three assets, reinforcing the accuracy of the estimation process.

Additionally, the predicted volatility over time for each asset is shown in **Figure 1**. The volatility predicted by the Circular Volatility Model (CVM) captures the cyclical nature of the angular data, and the model appears to perform well for all three assets.

The circular histograms (rose plots) in **Figure 2** also provide a clear view of the angular distribution of returns for each asset, showing the concentration of angular data around the estimated mean directions.

10.13. Discussion

In this study, we simulated angular returns for three assets using the von Mises distribution, estimated the distribution parameters using maximum likelihood estimation, and modeled the volatility of the assets using a Circular Volatility Model. The results demonstrate that the parameter estimation is accurate, as evidenced by the low errors in the estimated parameters and the goodness-of-fit tests. The Circular Volatility Model also performed well in capturing the volatility patterns in the angular data.

These findings show the utility of the von Mises distribution and CVM in analyzing angular data, particularly for applications in finance, biology, and other fields where circular data plays a significant role.

11. Conclusion

This paper introduces a novel circular volatility model for analyzing cross-correlated financial networks. The model incorporates directional dependencies between asset returns and extends traditional volatility models into the circular domain. Using the von Mises distribution, we propose maximum likelihood estimation techniques and introduce a circular covariance structure to capture interdependencies between assets. A goodness-of-fit test based on the circular chi-squared statistic is also proposed. The Circular Volatility Model offers new insights into the periodic behavior of financial systems and has potential applications in various fields beyond finance.

The Circular Covariance Structure offers a powerful tool for analyzing interdependencies between assets in financial markets, especially where periodicity and cyclic behavior are key drivers of price movements and volatility. By adapting traditional covariance to the circular domain, this model provides a more accurate representation of cross-asset dynamics and offers novel insights that linear methods might overlook. The applications discussed highlight the potential for this model to enhance risk management, optimize trading strategies, and improve our understanding of market behavior in both national and international contexts.

12. Future work

Future research can explore extensions of the CVM, such as incorporating higher-order circular dependencies, integrating external covariates, or developing Bayesian estimation frameworks to further enhance the model's applicability and robustness in diverse financial contexts. Several avenues for future work that can build upon the findings of this research:

- Application to real-world financial data: While the current study focuses on simulated data, applying the Circular Volatility Model (CVM) to real-world financial data, such as currency exchange rates or asset prices, would provide further validation of the model's practical utility.
- Extension to multivariate circular models: Future work could explore the extension of the CVM to multivariate frameworks, where multiple angular variables are analyzed simultaneously. This would enable the modeling of

complex dependencies in financial networks or other systems with interdependent angular data.

- Incorporating external covariates: The inclusion of external covariates, such as economic indicators or market sentiment, could enhance the predictive power of the CVM. Exploring how these covariates influence volatility in angular data would be a valuable extension of the current model.
- Generalization to other domains: Beyond financial networks, the CVM framework can be adapted to other fields such as meteorology (e.g., wind direction analysis), biology (e.g., animal movement), or geophysics (e.g., earthquake directionality). Future research can explore these applications and refine the model to address domain-specific challenges.
- Bayesian approaches: Implementing Bayesian versions of the Circular Volatility Model could offer additional insights into parameter uncertainty, providing a probabilistic framework that complements the current MLE-based approach.

Future research can continue to explore and expand the applicability of the Circular Volatility Model to different domains, enhancing our understanding of circular dynamics and volatility in various contexts.

Author contributions: Conceptualization, DC and SS; methodology, DC and SS; software, DC and SS; validation, DC and SS; formal analysis, DC and SS; investigation, DC and SS; resources, DC and SS; data curation, DC and SS; writing—original draft preparation, DC and SS; writing—review and editing, DC and SS; visualization, DC and SS.

Code availability statement: The R code used to generate the results in this paper is available at \url{https://github.com/debashisdotchatterjee/Circular-Dynamics-in-Financial-Networks}. The code is released under the Public license.

Conflict of interest: The authors declare no conflict of interest.

References

1. Abdullah M, Chowdhury MAF, Sulong Z. Asymmetric efficiency and connectedness among green stocks, halal tourism stocks, cryptocurrencies, and commodities: Portfolio hedging implications. Resources Policy. 2023; 81: 103419. doi: 10.1016/j.resourpol.2023.103419
2. Adams Z, Glück T. Financialization in commodity markets: A passing trend or the new normal? Journal of Banking & Finance. 2015; 60: 93-111. doi: 10.1016/j.jbankfin.2015.07.008
3. Ali Z, Ullah I, Ahmad A. Financial markets interdependence: Evidence from South and East Asian countries. Pakistan Social Sciences Review. 2022; 6(2). doi:10.35484/pssr.2022(6-ii)71
4. Almeida D, Dionísio A, Vieira I, et al. COVID-19 Effects on the Relationship between Cryptocurrencies: Can It Be Contagion? Insights from Econophysics Approaches. Entropy. 2023; 25(1): 98. doi: 10.3390/e25010098
5. Balcilar M, Ozdemir H, Agan B. Effects of COVID-19 on cryptocurrency and emerging market connectedness: Empirical evidence from quantile, frequency, and lasso networks. Physica A: Statistical Mechanics and its Applications. 2022; 604: 127885. doi: 10.1016/j.physa.2022.127885
6. Baldi L, Peri M, Vandone D. Stock markets' bubbles burst and volatility spillovers in agricultural commodity markets. Research in International Business and Finance. 2016; 38: 277-285. doi: 10.1016/j.ribaf.2016.04.020
7. Bollerslev T. Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics. 1986; 31(3): 307-327. doi: 10.1016/0304-4076(86)90063-1

8. Bouri E, Gabauer D, Gupta R, et al. Volatility connectedness of major cryptocurrencies: The role of investor happiness. Journal of Behavioral and Experimental Finance. 2021; 30: 100463. doi: 10.1016/j.jbef.2021.100463
9. Chen Y, Yang S, Lin F. Foreign institutional industrial herding in Taiwan stock market. Managerial Finance. 2012; 38(3): 325-340. doi: 10.1108/03074351211201442
10. Chiang TC, Zheng D. An empirical analysis of herd behavior in global stock markets. Journal of Banking & Finance. 2010; 34(8): 1911-1921. doi: 10.1016/j.jbankfin.2009.12.014
11. Fang T, Su Z. Does uncertainty matter for US financial market volatility spillovers? Empirical evidence from a nonlinear Granger causality network. Applied Economics Letters. 2020; 28(21): 1877-1883. doi: 10.1080/13504851.2020.1854656
12. Figueiredo AMS. Goodness-of-fit for a concentrated von Mises-Fisher distribution. Computational Statistics. 2011; 27(1): 69-82. doi: 10.1007/s00180-011-0238-4
13. Fisher NI, Lee AJ. Time Series Analysis of Circular Data. Journal of the Royal Statistical Society Series B: Statistical Methodology. 1994; 56(2): 327-339. doi: 10.1111/j.2517-6161.1994.tb01981.x
14. Gai P, Kapadia S. Contagion in financial networks. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences. 2010; 466(2120): 2401-2423. doi: 10.1098/rspa.2009.0410
15. Haldane AG, May RM. Systemic risk in banking ecosystems. Nature. 2011; 469(7330): 351-355. doi: 10.1038/nature09659
16. Harvey A, Hurn S, Palumbo D, et al. Modelling circular time series. Journal of Econometrics. 2024; 239(1): 105450. doi: 10.1016/j.jeconom.2023.02.016
17. Holzmann H, Munk A, Suster M, et al. Hidden Markov models for circular and linear-circular time series. Environmental and Ecological Statistics. 2006; 13(3): 325-347. doi: 10.1007/s10651-006-0015-7
18. Lin M, Duan L. Financial Risk Information Spreading on Metapopulation Networks. Complexity. 2021; 2021(1). doi: 10.1155/2021/6654169
19. Mardia KV, Voss J. Some Fundamental Properties of a Multivariate von Mises Distribution. Communications in Statistics - Theory and Methods. 2014; 43(6): 1132-1144. doi: 10.1080/03610926.2012.670353
20. Mei-jun L, Guang-xi C. Dynamics of asymmetric multifractal cross-correlations between cryptocurrencies and global stock markets: Role of gold and portfolio implications. Chaos, Solitons & Fractals. 2024; 182: 114739. doi: 10.1016/j.chaos.2024.114739
21. Mensi W, Rehman MU, Vo XV, et al. Spillovers and multiscale relationships among cryptocurrencies: A portfolio implication using high frequency data. Economic Analysis and Policy. 2024; 82: 449-479. doi: 10.1016/j.eap.2024.03.021
22. Okorie DI, Lin B. Crude oil price and cryptocurrencies: Evidence of volatility connectedness and hedging strategy. Energy Economics. 2020; 87: 104703. doi: 10.1016/j.eneco.2020.104703
23. Othman AHA, Kassim S, Rosman RB, et al. Prediction accuracy improvement for Bitcoin market prices based on symmetric volatility information using artificial neural network approach. Journal of Revenue and Pricing Management. 2020; 19(5): 314-330. doi: 10.1057/s41272-020-00229-3
24. Ren J, Vaughan RG. Spaced Antenna Design in Directional Scenarios Using the Von Mises Distribution. 2009 IEEE 70th Vehicular Technology Conference Fall. 2009; 1-5. doi: 10.1109/vetecf.2009.5379012
25. Rocha Filho TM, Rocha PMM. Evidence of inefficiency of the Brazilian stock market: The IBOVESPA future contracts. Physica A: Statistical Mechanics and its Applications. 2020; 543: 123200. doi: 10.1016/j.physa.2019.123200
26. So MKP, Chan LSH, Chu AMY. Financial Network Connectedness and Systemic Risk During the COVID-19 Pandemic. Asia-Pacific Financial Markets. 2021; 28(4): 649-665. doi: 10.1007/s10690-021-09340-w
27. Tao C, Zhong GY, Li JC. Dynamic correlation and risk resonance among industries of Chinese stock market: New evidence from time–frequency domain and complex network perspectives. Physica A: Statistical Mechanics and its Applications. 2023; 614: 128558. doi: 10.1016/j.physa.2023.128558
28. Zhu P, Tang Y, Wei Y, et al. Multidimensional risk spillovers among crude oil, the US and Chinese stock markets: Evidence during the COVID-19 epidemic. Energy. 2021; 231: 120949. doi: 10.1016/j.energy.2021.120949

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Article

Evaluating the causal link between FDI inflows and domestic interest rate in Nigeria

Innocent Chile Nzeh[1,*], Chizoma Olivia Osuagwu[2], Obiageli Joy Oparaojiaku[3], Chibundu Emeka Ikechi[4]

[1] Department of Cooperative and Rural Development, University of Agriculture and Environmental Sciences Umuagwo, PMB 110001, Imo State, Nigeria
[2] Department of Agricultural Economics, University of Agriculture and Environmental Sciences, Umuagwo, PMB 110001, Imo state, Nigeria
[3] Department of Agricultural Extension, University of Agriculture and Environmental Sciences Umuagwo, PMB 110001, Imo State, Nigeria
[4] Agriculture and Rural Development Department, Ohaji-Egbema Local Government Council, Mnahu-Egbema, PMB 464113, Imo State, Nigeria
*** Corresponding author:** Innocent Chile Nzeh, nzechile@yahoo.com

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Abstract: Investigating the relationship between domestic interest rate and inflows from foreign direct investment (FDI) in a country is paramount for policy formulation. While a preponderance of extant literature has evaluated the impact of interest rate on the penetration of FDI owing to existing theories that support such link, studies that focus on the role of FDI inflows in influencing domestic interest rate is scanty. Dearth of studies in this area limits an understanding of the actual link between the two variables. This study therefore adds to the existing literature by verifying both theoretical and conceptual views concerning how FDI inflows and domestic interest rate are related in Nigeria. In addressing the identified gap in knowledge, the study used the vector error correction model (VECM) Granger causality with annual series which covered the period from 1981 to 2022. Finding indicates a bi-directional causality existing between domestic interest rate and FDI inflows. The paper thus concludes that much as domestic interest rate influenced FDI inflows (supporting the theoretical postulations), a reverse causality running from FDI inflows to domestic interest rate was equally revealed to exist. The study thus recommends that instead of manipulating the monetary policy instruments to attract FDI and as well handle the consequences accompanying its massive penetration, efforts should be directed at providing institutional reforms and upgrading the infrastructure in the country.

Keywords: FDI inflows; interest rate; monetary policy; VECM

1. Introduction

In recent times, the Nigerian economy has witnessed huge penetrations of foreign capital that have complemented its domestic resources. Such capital penetrations come in diverse forms which include: foreign direct investment (FDI), external debt, foreign portfolio investment (FPI), remittances, among others [1]. Among these sources of capital inflows, it should be noted that attracting FDI inflows has often been the focus of most countries; especially developing countries like Nigeria that need to improve its productive capacity. As observed by [2], FDI inflows into an economy complement the supply of funds for investment, thus encouraging capital formation. It stimulates local investment as it provides linkages in the production chain when foreign firms purchase inputs that are produced domestically by local firms. Zvezdanovic [3] observed that FDI inflows have assisted many poor countries to improve their economic growth by providing developmental projects that encourage productivity and job provision for the citizens of the host country. From another perspective, Karau and Ng'ang'a [4] noted that FDI inflows strengthen the balance of payments position of the host country as they raise exports and in addition, lead to the transfer of technology just as they encourage new management techniques. If FDI inflows is this important to an economy, it them implies that adequate research effort has to be devoted to finding its roles in the economy.

The Nigerian economy still struggles to stand on its toes to compete favourably with its peers in terms of sustainable domestic productive capacity. Several attempts have been made to attract FDI in order to shore up the country's productive resources, yet the country still lags behind in terms of becoming self-sustaining in domestic productivity. This has encouraged massive importation, leading to exchange rate depreciation and high rate of unemployment. In examining the determinants of FDI inflows, interest rate has received so much emphasis. On the theoretical basis, the Mundell-Flemming model which was jointly developed by the duo of Robert Mundell and Marcus Fleming in the 1960s laid much emphasis on the role of domestic interest rate in attracting FDI inflows into an economy. There has been this observation the rise in a country's interest rate above the prevailing world interest rate, would position such country to attract FDI inflows. This theoretical viewpoint and others before and after it has been the basis for much of the empirical works that have examined how interest rate influences FDI inflows. Such studies tend to consider the nexus between the two variables to be one-directional, flowing from interest rate to FDI inflows. However, in recent times, some scholars such as [5] and [6] have come to the conclusion that by raising domestic money supply, FDI inflows has the tendency to lower domestic interest rate. Such emerging views, when aligned with the traditional view which holds that interest rate causes FDI inflows; implies that there is a possibility of a reverse causality running from FDI inflows to domestic interest rate. If such is the case, it has some policy implications, especially for a country like Nigeria which relies much on monetary policy measures to stabilize the macroeconomic environment.

The focus of the present paper is to evaluate the causal link between FDI inflows and domestic interest rate in Nigeria. Extant literatures in Nigeria have mainly concentrated on how domestic interest rate influences FDI inflows [7–9]. Some studies have equally focused on FDI inflows and economic growth nexus, while others aimed at examining the nexus between the official rate and domestic investment [10,11]. To the best of the knowledge of the authors, the literature has been silent on whether there is a possibility for the existence of a reverse causality running from FDI inflows to domestic interest rate in Nigeria. A focus on this area would be necessary to provide a balanced argument for the actual relationship between the two variables. Concentrating only on how interest rate influences FDI inflows implies a one-way causal link running from interest rate to FDI inflows and such could impede monetary policy implementation. This present paper therefore contributes to the literature by integrating both the theoretical propositions regarding the role of domestic interest rate in attracting FDI inflows and the conceptual views regarding the possibility of FDI inflows to influence domestic interest rate. Such approach provides a balanced argument regarding the relationship between the two variables and as such findings could be helpful in framing up appropriate policies to move the two variables in the desired direction.

To evaluate this, the study applied the vector error correction model (VECM) form of Granger causality. The motivation for choosing Nigeria in this paper is because the country is among the highest recipients of capital inflows in Africa and the authors argue that fluctuations in FDI inflows could pose a threat to price stability which is the main monetary policy target of the monetary authorities. Thus, the main objectives of the study are to evaluate both the short-run and long-run causal link between FDI inflows and real interest rate in Nigeria. The study is guided by the null hypothesis which states that there is an absence of a causal link between domestic interest rate and FDI inflows in either of the time horizons.

1.1. Stylized facts

Figure 1 indicates that as FDI inflows trended upward, real interest rate trended downward and vice-versa. For instance, from 1990 to 2005 when real interest rate was high, FDI inflows was low. It was only in 2007 through 2009 when the two variables moved in similar direction, but from 2010 to 2012 when FDI inflows was high, real interest rate was low. From 2013 through the entire study period, real interest rate was high while FDI inflows trended low. Worthy of note is that FDI inflows attained its peak in 2009 and 2011, respectively. However, after 2011 it continued to trend downward. The message that the information on **Figure 1** is passing is that the relationship between domestic interest rate and FDI inflows is negative, thus implying that FDI inflows; just like every other components of capital inflows, improves the liquidity position of the country which ends up lowering domestic interest rate. On the other hand, the periods from 2007 through 2009 present an abnormal case compared to other periods as both inflows from FDI and interest rate moved in similar direction. The study contends that booming capital market within this period with its accompanying high domestic interest rate attracted much capital inflows into the economy until the bubble got busted on the back of the financial recession that ensued later. It is noticed that from 2010 interest rate began to trend downwards while the fall in FDI inflows began in 2012.

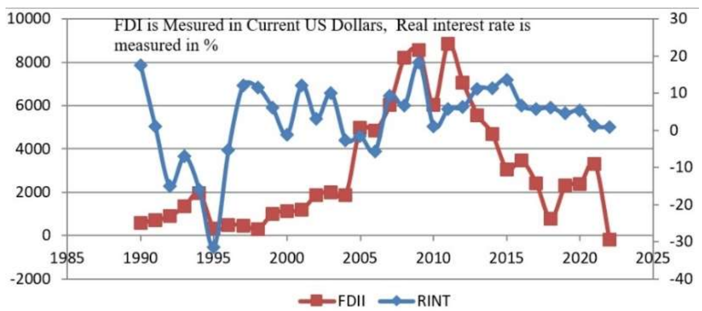

Figure 1. Movement in FDI and real interest rate.

Source: WDI (2022).

Note: FDII—foreign direct investment inflows, RINT—real interest rate.

Figure 2 shows the nexus among real interest rate, FDI inflows and exchange rate. As has been noted earlier, if there is an increase in the prevailing interest rate in an economy, such development encourages foreign investors to push more investments into the economy. However, low domestic interest rate could have an adverse effect on the penetration of FDI. The transmission mechanism through which this takes place is the exchange rate. Rising domestic interest rate encourages an appreciation of a country's local currency which could attract FDI into such economy. This possibility is shown in the direction of the arrow in **Figure 2**. From another perspective, a low domestic interest rate would lead to the depreciation of domestic currency which may retard FDI inflows as shown the direction of the arrow.

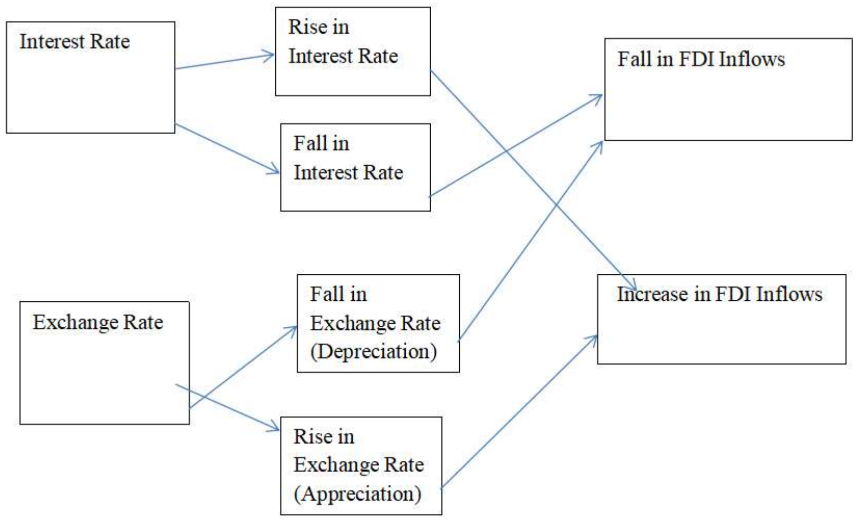

Figure 2. Nexus among FDI inflows, real exchange rate and real interest rate.
Source: Modified from [12].

2. Literature review

2.1. Conceptual issues

The term foreign direct investment has received various conceptual definitions [13]. Views FDI to represent investment made by a country's residents in a foreign company over which foreign owners exercise effective control. FDI is also defined as an investment made by multinational corporations in oversea countries with the aim of having control over the assets and as well manages the production activities in those countries. This definition finds support in the view of [14] who noted that FDI takes place through the establishment of a business operation in in another country by way of forming a joint venture in the host economy. It also involves building a new wholly-owned affiliate or the acquisition of a local company. In another vein, Devereux and Yetman [15] observed that two types of FDI have been identified in theory, namely: horizontal (market-seeking) and vertical. While horizontal FDI refers to the establishment of identical plants in foreign locations in order to supply certain goods in a foreign country, the aim of vertical FDI is to search for the lowest possible cost of production overseas. Apere and Akarara [11] defined FDI as the investment behaviour in which the country investing channels capital for production and operation in the host country in order to own part of management rights. In the opinion of [12], FDI generally could be in the form of acquisition, joint ventures, green-field investment and reinvested company earnings. Ragazzi [13] included other forms of FDI such as licensing, franchising and turnkey agreements.

In another vein, the relevance of interest rate to monetary policy setting has spurred interests in providing conceptual meaning to it. Keynes [14] earlier defined interest rates as the cost associated with borrowing capital over a specified period of time. Interest rate plays vital role as an indicator of future inflationary trend as well as any anticipated change in a country's purchasing power of money. In a similar vein, Devereux and Yetman [15] defined interest rates as the price which a borrower pays for using money or capital that he does not own. Normally, interest rates are predetermined by the interplay of demand and supply function of capital. Apart from this, interest rates in an economy are determined by the actions of the monetary authorities of a country. Cuthbertson [16] observed that interest rates operate in a similar way like other prices by acting as market clearing mechanism and the rationing of the amount of available credit. Kasemo [17] was of the opinion that interest rates are determined in the debt markets or credit markets just the same way the stock prices are determined. Interest rate could be nominal or real. Nominal interest rate is the actual price which borrowers pay to lenders without putting into consideration, other economic factors. However, real interest rate considers the impact of the price level or inflation which is attuned to reality and thus justifies its adoption in this present paper.

2.2. Theoretical issues

Some theoretical issues concerning the factors that determine capital inflows have been emphasized in literature. The Mundell-Flemming model which was jointly introduced by Robert Mundell and Marcus Fleming in the 1960s contended that the prevailing interest rate in an economy is influenced by the world interest rate. Thus, in any country where the domestic interest rate is higher than the world interest rate, such country has the opportunity to experience an increase in capital inflows into its domestic economy. Such inflows will persist till the country's interest rate equates the international rate. On the other hand, if domestic interest rate declines, such will lead to outflow of capital from the domestic economy which will persist until the domestic rate aligns with the international rate. From another perspective, the monopolistic advantage theory was developed by [18] as an extension of the work of [19]. The theory was based on the premise that firms which operate in foreign countries are faced with competition with host countries' firms that already have existing advantages with respect to language, consumer preferences, legal systems and culture. In order to penetrate the host country, these constraints must be dismantled through the acquisition of some form of market power to enable firms make profits. Lall [20] observed that the source of market power can only be through conditions of imperfect competition. Market power can be acquired through the possession of patent-protected products, economies of scale, superior technology, management skills, brand names and cheaper sources of finance.

Buckley and Casson [21] explained what happens when the external market condition facing the multinational corporations (MNCs) fails to yield efficient environment that can necessitate profit through the use of brand name, technological know-how and production processes. Under this condition, the firm may wish to create internal market through investing in numerous countries and hence create the needed market to achieve its aims. Buckley and Casson [21] came up with a different notion of FDI which lays emphasis on inputs and technology at the intermediate level. Thus, in the discussion of the determinants of inflows, there was a shift of emphasis on international investment theory away from country-specific. If a firm engages in research and development and thus develops new technology, it could be difficult for the firm to engage in technological transfer to other firms that use unrelated technology because the transaction costs may be too expensive for them. In the face of this limitation, a firm may decide to internalize through the adoption of forward and backward integration. This could mean that a subsidiary's output can serve as production input of another or that other subsidiaries may use the technology developed by another subsidiary.

2.3. Review of empirical literature

Studies across different countries have examined how FDI inflows is linked to interest rate with varying results. In Zimbabwe, Anna et al. [22] indicated that interest rates had no significant impact on FDI inflows. However, a cross-country study involving five Association of South East Asian Nations (ASEAN-5) comprising Indonesia, Malaysia, Philippine, Singapore and Thailand by [23] revealed that interest rates exerted an adverse effect on FDI inflows in Thailand, Indonesia and Malaysia. In Sierra Leone, Faroh and Shen [24] showed that interest rate had no significant influence on FDI inflows. This contrasted with a finding in India by [25] which indicated that interest rate impacted FDI inflows significantly. Equally in Sierra Leone, Fornah and Yuehua [26] indicated that interest rate failed to exert a significant influence on the penetration of FDI and this corroborates the finding by [24]. In Pakistan, Ditta and Hassan [27] showed that both interest rate and exchange rate impacted positively and significantly on FDI inflows. However, findings by [28] showed that a negative and insignificant link exists between interest rate and FDI inflows.

A study in China by [29] found that real interest rate led to higher FDI inflows, while findings by [4] in Kenya revealed that interest rates had positive link with FDI inflows. In Ghana, Kombui and Kotey [30] indicated that interest rate Granger caused FDI inflows. In Nigeria, finding by [7] revealed that differences in interest rate exerted non-significant influence on the penetration of FDI. However, while [8] showed that interest rate impacted positively on FDI inflows in the short-run, the long-run result indicates that its impact is negative. Another study in Nigeria by [9] indicated that interest rate impacted negatively on FDI inflows even though the outcome is not significant

A study focusing on Iraq by [31] showed that FDI inflows was influenced positively by interest rate, while a study involving Brazil, China, Turkey and Poland by [32] revealed that a policy which reduced interest rate before and during COVID-19 led to rising FDI inflows, while a policy that raised it after the pandemic constrained FDI inflows. A study in sub-Sahara African countries by [12] showed that a fall in interest rate attracted FDI inflows in the short-run, but the fall resulted in the decline of FDI inflows in the long-run. In Switzerland and Sweden, [33] found that negative interest rates did not have effect on the penetration of FDI in the both economies. In Bangladesh, Morshed and Hossain [34] did not find any causal relationship between FDI inflows and interest rate but a study in Nigeria by [35] indicated that in the short-run, domestic interest rate was impacted negatively by FDI inflows even though the impact was not significant. A study on emerging markets and developing economies by [36] indicated FDI inflows was adversely influenced by real interest rate.

3. Methodology

3.1. Data and sources

The present paper employed yearly data covering 1981–2022 to evaluate the nature of causal relationship that exists between FDI inflows and interest rate in Nigeria. For the sake of normalization and to ease interpretation, FDI inflows, real exchange rate, oil revenue and broad money supply are in log form. Included variables are shown in **Table 1** in addition to their sources and measurement.

Table 1. Variable sources and measurement.

Variables	Definition	Measurements	Source
RINTR	Real interest rate	Measured in percentage	WDI (2022)
FDII	Foreign direct investment inflows	Measured in current US Dollars	WDI (2022)
REXCHR	Real effective exchange rate	Exchange rate of naira to US Dollars measured in 2010 base year	WDI (2022)
CRPRV	Credit to the private sector	Measured as a percentage of GDP	WDI (2022)
OILR	Oil revenue	Measured in Billions of Naira	CBN Bulletin (2021)
CPI	Consumer price index	Measured using 2010 as the base year	WDI (2022)
M2	Broad money supply	Measured in current local currency unit	WDI (2022)

3.2. Model specification

In order to test for the causality between domestic interest rate and FDI inflows, this present paper used the vector error correction model (VECM) which is suitable when the series are stationary at first difference and are cointegrated. While causality in short-run is evaluated under the Wald test, causality in the long-run is evaluated by examining the sign and significance of the error correction model's coefficient in each equation. The VECM representation of a standard VAR is specified as follows:

$$\Delta\gamma_t = \omega + \sum_{i=1}^{n} \sigma_i\gamma_{t-1} + \pi ECM_{t-1} + \varepsilon_t \tag{1}$$

where,

Δ = differencing operator, $\Delta\gamma_t = \gamma_t - \gamma_{t-1}$, γ_t = $(nx1)$ column vector of the endogenous variables, φ= $(nx1)$ vector of constant, σ =$(3x3)$ coefficient matrices, π= $(3x1)$ vector of coefficients for each of the error correction terms.

The VECM Granger is thus specified as follows:

$$\Delta RINTR_t = \psi_0 + \sum_{i-1}^{p} \psi_1 \Delta RINTR_{t-1} + \sum_{t=1}^{p} \psi_2 \Delta LFDII_{t-1} + \sum_{t=1}^{p} \psi_3 \Delta LREXCHR_{t-1} + \sum_{t=1}^{P} \psi_4 \Delta CRPRV_{t-1} \tag{2}$$

$$+ \sum_{t=1}^{p} \psi_5 \Delta LOILR_{t-1} + \sum_{t=1}^{p} \psi_6 \Delta CPI_{t-1} . + \sum_{t=1}^{p} \psi_7 \Delta LM2_{t-1} + \psi_8 ECT_t + \varepsilon_t$$

$$\Delta LFDII_t = \gamma_0 + \sum_{t=1}^{p} \gamma_1 \Delta LFDII_{t-1} + \sum_{i-1}^{p} \gamma_2 \Delta RINTR_{t-1} + \sum_{t=1}^{p} \gamma_3 \Delta LREXCHR_{t-1} + \sum_{t=1}^{P} \gamma_4 \Delta CRPRV_{t-1} + \tag{3}$$
$$\sum_{t=1}^{p} \gamma_5 \Delta LOILR_{t-1} + \sum_{t=1}^{p} \gamma_6 \Delta CPI_{t-1} . + \sum_{t=1}^{p} \gamma_7 \Delta LM2_{t-1} + \gamma_8 ECT_t + \varepsilon_t$$

$$\Delta LREXCHR_t = \lambda_0 + \sum_{t=1}^{p} \lambda_1 \Delta LREXCHR_{t-1} + \sum_{t=1}^{p} \lambda_2 \Delta LFDII_{t-1} + \sum_{i-1}^{p} \lambda_3 \Delta RINTR_{t-1} + \sum_{t=1}^{P} \lambda_4 \Delta CRPRV_{t-1} + \tag{4}$$
$$\sum_{t=1}^{p} \lambda_5 \Delta LOILR_{t-1} + \sum_{t=1}^{p} \lambda_6 \Delta CPI_{t-1} . + \sum_{t=1}^{p} \lambda_7 \Delta LM2_{t-1} + \lambda_8 ECT_t + \varepsilon_t$$

$$\Delta CRPRV_t = \pi_0 + \sum_{t=1}^{P} \pi_1 \Delta CRPRV_{t-1} + \sum_{t=1}^{p} \pi_2 \Delta LREXCHR_{t-1} + \sum_{t=1}^{p} \pi_3 \Delta LFDII_{t-1} + \sum_{i-1}^{p} \pi_4 \Delta RINTR_{t-1} + \tag{5}$$
$$\sum_{t=1}^{p} \pi_5 \Delta LOILR_{t-1} + \sum_{t=1}^{p} \pi_6 \Delta CPI_{t-1} . + \sum_{t=1}^{p} \pi_7 \Delta LM2_{t-1} + \pi_8 ECT_t + \varepsilon_t$$

$$\Delta LOILR_t = \xi_0 + \sum_{t=1}^{p} \xi_1 \Delta LOILR_{t-1} + \sum_{t=1}^{P} \xi_2 \Delta\, CRPRV_{t-1} + \sum_{t=1}^{p} \xi_3 \Delta LREXCHR_{t-1} + \sum_{t=1}^{p} \xi_4 \Delta\, LFDII_{t-1} + \sum_{i-1}^{p} \xi_5\, \Delta RINTR_{t-1} + \sum_{t=1}^{p} \xi_6 \Delta CPI_{t-1}. + \sum_{t=1}^{p} \xi_7 \Delta LM2_{t-1} + \xi_8 ECT_t + \varepsilon_t \tag{6}$$

$$\Delta CPI_t = \delta_0 + \sum_{t=1}^{p} \delta_1 \Delta CPI_{t-1} + \sum_{t=1}^{p} \delta_2 \Delta LOILR_{t-1} + \sum_{t=1}^{P} \delta_3 \Delta\, CRPRV_{t-1} + \sum_{t=1}^{p} \delta_4\, \Delta LREXCHR_{t-1} + \sum_{t=1}^{p} \delta_5 \Delta\, LFDII_{t-1} + \sum_{i-1}^{p} \delta_6\, \Delta RINTR_{t-1} + \sum_{t=1}^{p} \eta_7 \Delta LM2_{t-1} + \delta_8 ECT_t + \varepsilon_t \tag{7}$$

$$\Delta LM2_t = \eta_0 + \sum_{t=1}^{p} \eta_1 \Delta LM2_{t-1} + \sum_{t=1}^{p} \eta_2 \Delta CPI_{t-1} + \sum_{t=1}^{p} \eta_3 \Delta LOILR_{t-1} + \sum_{t=1}^{P} \eta_4 \Delta\, CRPRV_{t-1} + \sum_{t=1}^{p} \eta_5\, \Delta LREXCHR_{t-1} + \sum_{t=1}^{p} \eta_6\, \Delta LFDII_{t-1} + \sum_{i-1}^{p} \pi_7\, \Delta RINTR_{t-1} + \pi_8 ECT_t + \varepsilon_t \tag{8}$$

where: *RINTR*= real interest rate, *LFDII* = log of foreign direct investment inflows, *LEXCHR* = log of real exchange rate, *CRPRV*= credit to the private sector, *LOILR*= log of oil revenue, *CPI* = consumer price index, *LM2* = log of broad money supply, Δ = first difference operator, *t* = trend value, *ECT* = error correction term, ε_t are the stochastic terms assumed not to be correlated with one another as well as being normally distributed with zero mean. The coefficients ψ_1 to ψ_7, γ_1 to γ_7, λ_1 to λ_7, π_1 to π_7, ξ_1 to ξ_7, δ_1 to δ_7 and η_1 to η_7 in equation 2 through equation 8 measure the short-run causality, while the coefficient of the ECT in each equation measures the long-run causality.

4. Results and discussions

Descriptive statistics is carried out to examine how the included variables behave. Results in **Table 2** show an evidence of proximity between the median and mean of every series. This reveals the symmetric nature of the series. A distribution is said to be symmetrical when the values of the variables appear at regular frequencies and usually the median, mean and mode all occur at the same point. The variable that has the highest mean (82.72) is the CPI. On the other hand, with the mean value of 0.46, RINTR was revealed to have the lowest mean. Apart from the CPI whose standard deviation is high, other variables have relatively low standard deviation, implying that the deviations from their mean values are very small. The range of CPI was equally the highest among the variables which implies that it experienced the highest volatility compared to others within the period of study. Evidence reveals that the variable with the least range is RINTR, indicating that it exhibited the least volatility within the period. In terms of skewness, it is found that a positive skewness was found in the consumer price index and real exchange rate, while a negative skewness was found in other variables. With respect to Kurtosis, it is found that every variable is heavy-tailed as their values are positive.

Table 2. Descriptive statistics.

	RINTR	FDII	REXCHR	CRPRV	OILR	CPI	M2
Mean	0.46	8.94	2.08	30.61	2.59	82.72	11.72
Median	3.66	9.20	2.00	28.18	2.97	37.45	12.06
Maximum	18.18	9.94	2.72	46.30	3.94	421.07	13.64
Minimum	−65.85	0.00	1.69	0.00	0.00	0.48	0.00
Std. Dev.	14.08	1.49	0.25	9.85	1.19	105.56	2.18
Skewness	−2.75	−5.24	1.02	−0.37	−0.63	1.57	−3.70
Kurtosis	13.23	32.12	3.21	3.30	2.10	4.79	20.73
Jarque-Bera	236.32	1676.7	7.40	1.15	4.22	23.00	646.20
Probability	0.00	0.00	0.02	0.56	0.12	0.00	0.00
Sum	19.51	375.7	87.36	1285.7	109.01	3474.3	492.5
Sum Sq. Dev.	8133.1	91.56	2.68	3984.8	58.86	456898.7	196.4
Observations	42	42	42	42	42	42	42

In order to ascertain the degree of correlation that exists among the series, the correlation matrix test was conducted. Information in **Table 3** revealed the existence of a low positive correlation between RINTR and the rest of the series with the exception of REXCHR which has low and negative correlation. A strong and positive correlation was also found to exist between FDI inflows and M2 and between FDI inflows and CRPRV. However, the correlation between FDI inflows and CPI and REXCHR is negative and weak. It is found that while the correlation between REXCHR and other variables is negative and weak, a relatively strong correlation was found to exist between CRPRV and FDI inflows, M2 and OILR. In summary, the low correlation between RINTR and other variables is an indication of the existence of a low multicolinearity among them.

Table 3. Correlation matrix.

	RINTR	FDII	REXCHR	CRPRV	OILR	CPI	M2
RINTR	1	0.09	−0.19	0.40	0.37	0.25	0.23
FDII	0.09	1	−0.19	0.67	0.53	−0.33	0.93
REXCHR	−0.19	−0.19	1	−0.19	−0.44	−0.09	−0.21
CRPRV	0.40	0.67	−0.19	1	0.66	0.40	0.83

Table 3. (*Continued*).

	RINTR	FDII	REXCHR	CRPRV	OILR	CPI	M2
OILR	0.37	0.53	−0.44	0.66	1	0.13	0.68
CPI	0.25	−0.33	−0.09	0.40	0.13	1	−0.04
M2	0.23	0.93	−0.21	0.83	0.68	−0.04	1

Next the paper conducted a cointegration test to ascertain the order of integration of the series. A major pre-requisite in the time series analysis is that the series have to be stationary in order not to obtain spurious results. In this study, stationarity test was conducted through the frameworks of the Augmented Dickey Fuller (ADF) and the Phillip Perron (PP). Analyses are based on the null hypothesis which states that the series are not stationary (have unit root) which is evaluated at chosen level of significance. If the t-statistics is lower than the chosen level of significance, then there is every reason to accept the null, otherwise it is rejected. **Tables 4** and **5** below display the summary of results of the stationarity tests under the ADF and PP, respectively. In **Tables 4** and **5**, evidence shows that under both the ADF and PP, RINTR and CPI are stationary at level I (0). However, other series are stationary only after the first difference I (1). Thus, there is an admixture of order of integration of the variables which makes the ARDL appropriate to be used to examine the cointegrating relationship among them.

Table 4. ADF level and first difference results of stationarity.

Variable	ADF Level t-stat	ADF Level Critical value at 5%	ADF First Diff. t-stat	ADF First Diff. Critical value at 5%	Order of Integration
RINTR	−7.57	−2.93*	−10.22	−2.93*	I (0)
FDII	−1.55	−2.93	−7.14	−2.93*	I (1)
REXCHR	−2.15	−2.93	−4.29	−2.93*	I (1)
CRPRV	−1.39	−2.93	−5.23	−2.93*	I (1)
OILR	−1.66	−2.93	−6.11	−2.93*	I (1)
CPI	−3.55	−2.93*	−20.33	−2.93	I (0)
M2	−1.65	−2.93	−2.94	−2.93*	I (1)

Table 5. PP level and first difference results of stationarity.

Variable	PP Level t-stat	PP Level Critical value at 5%	PP First Diff. t-stat	PP First Diff. Critical value at 5%	Order of Integration
RINTR	−7.34	−2.93*	−25.21	−2.93*	I (0)
FDII	−1.54	−2.93	−7.14	−2.93*	I (1)
REXCHR	−2.04	−2.93	−4.29	−2.93*	I (1)
CRPRV	−1.60	−2.93	−4.24	−2.93*	I (1)
OILR	-1.67	−2.93	−6.18	−2.93*	I (1)
CPI	−3.87	−2.94*	−10.78	−2.93*	I (0)
M2	−1.96	−2.93	−3.18	−2.93*	I (1)

Next, the study went on to examine the cointegration among the series. The result of cointegration in **Table 6** indicates that the computed F-statistic is 6.70, while the upper critical bounds I (1) at the 5% level is 3.61 which is lower than the computed F-statistic. The study thus concludes that the series are co-integrated at the chosen level of significance.

Table 6. ARDL bound tests result for model 2.

Test Statistic	Value	K
F-statistic	6.70	6
Critical Value Bounds		
Significance	I0 Bound	I1 Bound
10%	2.12	3.23
5%	2.45	3.61
2.5%	2.75	3.99
1%	3.15	4.43

The Granger causality results in **Table 7** reveal that while FDI inflows Granger caused RINTR at the 10% level of significance, at the 5% level, RINTR also Granger caused FDI inflows. It thus indicates that a bi-directional link exists between the two variables. The study found that CRPRV Granger caused FDI inflows and M2 at the 10% level of significance without a feedback. It also Granger caused CPI at the 5% level of significance with no feedback. Thus, an un-directional relationship running from CRPRV to FDI inflows, M2 and CPI is revealed in the study. OILR was found to Granger cause RINTR and FDI inflows at the 10% level without a feedback. Findings equally indicate the existence of a one-way (uni-directional) causality that runs from OILR to RINTR and FDI inflows. However, at the 5% level, OILR was found to Granger cause both M2 and CRPRV without a feedback.

Table 7. Results of the causality between FDI and interest rate (Model 3).

VECM Granger Causality/Block Exogeneity Wald Tests							
Ind. Variable	Dependent Variable						
	D(RINTR)	D(LFDII)	D(LREXCHR)	D(CRPRV)	D(LOILR)	D(CPI)	D(LM2)
D(RINTR)	-	5.17(0.07) **	0.62(0.73)	3.07(0.21)	1.57(0.45)	0.63(0.72)	3.30(0.19)
D(LFDII)	5.70(0.05)*	-	1.13(0.56)	1.44(0.48)	4.23(0.12)	0.58(0.74)	0.03(0.98)
D(RLEXCHR)	1.78(0.41)	0.8(0.66)	-	1.21(0.54)	1.09(0.57)	0.46(0.79)	0.14(0.93)
D(CRPRV)	2.23(0.32)	9.22(0.009) **	1.78(0.4091)	-	2.78(0.24)	8.57(0.01) *	5.24(0.07) **
D(LOILR)	5.07(0.07) **	4.68(0.09) **		12.13(0.002) *	-	0.92(0.62)	19.15(0.00) *
D(CPI)	0.20(0.90)	1.48(0.47)	3.16(0.20)	0.33(0.84)	13.61(0.00) *	-	0.41(0.81)
D(LM2)	0.94(0.62)	1.61(0.44)	0.99(0.60)	0.72(0.69)	0.50(0.77)	4.58(0.101)	-

Note: Figures with asterisks * and ** indicate that the null hypothesis of an absence of causality is rejected at 5% and 10% level, respectively.

Table 8 shows the results of the long-run causality among the variables. Findings indicate that at the 10% level, other variables Granger caused RINTR. On the other hand, other variables Granger caused FDI inflows, M2 and CRPRV. By implication, the long-run causality reveals a bi-directional causal relationship between RINTR and FDI inflows, thus confirming the short-run result.

Table 8. Results of long-run causality.

Variable	ECMt-1/*P*-value	Decision
ΔRINTR	−0.75/0.09**	Existence of causality
ΔLFDII	−0.03/0.01*	Existence of causality
ΔLEXCHR	0.00/0.004	No causality
ΔCRPRV	−0.09/0.06**	Existence of causality
ΔLOILR	0.01/0.01	No causality
ΔCPI	0.158/0.09	No causality
ΔLM2	−0.01/0.01*	Existence of causality

Note: Figures with asterisks * and ** indicate that the null hypothesis of an absence of causality is rejected at 5% and 10% level, respectively.

Discussion of findings

A bi-directional causal relationship is found to exit between FDI inflows and real interest rate in Nigeria. By implication as FDI inflows Granger caused RINTR, it also Granger caused FDI inflows. These results have thus corroborated both the theoretical and conceptual views raised in this study. Theoretically, the Mundell-Flemming model suggests that in any country where the domestic interest rate is high, such country has the tendency to attract capital inflows, implying that interest rate should influence FDI inflows. Also, conceptually FDI inflows has been said to raise money supply which could end up reducing real interest rate; implying that FDI inflows should cause real interest rate. In periods of rising capital inflows such as inflows occasioned by high FDI inflows, money supply is usually high; encouraging improved liquidity in the banking sector. Such situation has the tendency to result into high inflation as the increased liquidity gives banks the leverage to offer more loans to investors and other economic agents. Since inflation-targeting is a major monetary policy thrust of the monetary authorities, the Central Bank of Nigeria (CBN) usually intervenes to mitigate the inflationary impact of the inflows through, among others by raising the monetary policy rate (MPR). Such contractionary monetary policy measure often pushes domestic interest rate up because the monetary policy rate is the benchmark rate that influences other interest rates. Furthermore, exchange rate policy intervention of the monetary authorities during periods of rising FDI inflows such as buying of foreign currency often results into increase in high powered money and, hence improved liquidity in the banking sector. The reverse is the case if there is a fall in FDI inflows. Therefore, it is apposite to state that the transmission mechanism through which FDI inflows Granger cause real interest rate is through the monetary policy intervention of the CBN which seeks to reduce or increase the reserve position of the deposit money banks. In another vein, high interest rate in Nigeria is a phenomenon that has been of much concern to domestic investors. There have been calls by manufacturers for a reduction in interest rate to enable them access cheap funds, but such demands has not been able to be addressed. The reasons often given by the monetary authorities for the continuous rise in interest is that so long as the economy keeps experiencing high inflation, it will be difficult to officially reduce interest rate. This much accounts for part of the reasons why high interest rate persists in the economy. The implication of high interest rate is that it offers foreign investors an opportunity to push more investments into the economy because with high interest rate, returns on investment is usually high and this is the major issue raised by the Mundell-Flemming hypothesis. It has been noted in several quarters that since interest rate in advanced economies is usually very low, foreign investors often seek opportunities in developing countries such as African countries where interest is relatively high. During the global financial meltdown that happened around 2008, the capital markets of several African countries including Nigeria experienced a glut in liquidity as foreign investors were attracted by the high interest rate which was a phenomenon then.

Findings in this present paper is partly supported by a study in Ghana by [30] which revealed that real interest rate Granger caused FDI inflows without a feedback. Also in Nigeria, Ezirim and Ezirim [8] revealed the existence of a one-way causal relationship that runs from real interest rate to FDI inflows. The reason for the divergent results between the present study and that of [8] could be because the two studies used different sample periods. While Ezirim and Ezirim [8] used dataset that spanned the period between 1986 and 2018, the dataset used in the present study ranged between 1981 and 2022. Also, while the present study used net inflows in current US Dollars to proxy FDI inflows, the proxy used by [8] is FDI ratio to GDP. Findings also revealed that causality runs from credit to the private sector to M2 without a feedback. This has thus revealed the sensitive nature of credit granted to the private sector in Nigeria regarding its impact on money supply. High private sector to the private sector raises money supply which stimulates domestic demand and which may give rise to price increases. It is against this backdrop that the CBN often intervenes through the implementation of contractionary monetary policy aimed at curtailing the growth in money supply. The oil sector is the main source of revenue in Nigeria such that when oil price rises, the CBN employs monetary policy instruments to reduce the monetary impact of the oil price rise. Finding in this present study has therefore confirmed the role of oil revenue in influencing real interest rate and money supply as evidence has shown that oil revenue Granger caused both variables without a feedback.

Table 9 displays the results of post-diagnostic tests. Findings prove that the VEC residual heteroskedasticity test result has a p-value of 0.38 that is greater than the 5% level. Thus, the study has every reason not to reject the null hypothesis that there is an absence of heteroskedasticity in the error terms. The result of the VEC residual serial correlation also indicates that with a p-value of 0.75 that is greater than the 5% level, there is every reason to accept the null hypothesis of no serial correlation in the variables. The normality result indicates that the Jarque-Bera test has a p-value of 0.37 which is greater than the 5% level of, indicating that the errors are normally distributed. The stability test in **Figure 3** indicates that some of the roots of the equation are not within unit circle which reveals that the model is not stable. The instability in the model could be attributed to many factors such as policy issues and other factors which impact on the variables as the study extended to the periods of pre-structural adjustment programme (SAP), military regimes and civilian regimes as well as exogenous shocks arising from the global financial meltdown that occurred in 2009 and the COVID-19 pandemic.

Table 9. Results of post diagnostics.

Test	*P*-value	Null Hypothesis	Conclusion
VEC Residual Heteroskedasticity Tests	0.38	No Heteroskedasticity	Accept
VEC Residual Serial Correlation LM Tests	0.75 at lag 1	No Serial Correlation	Accept
VEC Residual Normality Tests: Jarque-Bera	0.37 at lag 1	Normally distributed	Accept

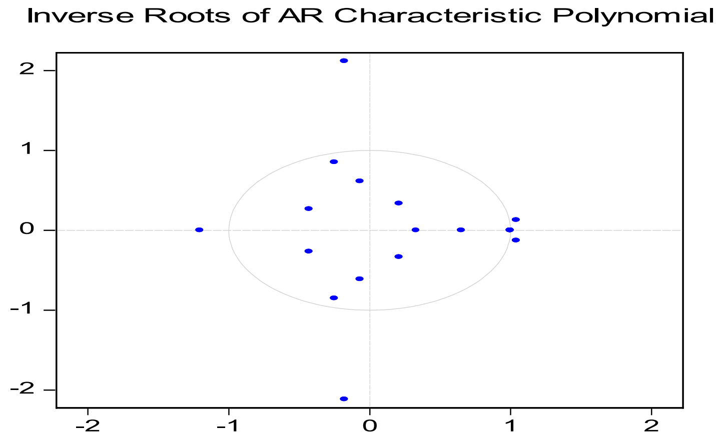

Figure 3. Test of stability.

5. Conclusion

This study was motivated by the divergent views; both theoretically and conceptually regarding the relationship that exists between FDI inflows and interest rate. While theoretically, interest rate is expected to impact on FDI inflows, some emerging opinions emphasize that FDI inflows should impact on interest rate through its impact on money supply. Findings so far have proven the existence of a one-way causal relationship between FDI inflows and real interest rate which supports both the theoretical and conceptual views. The outcome so far has confirmed that much as interest rate influences FDI inflows in Nigeria, a reverse causality running from FDI inflows to interest rate equally exists. This finding has some policy implications. First, while it is economically wise to raise domestic interest rate in order to shore up FDI, such has the tendency to raise money supply which could eventually lead to fall in interest rate. Fall in interest rate may retard the penetration of FDI but could rather encourage outflows of domestic investment that may affect the liquidity position of the economy. Second, if the rise in FDI inflows persists, the intervention of the monetary authorities to curtail its inflationary impact through increase in the policy rate may lead to further penetration of FDI inflows into the economy. This is because when the policy rate is increased, other domestic rates also rise and foreign investors may want to seize the opportunity to push more investments into the economy. The aftermath of this scenario is that another round of policy intervention may ensue which could put pressure on the monetary authorities. Third, if the continuous penetration of FDI leads to a fall in interest rate through raising money supply, such may result into high inflation if there is no commitment on the part of the monetary authorities to intervene. However, fall in interest rate leads to lower cost of capital which helps to improve productivity in the economy. In all these scenarios, the monetary authorities are in a dilemma.

Consequently, the study recommends that other measures that enhance FDI inflows such as institutional reforms and upgrading of critical infrastructure has to be put in place instead of relying on the manipulation of the monetary policy instruments to attract FDI inflows as well as influencing them with the aim of handling the consequences of fluctuating FDI inflows. For future research, the paper suggests that a decomposition of the various types of capital inflows should be done and their individual impact on domestic interest rate should be evaluated. This is paramount as it will reveal which aspect of capital inflows has more influence on interest rate for the sake of policy simulation.

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References

1. Adekunle W, Suliamon MD. Are-examination of the relationship between foreign capital inflows and economic growth in Nigeria. Munich Personal RePEc Archive (MPRA) Paper No 87754. 2018.
2. Adu BE, Ntim AL. Causality analysis of foreign direct investment, exchange rate and interest rate volatility in Ghana. Journal of Economics and Sustainable Development. 2014; 5(9): 17-25.
3. Zvezdanovic J. Analyzing FDI economic effects and determinants in SEE economies: Comparative approaches. Our Economy. 2013; 59(3-4): 67-75.
4. Karau J. Ng’ang’a P. Effects of macroeconomic factors on FDI in Kenya. European Journal of Business and Management, 2019; 11(3): 55-62.
5. Ali S. Nazar R. Impact of foreign capital inflows and money supply on exchange rate: A case study of Pakistan. Review of Economics and Development Studies. 2017; 3(1): 83-90.
6. Adrian T. Policy responses to capital flows. Available online: https://www.imf.org/en/News/Articles/2018/11/15/sp101118-policy-responses-to-capitalflows (accessed on 3 May 2024).
7. Adejoke M. Adewale AM, Olopade BC, et al. Effect of exchange rate on foreign direct investment in Nigeria. ABUAD Journal of Social and Management Sciences (AJSMS). 2024; 5(2): 302-318.
8. Ezirim CB, Ezirim UI. Determination of foreign direct investment causation and shocks: empirical evidence from Nigeria. International Journal of Business and Economics Perspectives. 2020; 15(1).
9. Karimo TM. Impact of interest rate differential and exchange rate movement on the dynamics of Nigeria’s international private capital flows. CBN Journal of Applied Statistics. 2020; 11(2): 29-63.
10. Ibrahim E. Foreign Direct Investment (FDI) and Economic Growth in Nigeria. Journal of Poverty, Investment and Development. 2024; 9(2): 13-25.
11. Apere TO, Akarara, EA. Interest rate and investment nexus in Nigeria. International Journal of Advanced Studies in Business Strategies and Management. 2024; 11(1).

12. Isiaka NA, Osifalujo BB, Taiwo OK. Interest rate and foreign direct investment in Sub Sahara African countries: The comparison between asymmetric and symmetric effects (ARDL and NARDL approach). International Journal of Economics, Social Science, Entrepreneurship and Technology, 2022; 1(3): 186-200.
13. Ragazzi G. Theories of the determinants of foreign direct investment. Staff Papers - International Monetary Fund. 1973; 20(2): 471. doi: 10.2307/3866325
14. Keynes, JM. General theory. Cambridge University: London; 1936.
15. Devereux, MB, Yetman J. Price setting and exchange rate pass through: Theory and evidence. In the book price adjustment and monetary policy. 2002; 347-371.
16. Cuthbertson K. Quantity financial economics: Stocks, bonds and foreign exchange. Chichester: Wiley Corporation; 2004.
17. Kasemo SM. The relationship between interest rates and real estate developments cost in Mombasa country. Nairobi: University of Nairob; 2015.
18. Kindleberger CP. American business abroad. Yale University Press; 1969.
19. Hymer SH. The international operations of national firms: A study of direct foreign investment [PhD thesis]. Cambridge, Mass, Mass: The MIT Press; 1976.
20. Lall S. Theories of direct private foreign investment and multinational behaviour. Economic and Political Weekly, Special Issue. 1976; 11, 31-33.
21. Buckley PJ, Casson M. The future of the multinational enterprises. London: Macmillan; 1976.
22. Anna C, Karambakuwa RT, Felex T, et al. The impact of interest rates on foreign direct investment: A case study of the Zimbabwean economy (February 2009-June 2011). International Journal of Management Sciences and Business Research. 2012; 1(5).
23. Siddiqui HA, Aumeboonsuke V. Role of interest rate in attracting the FDI: study on ASEAN 5 economy. International Journal of Technical Research and Applications. 2014; 2(3): 59-70.
24. Faroh A, Shen H. Impact of interest rates on foreign direct investment: Case study Sierra Leone economy. International Journal of Business Management and Economic Research. 2015; 6(1): 124-132.
25. Tripathi V, Seth R, Bhandari V. Foreign direct investment and macroeconomic factors: Evidence from the Indian economy. Asia-Pacific Journal of Management Research and Innovation. 2015; 11(1): 46-56.
26. Fornah S, Yuehua Z. Empirical analysis on the influence of interest rates on foreign direct tnvestment in Sierra Leone. International Journal of Research in Business Studies and Management. 2017; 4(12): 28-35.
27. Ditta A, Hassan A. Nexus of economic misery, interest rate, exchange rate and foreign direct investment in Pakistan. Bulletin of Business and Economics. 2017; 6(1): 35-44.
28. Benson E, Eya CI, Yunusa A. Effect of exchange and interest rates on foreign direct investment in Nigeria. International Journal of Contemporary Research and Review. 2019; 10(7): 72-85.
29. Vidhya K, Ahamed SB. The relationship of interest rate, exchange rate, GDP and FDI with respect to Chinese economy. International Journal of Innovative Technology and Exploring Engineering. 2019; 8(11): 273-277.
30. Kombui D, Kotey RA. Foreign direct investment in an emerging economy: Exploring the determinants and causal Linkages. Academic Journal of Economic Studies. 2019; 5(1): 51-62.
31. Salim AD. Monetary Policy and its role in attracting foreign direct investment: A study in the Iraqi economy for the period (2004-2018). Journal of Business and Management Studies. 2020; 4(4): 133-148.
32. Karahan O, Bayır M. The effects of monetary policies on foreign direct investment inflows in emerging economies: Some policy implications for post-COVID-19. Karahan and Bayır Future Business Journal. 2022; 8(1): 39.
33. Wawrosz P, Traksel S. Negative interest rates and its impact on GDP, FDI and banks' financial performance: The cases of Switzerland and Sweden. International Journal of Financial Studies. 2023; 11: 69.
34. Morshed N, Hossain MR. Causality analysis of the determinants of FDI in Bangladesh: fresh evidence from VAR, VECM and Granger causality approach. SN Business and Economics. 2022; 2(64).
35. Nzeh IC, Uzoechina BI, Ozoh JN, et al. Examining the impact of financial openness on domestic interest rate in Nigeria. ACTA VŠFS. 2023; 1(17): 23-37.
36. Tiberto BP, de Mendonça HF. Effects of sustainable monetary and fiscal policy on FDI inflows to emerging markets and developing economies. Banco Central do Brazil Working Paper Series 575. 2023.

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Review

How does digital transformation affect corporate governance paradigms? A synthesis of the literature

Yulong Sun[1,2,*], **Juancan Guo**[3]

[1] China Academy of Corporate Governance, Nankai University, Tianjin 300071, China
[2] Business School, Nankai University, Tianjin 300071, China
[3] School of Economics and Management, North China Electric Power University, Beijing 102206, China
*** Corresponding author:** Yulong Sun, sunyul1995@163.com

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Abstract: At present, digital transformation has become a vital way for companies to achieve sustainable growth. This paper reviews the literature on the correlation between digital transformation and corporate governance paradigm, analyzes the specific impact mechanism of AI technology and big data technology in the field of corporate governance, and explores the influence effect of the most popular ChatGPT technology on corporate governance from the perspective of business practice. It is found that digital transformation has an important impact on stakeholder management, information disclosure, green governance and other aspects of corporate governance. The purpose of the study is to provide a new reference for the construction of corporate governance paradigm and help companies achieve long-term development in the digital wave.

Keywords: digital transformation; digital technology; corporate governance

1. Introduction

With the rapid development of the digital economy, advanced digital technologies have emerged, and important strategic decisions of digital transformation (DT) have been implemented by companies, thus completing the deep integration of digital technologies and companies, and ultimately achieving sustainable development [1]. At the same time, corporate governance is an important factor affecting the sustainable development of a company because of involving a set of institutional arrangements such as supervision, incentive and coordination [2]. Therefore, it is of great significance to explore the influence mechanism of DT on corporate governance.

Advanced digital technologies could be used by DT to collect, store and analyze data, such as Chat GPT technology, big data technology and machine learning technology, which could enhance the corporate governance effect from multiple dimensions, provide sustainable technical support for the development of the company, and achieve large-scale changes in the production and governance system [3]. On the one hand, the implementation of digital transformation strategy can promote the flow of information across fields and departments, improve the sharing of resources between internal and external enterprises, help meet the needs of multi-stakeholders, and affect stakeholder governance. On the other hand, in the area of corporate governance, the use of digital technology monitoring at the management stage has greatly improved the scientific nature of strategic decisions, including high-quality information disclosure. In addition, digital technologies can be used as an important means of energy saving initiatives, enhancing systematic lifecycle management of the company and enhancing the green governance effect of the company. However, the long-term investment of large resources is required by digital transformation, and the initial investment cannot generate a rapid return, which may lead to resource scale effects and increase financial risk [4].

The research on the impact of digital transformation in the field of corporate governance is mainly divided into two aspects: one is the internal strategy and operation of the company, such as production efficiency [5] and corporate environmental performance [6,7], corporate innovation [8]. The second is the external factors of the company, such as business risk [9] and social responsibility [10]. In terms of the selection of research samples, Chinese companies are more likely to be selected as research objects [11,12] to analyze the influence mechanism of DT.

In general, this paper uses the method of literature review to study the correlation between DT and corporate governance. On the one hand, it summarizes the existing theoretical basis, builds the theoretical framework of this paper, and uses VOSviewer software to carry out literature measurement and generate a visual map. On the other hand, taking digital technology as the entry point, the influence effect and influence path of DT in the field of corporate governance are explored. Finally, the purpose of this paper is to assist company executives to further deepen their understanding of the influence mechanism of DT and adjust the management structure, so as to help the company obtain the development dividend of The Times and provide an important reference for the construction of a new corporate governance paradigm in the digital era.

The contribution of this study is as follows: on the one hand, it enriches the perspective of DT-related research and provides relevant reference for enterprises to implement digital transformation. At the macro level, this paper discusses the impact of enterprise DT from the perspective of corporate governance, including corporate stakeholder management, information disclosure and green governance, which helps to further clarify the importance of DT in the business process and provide theoretical support for enterprises to promote DT reasonably. At the micro level, this study takes digital technology as the entry point and explores the impact of digital transformation in the field of corporate governance through literature review. On the other hand, it deepens the relevant research on the influencing factors of corporate governance and provides new thinking for the digital intelligence of corporate governance. DT is an important way for enterprises to adapt to the development of the digital age and master key resources, and provides new ideas for enterprises to achieve sustainable development. At the same time, it enriches the research methods of literature review. Based on a large number of sample data in the WOS database, VOSviewer was used to summarize large-scale data [13] and perform bibliometric visualization, providing a new perspective for research in this field. In addition, this study has important practical significance, which provides an important reference for companies to build a new governance paradigm in the digital age, and helps companies to obtain dividends in the digital age and achieve sustainable development.

2. Methodology

In order to realize the purpose of this study, the relevant database is extended to improve the universality and relevance of this paper. WOS database is utilized in this paper, the keywords of which are set as “Digital transformation and corporate governance”, and the publication time of relevant literatures is defined as 2014–2023. The research area is business economics, environmental sciences ecology and science technology other topics. A total of 167 papers are found to be highly relevant to this study. Among the 167 papers, China, Russia, France and Italy are the most frequently studied countries or regions.

Through bibliometric analysis, this paper can better evaluate the relevant literature and draw common conclusions. This approach is not only beneficial for data processing, but also has a positive impact on academic output [14]. In terms of research methods, this paper uses VOSviewer to dig the internal relations between literatures related to this topic in recent ten years, generate bibliometric maps, and conduct bibliometric analysis for the propose of highlighting the research trends on the topic of “Digital transformation and corporate governance” from 2014 to 2023 (on 167 papers from the WOS database).

As an important database for global access to academic information, WOS database involves information in the fields of natural sciences, social sciences, arts and humanities, and includes nearly 9000 of the most prestigious high-impact research journals from around the world. It is of great significance to take the relevant literature collected by WOS as samples for research and analysis.

3. Theoretical framework

In the existing literature, a large number of theories are used to analyze and study the relationship between DT and corporate governance, among which resource-based theory, stakeholder theory and dynamic capability theory are the most frequently used theories. The above theories also provide a theoretical basis for DT to exert its unique advantages in the field of green governance. Details are shown in **Figure 1**.

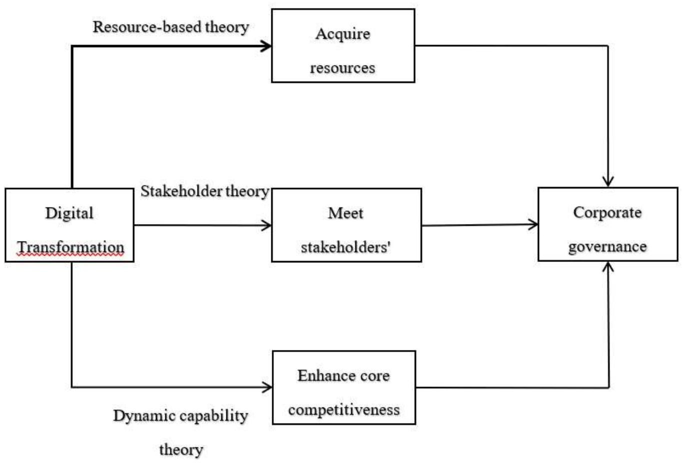


Figure 1. The theoretical framework of DT effect.

3.1. Resource-based theory

According to the resource-based theory, the core competence of an enterprise is determined by the heterogeneous resources possessed by the enterprise. DT is a key means for companies to make full use of the heterogeneous resources of data elements to create competitive advantages for the sustainable development of the company [15,16]. On the one hand, more favorable information resources are provided to enterprises with the support of DT [17]. With the help of digital technology, companies can accelerate the speed of information transmission and promote internal information exchange and sharing [18], which is conducive to breaking the "digital gap" in internal management of enterprises and strengthening communication and cooperation between departments. Ultimately, the efficiency of resource circulation and allocation can be improved, and the stability of financial resources of enterprises can be enhanced [19]. In addition, DT can also provide sufficient information support for the operation, management and strategic decision-making of enterprises. In addition, digital transformation can help promote the positive impact of enterprises on the environment, improve resource utilization, and build a more environmentally friendly circular economy development model [20,21].

On the other hand, under the current situation that national policies constantly emphasize innovation-driven and sustainable development, the cost of collecting and analyzing environmental information for enterprises could be reduced by digital transformation, so that the change direction of the latest policies and market dynamics could be quickly grasped by enterprises, and more preferential government policies could be obtained by enterprises [22], such as sufficient financial support, financing loans, which is beneficial to reduce the financing difficulty of R&D activities in the field of corporate governance. Moreover, company executives can accurately understand customer needs through the implementation of DT [23] and timely adjust business strategies, which will reduce production costs and improve productivity of enterprises. In summary, according to the resource-based theory, DT can create competitive advantages for companies, and improve the level of corporate risk taking and the effect of corporate governance [24].

3.2. Dynamic capability theory

Dynamic capability refers to the ability of an organization to create, adjust and expand competitive advantages for specific purposes [25]. Dynamic capabilities can help companies quickly adapt to complex and changing environmental situations of distance column changes and make dynamic adjustments. When it comes to scaling up operations, dynamic capabilities can create and maintain significant advantages over other competitors [25]. Therefore, dynamic capability plays a positive role in the business development of enterprises in the rapidly changing environment [26].

Research shows that dynamic capabilities play a crucial role in the DT of enterprises [27]. DT is not simply digitization based on existing capabilities [28]. Enterprises need to make full use of existing capabilities, while constantly exploring new capabilities in the process of business development, such as using perception capabilities to uncover opportunities and challenges in the internal management and external competitive environment of the company [29], so as to maintain dynamic innovation capabilities and competitiveness, which is conducive to the DT of the company. As for the solution of maintaining dynamic capability, company executives need to maintain keen judgment and innovation awareness in a complex and changing competitive environment, and promote green governance effect [30] by increasing

R&D investment, so as to gain competitive advantages in green products and services, and ultimately further enhance the effect of green governance.

Relevant researches on dynamic capability theory are mainly divided into three aspects, namely basic theory, process theory and hierarchy theory, among which the basic theory is the most widely used [31]. The influence of DT on corporate green governance can be explored based on the internal relationship between dynamic capability theory and corporate sustainable development strategy. Dynamic capability can be divided into green innovation capability and social responsibility capability, thus enriching the basic components of dynamic capability theory [32]. Energy conservation and emission reduction are advocated by green innovation, which provides environmental and social benefits [33]. Social responsibility refers to a company's ability to continuously improve and optimize its digital infrastructure based on customer feedback to meet the interests of all parties. Actively fulfilling social responsibility is conducive to improving the reputation and image of the company, thus enhancing the value of the brand [34].

3.3. Stakeholder theory

According to stakeholder theory, enterprises should take into account the interests of internal and external stakeholders and be responsible to all stakeholders, including shareholders. Stakeholder theory has been used by more and more existing literature to explore the effect of corporate governance. On the one side, there is a correlation between information disclosure and corporate financial performance. Constantinescu et al. [35] found that the information disclosure is positively correlated with the value of the company. A higher degree of information disclosure contributes to the enhancement and improvement of corporate transparency and financial performance [36,37], which is conducive to improving the degree of trust of shareholders and stakeholders. [38] found that positive signals about future financial performance can be released by corporate social responsibility reporting.

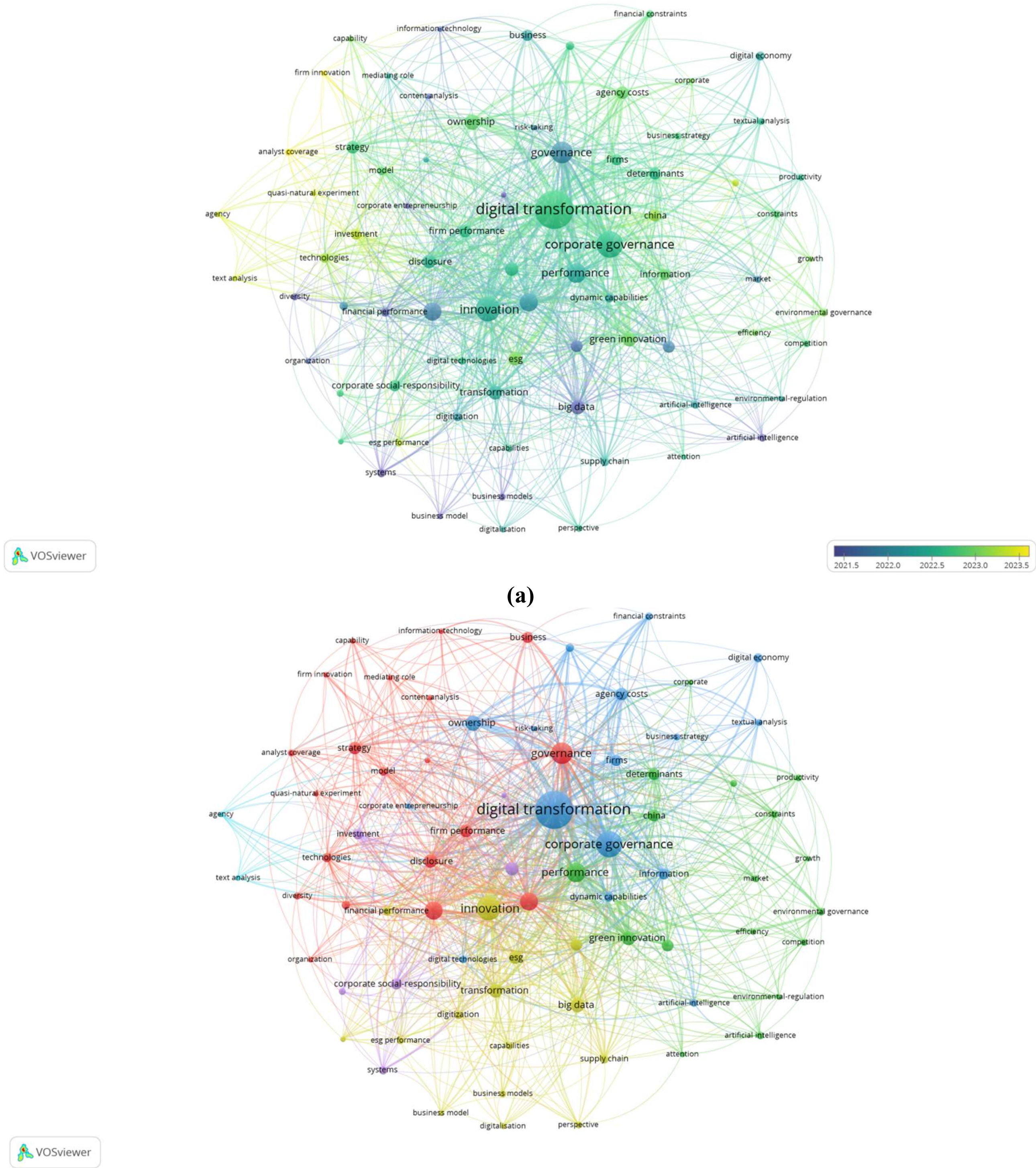
On the other side, Stakeholder theory can better promote DT to exert its influence in the field of corporate governance. In order to meet the interests of shareholders, investors and other people, companies can implement DT to reduce internal control costs and information costs, promote organizational efficiency [39] and decentralization reform [40], improve corporate supervision efficiency and optimize organizational authorization decisions [41].

In addition, stakeholder satisfaction can be improved by the high-quality information disclosure [42]. And puts more emphasis on the impact of information disclosure on consumers [43]. Empirical research finds that high-quality information disclosure has a positive effect on corporate development, which helps to enhance brand value and give consumers sufficient confidence.

4. Results

4.1. More and more scholars participate in exploring corporate governance

The connections between DT and corporate governance have been found in the literature since 2014 until 2023.



(a)

(b)

Figure 2. A bibliometric analysis of research themes on ‘digital transformation and corporate governance’. **(a)** visualization of the paper network confirming the main themes of research; **(b)** evolutions of research trends between 2014 and 2023 based on WOS database.

Figure 2 gives a holistic overview of the past research based on keywords’ cooccurrences with ‘digital transformation and corporate governance’. The analysis reveals a discernible prominence reflecting four principal domains of investigation, namely three significant clusters that can be identified based on visualization in **Figure 2a,** such as a) digital transformation, b) disclosure, c) firm performance, d) environmental performance. We can see that studies on financial performance, green innovation, artificial intelligence, big data and information disclosure are hot. (**Figure 2b**). Thus, it can be seen that the existing literature on DT and corporate governance field research is more comprehensive, and its view of research is also different.

The colours in the top panel indicate the themes of research that the papers are discussing, while the colours in the bottom panel indicate the year of publication. N = 167 papers.

4.2. Digital technology and corporate governance

Based on the literature review from 2014 to 2023, combined with the relationship between digital technology and corporate governance, this paper subdivides digital technology into widely used big data technology and artificial intelligence technology, and explores its impact on the field of corporate governance in combination with the latest hot ChatGPT technology. Big data technology is an important part of digital technology, it can mine a large number of data samples, analyze the operating rules and characteristics of a specific field, and draw conclusions that are universal. The application of big data technology can better explore the universality of DT effect in the field of corporate governance. On the other hand, artificial intelligence technology plays an important role in digital technology. It is based on big data and uses unique algorithms to analyze data and build models. The application of artificial intelligence technology helps company executives improve decision-making through data models. Finally, at the practical level, ChatGPT is a vivid embodiment of AI technology, which is also the hottest digital technology at this stage. By summarizing the influence mechanism of ChatGPT in the field of corporate governance, we can enrich the influence forms of DT and strengthen the practicability of ChatGPT in business activities.

4.2.1. Big data technology and corporate governance

Human production and lifestyle have been profoundly reformed and influenced by the fourth Industrial Revolution in the digital age [44]. The economic and social activities of many actors, including companies, are recorded in digital form, forming various forms of big data, which contains a large number of interrelated dynamic information of microeconomic actors.

Compared with traditional data, big data has three characteristics of influence in the field of corporate governance: First, the scale is large, which means the sample size and the number of variables are large [45]. A lot of big data is the dynamic behavioral big data of a large number of interrelated microeconomic subjects (such as consumers, producers, investors, etc.). The larger the data scale, the greater the universality of research conclusions, and the richer the information resources provided for corporate decision-making. The second is high efficiency, which means that high-frequency data and even real-time data are available [46]. The third is the diversification of data resources. Through big data technology, the acquisition of corresponding data information provides information advantages for improving the effect of corporate governance, including structured data and various forms of unstructured data, such as text, graphics, audio, video, etc.

Big data technology can provide a new perspective for corporate governance research. Ahmed et al. [47] believe that big data technology can better improve the governance of the board of directors and monitor the lag of audit reports. Besides, Choi and Park [48] pay more attention to the application of big data in the field of corporate green governance. It is found that big data analysis can positively affect the company's social responsibility performance. Sadasivam et al. [49] can use big data analysis to identify corporate governance fraud and avoid business risks.

4.2.2. AI technology and corporate governance

As one of the core development directions of the new generation of technological revolution, artificial intelligence technology has an impact on enterprises mainly in promoting enterprise technology upgrading and product iteration as well as enterprise management reform [50]. Overall, AI technology will have a significant impact on corporate governance. On the one hand, due to the professionalism of artificial intelligence technology, enterprises increase their investment in the application and research and development of artificial intelligence technology, which encourages enterprises to include core technical personnel in the corporate governance team and encourage them to participate in corporate governance activities, including the allocation and incentive in equity, the determination of salary levels, and the internal composition of "directors, supervisors, senior managers" positions, etc. [51,52].

On the other hand, AI provides an important technical means for improving the level of corporate governance [53]. Relying on powerful learning technology, accurate algorithms, neural network technology and other advanced technologies, AI can assist in analyzing the company's management and decision-making [54], greatly reducing the subjectivity of management decisions, improving the rationality of decisions, and effectively ensuring that the company's long-term and global development of major governance decisions more scientific and reasonable [55].

In recent years, domestic and foreign scholars have conducted research on artificial intelligence and corporate governance. The research focus can be summarized in two aspects: on the one hand, the influence mechanism of artificial intelligence on corporate governance. The powerful computing power of artificial intelligence can not only effectively reduce the problem of information asymmetry in corporate governance and reduce agency costs [56], but also provide technical support for scientific decision-making of company executives. Gao and Liu [57] believe that the application of artificial intelligence in enterprise management makes the behavior of managers tend to be "transparent", the transparency and efficiency of corporate operation and management are improving, and new governance methods are gradually emerging [58]. This also indicates that the development of artificial intelligence technology makes the intelligence of corporate governance feasible [59]. The "wisdom + data" decision model built based on artificial intelligence technology can support enterprises to make more accurate management decisions [60]. On the other hand, some scholars believe that artificial intelligence will negatively affect corporate governance. The application of artificial intelligence technology will increase the complexity of corporate governance, exacerbate institutional conflicts [61] and agency conflicts [62], and weaken the efficiency of corporate governance.

4.2.3. Business practice level: The impact of Chat GPT technology

In order to explore the impact of models such as ChatGPT on corporate governance paradigms, the main features of ChatGPT need to be understood.

ChatGPT has remarkable properties as a deep learning algorithm that mimics human cognition [63]. One is intelligence [64]. Since ChatGPT is now utilized all over the world and provided with free training and updates, the speed of improvement and iteration of ChatGPT will be further accelerated [65]. ChatGPT will gradually approach the level of human intelligence and surpass humans in some aspects, especially in the case of long repetitive work, humans may get tired, but this situation should not be considered when the machine is working [66]. Second is the wide range of applications. With access to public information on the Internet, ChatGPT can perform a variety of tasks, including generating structured knowledge, providing solutions to problems, etc., thereby helping to improve the decision-making ability and level of company executives [67]. The third is the ability of inductive speculation. As a generative artificial intelligence technology [68], the text content generated by ChatGPT comes from the large model to predict the probability of word and phrase occurrence in the Internet big data [69]. The inductive reasoning function of ChatGPT is realized, which provides significant reference for the strategic decision of the company.

At present, ChatGPT technology has been gradually applied to the business practice of companies [70]. For instance, ChatGPT has been appointed by CS India as the CEO of the organisation [71]. At the same time, ChatGPT has taken on supervisory and management functions within the company, assisting in the day-to-day operation and scaling of the company. João Ferrão dos Santos appointed ChatGPT as CEO of the company and set KPIs for ChatGPT by allocating a budget of $1000 per day. In response, ChatGPT suggested building corresponding e-commerce platforms and utilizing AI imagery and visual graphics tools to design clothing, thereby obtaining venture capital for businesses [72].

The selection of research methods, that is, qualitative analysis or quantitative analysis, has been debated in academic circles. Due to the vigorous development of big data technology, the valuable information contained in various unstructured data, especially text data, has attracted wide attention of scholars. Shiller [73] points out that, in contrast to structured economic data, text data contains a lot of information about company development and performance improvement. Therefore, it has become an effective way to extract psychological factors information of various economic entities including companies from text data. The main tool to extract psychological factors from text data is natural language processing technology, and the commonly used methods include word frequency method, topic method, etc. [74].

4.3. The impact of digital transformation on corporate governance

Digital transformation can alleviate the problem of information asymmetry and has a significant impact on improving the corporate governance effect. On the one hand, the digital transformation of enterprises alleviates the problem of information asymmetry and reduces the derived principal-agent cost [75]. The technological change brought about by digital transformation not only improves the corporate ecoenvironmental governance effect [76,77], also has a positive impact on the scale and business performance of enterprises [78]. For example, in the field of green governance, industrial data modeling and artificial intelligence technology are used to implement carbon emission detection and carbon footprint tracking, and systematic management of the entire life cycle of the company is strengthened, so as to improve the effect of green governance and the company's business performance [79]. Therefore, the impact of digital transformation in the field of green governance deserves attention. On the other hand, DT helps to improve the quality of information disclosure. High-quality information disclosure is conducive to improving regulatory efficiency [80] and the business value of enterprises [81]. The implementation of digital transformation can not only help reduce the operating costs of enterprises, including the costs of information search, negotiation and transaction monitoring [82], but also form a more centralized management model [83] and improve the quality of information supervision and audit [84], so as to improve the quality of information disclosure through various ways and effects, and thus have a significant impact on corporate governance.

Moreover, digital transformation can enhance external oversight. The company has accelerated the exchange of information with the external environment through DT [85]. The new business model has become more complex and specialized, and the traditional financial statement information can no longer meet the needs of internal and external stakeholders, including investors. There is also a large gap in the efficiency of information resource utilization among stakeholder groups [86]. Therefore, attention should be paid to the impact of DT on enterprise stakeholder management. Based on a review of a large number of relevant literatures, this paper divides the specific impact paths of DT on corporate governance into three aspects: stakeholder management, information disclosure and green governance.

4.3.1. Digital transformation affects stakeholder management

Stakeholder rights and interests have always been a hot topic in academic circles. Brooks et al. [87] divide the stakeholders of an enterprise into internal and external categories. Internal stakeholders include shareholders, boards, management and employees, while external stakeholders include consumers, creditors, governments and other businesses. The protection of stakeholders' rights and interests includes economic responsibilities to shareholders, as well as responsibilities to creditors, governments, customers, employees and other stakeholders. Internal stakeholders are different from external stakeholders. Internal stakeholders are the makers and implementers of corporate strategies, so they have more immediate and adequate access to information than external stakeholders. Bridoux and Vishwanathan [88] divide stakeholders into strong stakeholders and weak stakeholders. The strong stakeholders have stronger bargaining power, thus constrains managers' behaviors and strategic decisions.

This study analyzes the influence mechanism of DT from the perspectives of internal stakeholders (managers and employees) and external stakeholders (consumers and government).

From the perspective of internal stakeholders, on the one hand, digital transformation can help enterprises and employees achieve value co-creation. Digital transformation can clearly communicate business goals to employees. Enterprise digital transformation industrializes enterprise data elements, including a series of data elements that are closely related to enterprise objectives such as employee performance, responsibility and risk accumulated during enterprise operation. Digital transformation applies these data indicators to employee management to realize the close combination of internal performance evaluation and enterprise objectives. At the employee level, digital transformation clarifies the direction of the enterprise's efforts and the path of performance improvement. At the enterprise level, digital transformation activates the data elements of the enterprise, enables employees to create value through the digital performance appraisal mechanism, reduces employee management costs, and improves the incentive and constraint mechanism of the enterprise. At the same time, the digital elements provided by the enterprise will not only bring long-term development opportunities for the enterprise, but also greatly enhance the individual innovation and entrepreneurship ability and data analysis and processing ability of employees. Intrapreneurial employees use the market and customer data obtained from the enterprise's front-line operation to explore entrepreneurial opportunities that can improve their own economic benefits. In addition, employees' intrapreneurship can share and exchange information through a digital cloud platform to exercise operation and management capabilities in practice. It also provides a digital assessment path for companies to find outstanding employees with leadership potential and innovative thinking. On the other hand, the digital transformation of enterprises has a profound impact on the working patterns of managers. Managers use digital technology to make more intelligent decisions, so that the decision-making process can be tracked and justified. Digital transformation reduces the vertical and horizontal communication costs within the enterprise. The efficient information processing ability of digital technology helps managers to save time and effort to supervise the operation status of the organization, pay precise attention to the work of employees, and ensure that managers can effectively manage more subsidiary departments and employees. While the scale of enterprises is gradually expanding, the management range of managers can also be improved. Ensure a reasonable level of enterprise management and fast information transmission, so that enterprises are more democratic and dynamic.

From the perspective of external stakeholders, on the one hand, digital transformation can effectively eliminate the information asymmetry between consumers and production enterprises. The digital transformation of enterprises has shifted the traditional "manufacturing-inventory-sales" model to the "sales-based production" manufacturing model. Through big data technology, enterprises can standardize the number of orders and quickly respond to consumers' personalized and fragmented needs, improve production accuracy, enhance enterprises' personalized manufacturing capabilities and customization efficiency, and reduce information asymmetry and resource waste. It also increases consumer satisfaction. Finally, the company realizes the win-win situation of enterprises and consumers through DT. On the other hand, digital transformation helps enterprises to find quality suppliers more efficiently. Based on the characteristics of digital technology breaking through time and space constraints, enterprises establish their own digital supply market through digital transformation. At the same time, enterprises can choose and process a large number of order data, and select high-quality suppliers that meet market demand and consumer expectations in a wider range and a wider region. Then through the digital market online bidding, bidding and other forms to select high quality, low price, good credit suppliers, and complete the contract signing online. Digital supply market allows enterprises to significantly reduce the purchase cost, can better adapt to the rapidly changing market environment and customer needs, but also allows suppliers to save sales time and product storage costs, to achieve more intelligent inventory management.

In order to explore the influence mechanism of DT on stakeholder management, scholars have adopted various research methods to carry out empirical analysis. Due to differences in the selection of variables, the general model is constructed as follows:

$$right_{i,t} = \gamma_0 + \gamma_1 digital_{\mathrm{i,t}} + \gamma_2 X_{i,t} + \mu_t + \nu_i + \eta_j + \varepsilon_{i,t}$$

Among them, right is a set of stakeholder equity variables, including internal stakeholder equity (inright) and external stakeholder equity (exright). digital is the degree of digitization; *X* is the set of control variables; *i* stands for individual enterprise, *t* stands for time, j stands for industry; μ_t is the time fixed effect; ν_i is the fixed effect of enterprise; η_j is the fixed effect of the industry; $\varepsilon_{i,t}$ is a random interference term.

4.3.2. Digital transformation affects corporate information disclosure

High-quality information disclosure plays an important role in reducing information asymmetry, alleviating agency problems, optimizing resource allocation, and protecting small and medium investors [89,90]. However, in a semi-strong efficiency capital market, the strategic preference and insider manipulation of information disclosure [91] seriously affect the stability of the capital market. How to improve the quality of information disclosure has become an important and urgent practical problem. In the digital time, the implementation of digital transformation can solve this problem well.

The digital transformation of enterprises affects the quality of information disclosure in multiple aspects. On the one hand, Digital transformation reduces the company's operating costs. Digitization has created information, knowledge management and decision support systems that reduce the risk of information distortion [92]. As digital management concepts and internal control methods are embedded in daily operations, the transparency of financial management and internal control has increased significantly. This can reduce supervision costs for external stakeholders, as well as agency costs for internal stakeholders and managers [93] to improve capital allocation efficiency of enterprises [94] and the quality of information disclosure.

On the other hand, digital transformation can improve the efficiency of information supervision, thus improving the quality of corporate information disclosure. Using digital technology to supervise the information generation process can also reduce the space for human manipulation and the executives' behavior of opportunistic information disclosure [95]. Besides, DT enables companies to aggregate and process massive amounts of data at a lower cost and with a faster speed. This can reduce the cost of information processing and transmission, organizational coordination and control management, which can inhibit the concentration of power of executives [96], and ultimately improving the quality of information disclosure.

The existing literature discusses the factors affecting the quality of information disclosure from various aspects. From the perspective of internal factors, regulatory characteristics, internal governance [97], corporate characteristics [98,99], institutional investors [100,101], etc., have a significant impact on information disclosure. From the perspective of external factors, regulations of regulatory agencies will affect the content and characteristics of corporate information disclosure texts [102], and texts are more readable under non-mandatory disclosure requirements [103].

In order to explore the influence mechanism of DT on corporate information disclosure, scholars have adopted various research methods to carry out empirical analysis. The most common way to measure the quality of information disclosure is to measure the number of shares by stock production and regression slope fungibility. The higher the rate of change, the worse the quality of information disclosure; On the contrary, the better the quality of the disclosure [104]. The details are as follows:

$$ln|\Delta P_t/\mathrm{P}_{t-1}| = \alpha + \beta(Vol_t - Vol_0) + \mu_i$$

ΔP_t is the difference between ΔP_t and P_{t-1}; P_t is the closing price on day *t*; Volt is the trading volume on day *t*; Vol_0 is the annual daily trading volume; Delete values with annual trading days less than 100 days, β of negative value and ΔP_t value of 0, obtaining KimV = β × 1,000,000. The smaller the KimV value is, the higher the information disclosure quality is.

4.3.3. Digital transformation affects corporate green governance

As an important tool for the digital transformation of the enterprise, digital technology plays an auxiliary role in the business management and strategic decision-making. [105] adopt dual theory and fuzzy set qualitative comparative analysis (fsQCA) to combine digital tools and corporate responsibility to improve the green governance effect of enterprises. Guo et al. [106] believe that the updated iteration of digital technology and the improvement of digital infrastructure contribute to the online “paperless” work, reduce resource consumption and promote the green transformation of the company.

With the advancement of digital transformation, the connection strength within the company and between upstream and downstream partners has been further enhanced, and key resources such as information and technology have gradually accumulated, which is conducive to improving technological innovation, strengthening the correlation and collaboration in the supply chain, and making the company’s green activities more transparent and efficient. Corporate green governance is positively influenced by digital transformation [107].

On the one hand, digital transformation can promote product development and innovation of enterprises and improve corporate green governance. Based on cutting-edge digital technology, enterprises can shorten the research and development cycle, reduce the research and development cost of green products, accelerate the development progress of green differentiated products [108], and improve the efficiency of green innovation. In addition, the company is promoted by digitalization to carry out cooperative research and development activities with universities and research institutes, so as to integrate a large amount of resources needed in the field of green innovation, master core technologies and future development trends, and provide technical support for the green governance. On the other hand, digital transformation enhances the closeness between the company and its partners, thereby strengthening corporate green governance. By establishing intelligent integrated systems, Internet of Things and other digital intelligent platforms, companies can realize the full exchange and sharing of information, technology and other resources in the supply chain, so as to improve the allocation efficiency and ultimately promote digital transformation [109]. Additionally, digital transformation has advantages in information exchange and resource sharing, which stimulates downstream greening demand and further promotes upstream green governance. Furthermore, upstream enterprises carry out green governance by transforming and upgrading equipment and reducing pollutant emissions [110], which also improves the standards of downstream enterprises' production processes, thus enhancing the green governance effect of the entire supply chain.

Based on the above analysis, the model construction of the impact of digital transformation on corporate green governance is as follows:

$$Greengov_{i,t} = \beta_0 + \beta_1 Digital_{i,t} + \beta_2 Control_{i,t} + \sum Year + \sum Firm + \varepsilon_{i,t}$$

Greengov is considered as a proxy variable to measure a company's green governance, *Digital* is utilized to measure the degree of a company's digital transformation, *Control* is a selected series of control variables, *Year* and *Firm* are year fixed effects and individual fixed effects, while ε represents random interference terms.

5. Conclusion

The impact of digital transformation in the field of corporate governance has become a hot research topic, which is paid more and more attention by more and more scholars. The rapid development of digital economy has given birth to cutting-edge digital technology. Many studies focus more on specific digital technology as a typical representative of the digital transformation of the company and explore the impact mechanism on the corporate governance paradigm, such as big data technology and AI technology, and explore the impact of ChatGPT technology, which is the most popular at present. This paper reviews and summarizes relevant literature, points out the specific characteristics of digital technology and corresponding research methods from the micro level, and finds that digital technology has a positive effect on corporate governance, which provides reference and suggestions for building a new corporate governance paradigm and improving the effect of corporate governance.

Further research points out the specific impact path of digital transformation on corporate governance paradigm at the macro level, more attention is paid to the impact of digital transformation on corporate stakeholder management, information disclosure and green governance, and the corresponding variable design is adopted to build a basic model for empirical research. Through model construction, the influence mechanism of DT on these three levels is proposed in this paper, which provides a basis for further research by subsequent scholars and business practices of company executives in the digital era, and helps the company achieve sustainable long-term development.

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References

1. Ma D, Zhu Q. Innovation in emerging economies: Research on the digital economy driving high-quality green development. Journal of Business Research. 2022; 145: 801-813. doi: 10.1016/j.jbusres.2022.03.041
2. Boeva B, Zhivkova S, & Stoychev I. Corporate governance and the sustainable development,2021; https://revistia.com/index.php/ejes/article/view/7006.
3. Schwab K. The fourth industrial revolution: What it means, how to respond. Geneva, Switzerland: World Economic Forum; 2016.
4. Guo JT, Luo GY. How the tax burden affects the high-quality development of enterprises: the moderating effect based on the digital transformation of enterprises. Journal of Shenzhen University (Humanities and Social Sciences Edition). 2023; 40(04): 1–11.
5. Bharadwaj AS. A Resource-Based Perspective on Information Technology Capability and Firm Performance: An Empirical Investigation. MIS Quarterly. 2000; 24(1): 169. doi: 10.2307/3250983
6. Zhao X, Cai L. Digital transformation and corporate ESG: Evidence from China. Finance Research Letters. 2023; 58: 104310. doi: 10.1016/j.frl.2023.104310
7. Yang P, Hao X, Wang L, et al. Moving toward sustainable development: the influence of digital transformation on corporate ESG performance. Kybernetes. 2023; 53(2): 669-687. doi: 10.1108/k-03-2023-0521
8. Niu Y, Wen W, Wang S, et al. Breaking barriers to innovation: The power of digital transformation. Finance Research Letters. 2023; 51: 103457. doi: 10.1016/j.frl.2022.103457
9. Tian G, Li B, Cheng Y. Does digital transformation matter for corporate risk-taking? Finance Research Letters. 2022; 49: 103107. doi: 10.1016/j.frl.2022.103107
10. Xu Y, Wang L, Xiong Y, et al. Does digital transformation foster corporate social responsibility? Evidence from Chinese mining industry. Journal of Environmental Management. 2023; 344: 118646. doi: 10.1016/j.jenvman.2023.118646
11. Li S, Yang Z, Tian Y. Digital transformation and corporate performance: evidence from China. China Economic Journal. 2023; 16(3): 312-334. doi: 10.1080/17538963.2023.2254138
12. Gao F, Lin C, Zhai H. Digital Transformation, Corporate Innovation, and International Strategy: Empirical Evidence from Listed Companies in China. Sustainability. 2022; 14(13): 8137. doi: 10.3390/su14138137
13. Van Eck NJ, Waltman L. Software survey: VOSviewer, a computer program for bibliometric mapping. Scientometrics. 2009; 84(2): 523-538. doi: 10.1007/s11192-009-0146-3
14. Ellegaard O, Wallin JA. The bibliometric analysis of scholarly production: How great is the impact? Scientometrics. 2015; 105(3): 1809-1831. doi: 10.1007/s11192-015-1645-z
15. Giustiziero G, Kretschmer T, Somaya D, et al. Hyperspecialization and hyperscaling: A resource-based theory of the digital firm. Strategic Management Journal. 2022; 44(6): 1391-1424. doi: 10.1002/smj.3365
16. Zhang H, Zhang Q. How Does Digital Transformation Facilitate Enterprise Total Factor Productivity? The Multiple Mediators of Supplier Concentration and Customer Concentration. Sustainability. 2023; 15(3): 1896. doi: 10.3390/su15031896
17. Chen H, Tian Z. Environmental uncertainty, resource orchestration and digital transformation: A fuzzy-set QCA approach. Journal of Business Research. 2022; 139: 184-193. doi: 10.1016/j.jbusres.2021.09.048
18. Mooi EA, Frambach RT. Encouraging innovation in business relationships—A research note. Journal of Business Research. 2012; 65(7): 1025-1030. doi: 10.1016/j.jbusres.2011.03.016
19. Frynas JG, Mol MJ, Mellahi K. Management Innovation Made in China: Haier’s Rendanheyi. California Management Review. 2018; 61(1): 71-93. doi: 10.1177/0008125618790244
20. Asif M, Yang L, Hashim M. The Role of Digital Transformation, Corporate Culture, and Leadership in Enhancing Corporate Sustainable Performance in the Manufacturing Sector of China. Sustainability. 2024; 16(7): 2651. doi: 10.3390/su16072651
21. Long H, Feng GF, Chang CP. How does ESG performance promote corporate green innovation? Economic Change and Restructuring. 2023; 56(4): 2889-2913. doi: 10.1007/s10644-023-09536-2

22. Song M, Wang S, Zhang H. Could environmental regulation and R&D tax incentives affect green product innovation? Journal of Cleaner Production. 2020; 258: 120849. doi: 10.1016/j.jclepro.2020.120849
23. Cui J, Wang SW, Xin YC. Research on technical framework of smart grid data management from consortium blockchain perspective. In: Proceedings of the CSEE. 2020.
24. Li D, Shen W. Can Corporate Digitalization Promote Green Innovation? The Moderating Roles of Internal Control and Institutional Ownership. Sustainability. 2021; 13(24): 13983. doi: 10.3390/su132413983
25. Siachou E, Vrontis D, Trichina E. Can traditional organizations be digitally transformed by themselves? The moderating role of absorptive capacity and strategic interdependence. Journal of Business Research. 2021; 124: 408–421. doi: 10.1016/j.jbusres.2020.11.011
26. Li D, Liu J. Dynamic capabilities, environmental dynamism, and competitive advantage: Evidence from China. Journal of Business Research. 2014; 67(1): 2793–2799. doi: 10.1016/j.jbusres.2012.08.007
27. Matarazzo M, Penco L, Profumo G, et al. Digital transformation and customer value creation in Made in Italy SMEs: A dynamic capabilities perspective. Journal of Business Research. 2021; 123: 642–656. doi: 10.1016/j.jbusres.2020.10.033
28. Feng H, Wang F, Song G, et al. Digital Transformation on Enterprise Green Innovation: Effect and Transmission Mechanism. International Journal of Environmental Research and Public Health. 2022; 19(17): 10614. doi:10.3390/ijerph191710614
29. Ellström D, Holtström J, Berg E, et al. Dynamic capabilities for digital transformation. Journal of Strategy and Management. 2021; 15(2): 272–286. doi: 10.1108/jsma-04-2021-0089
30. Yousaf Z. Go for green: green innovation through green dynamic capabilities: accessing the mediating role of green practices and green value co-creation. Environmental Science and Pollution Research. 2021; 28(39): 54863-54875. doi: 10.1007/s11356-021-14343-1
31. Su X, Wang S, Li F. The Impact of Digital Transformation on ESG Performance Based on the Mediating Effect of Dynamic Capabilities. Sustainability. 2023; 15(18): 13506. doi: 10.3390/su151813506
32. Mele G, Capaldo G, Secundo G, et al. Revisiting the idea of knowledge-based dynamic capabilities for digital transformation. Journal of Knowledge Management. 2023; 28(2): 532-563. doi: 10.1108/jkm-02-2023-0121
33. Jiang Z, Lyu P, Ye L, et al. Green innovation transformation, economic sustainability and energy consumption during China's new normal stage. Journal of Cleaner Production. 2020; 273: 123044. doi: 10.1016/j.jclepro.2020.123044
34. Fosu E, Yi K, Asiedu D. The effect of CSR on corporate social performance: Mediating role of corporate image, green innovation and moderating role of corporate identity. Corporate Social Responsibility and Environmental Management. 2023; 31(1): 69-88. doi: 10.1002/csr.2553
35. Constantinescu D, Caraiania C, Lungua CI, et al. Environmental, social and governance disclosure associated with the firm value. Evidence from energy industry. Accounting and Management Information Systems. 2021, 20(1): 56-75.
36. Inamdar MM. Moderating role of ESG disclosures and its impact on firm financial performance. The Quarterly Review of Economics and Finance. 2024; 97: 101892. doi: 10.1016/j.qref.2024.101892
37. Veeravel V, Murugesan VP, Narayanamurthy V. Does ESG disclosure really influence the firm performance? Evidence from India. The Quarterly Review of Economics and Finance. 2024; 95: 193-202. doi: 10.1016/j.qref.2024.03.008
38. Lys T, Naughton JP, Wang C. Signaling through corporate accountability reporting. Journal of Accounting and Economics. 2015; 60(1): 56-72. doi: 10.1016/j.jacceco.2015.03.001
39. Yuan C, Xiao TS, Geng CX, Sheng Y. Digital Transformation and Division of Labor between Enterprises: Vertical Specialization or Vertical Integration. China Industrial Economics. 2021; (09): 137-155.
40. Liu Z, Yao YX, Zhang GS, Kuang HS. Firm's Digitalization, Specific Knowledge and Organizational Empowerment. China Industrial Economics. 2020; (09): 156-174.
41. Dobrajska M, Billinger S, Karim S. Delegation Within Hierarchies: How Information Processing and Knowledge Characteristics Influence the Allocation of Formal and Real Decision Authority. Organization Science. 2015; 26(3): 687-704. doi: 10.1287/orsc.2014.0954
42. Srivisal N, Jamprasert N, Sthienchoak J, et al. Environmental, social and governance and creditworthiness: Two contrary evidence from major Asian markets. Asian Academy of Management Journal of Accounting and Finance. 2021; 17(2): 161-187. doi: 10.21315/aamjaf2021.17.2.7
43. Nugroho DPD, Hsu Y, Hartauer C, et al. Investigating the Interconnection between Environmental, Social, and Governance (ESG), and Corporate Social Responsibility (CSR) Strategies: An Examination of the Influence on Consumer Behavior. Sustainability. 2024; 16(2): 614. doi: 10.3390/su16020614

44. Schwab K, Davis N. Shaping the future of the fourth industrial revolution. Crown Currency; 2018.
45. Fan J, Han F, Liu H. Challenges of Big Data analysis. National Science Review. 2014; 1(2): 293-314. doi: 10.1093/nsr/nwt032
46. Seddon JJJM, Currie WL. A model for unpacking big data analytics in high-frequency trading. Journal of Business Research. 2017; 70: 300-307. doi: 10.1016/j.jbusres.2016.08.003
47. Ahmed HMS, El-Halaby S, Albitar K. Board governance and audit report lag in the light of big data adoption: the case of Egypt[J]. International Journal of Accounting & Information Management, 2023, 31(1): 148-169.
48. Choi HY, Park J. Do data-driven CSR initiatives improve CSR performance? The importance of big data analytics capability. Technological Forecasting and Social Change. 2022; 182: 121802. doi: 10.1016/j.techfore.2022.121802
49. Sadasivam GS, Subrahmanyam M, Himachalam D, et al. Corporate governance fraud detection from annual reports using big data analytics. International Journal of Big Data Intelligence. 2016; 3(1): 51. doi: 10.1504/ijbdi.2016.073895
50. Cui X, Xu B, Razzaq A. Can Application of Artificial Intelligence in Enterprises Promote the Corporate Governance? Frontiers in Environmental Science. 2022; 10. doi: 10.3389/fenvs.2022.944467
51. Yu-tao Z, Jun-wen F, Xiu-lai W. Research on the Incentive Combination of Core Technical Personnel in the Hospital. In: Proceedings of the 2014 International Conference on Management of e-Commerce and e-Government.
52. Ma Y. Research on the Salary System Design of the Technical Personnel in State-owned Enterprises - Take Changqing Oilfield Exploration and Development Research Institute as an example. In: Proceedings of the 2017 International Conference on Innovations in Economic Management and Social Science (IEMSS 2017).
53. Cui X, Xu B, Razzaq A. Can application of artificial intelligence in enterprises promote the corporate governance? Frontiers in Environmental Science, 2022, 10: 944467.
54. Bag S, Gupta S, Kumar A, et al. An integrated artificial intelligence framework for knowledge creation and B2B marketing rational decision making for improving firm performance. Industrial Marketing Management. 2021; 92: 178-189. doi: 10.1016/j.indmarman.2020.12.001
55. Judijanto L, Asfahani A, Bakri AA, et al. AI-Supported Management through Leveraging Artificial Intelligence for Effective Decision Making. Journal of Artificial Intelligence and Development. 2022; 1(1): 59-68.
56. Wu W. The integration of blockchain technology and corporate governance-value, path and legal response. Secur. Mark. Her. 2021; 6.
57. Gao S, Liu J. The Impact of Artificial Intelligence on Enterprise Management Theory and its Response. Sci. Res; 2018.
58. Li W. Corporate Governance Reform in the Mobile Internet Era. Nankai Manag; 2014.
59. Zhang J. Intelligent Management: Theoretical Innovation and Technology Development. Fujian Forum Humanit. Soc. Sci; 2020. P. 10.
60. Horvitz EJ, Breese JS, Henrion M. Decision Theory in Expert Systems and Artificial Intelligence. Int. J. Approx. Reason. 1988; 2(3). doi:10.1016/ 0888-613x(88)90120-x
61. Doyle J, Ge W, McVay S. Determinants of weaknesses in internal control over financial reporting. Journal of Accounting and Economics. 2007; 44(1-2): 193-223. doi: 10.1016/j.jacceco.2006.10.003
62. Gu H, Bian F, Elahi E. Impact of Availability of Grandparents' Care on Birth in Working Women: An Empirical Analysis Based on Data of Chinese Dynamic Labour Force. Child. Youth Serv. Rev. 2021; 121: 105859. doi:10.1016/j. childyouth.2020.105859
63. Algabri HKM. Artificial Intelligence And ChatGPT. Academic Guru Publishing House. 2024.
64. Levantino FP. Generative and AI-powered oracles: "What will they say about you?" Computer Law & Security Review. 2023; 51: 105898. doi: 10.1016/j.clsr.2023.105898
65. Wu T, He S, Liu J, et al. A Brief Overview of ChatGPT: The History, Status Quo and Potential Future Development. IEEE/CAA Journal of Automatica Sinica. 2023; 10(5): 1122-1136. doi: 10.1109/jas.2023.123618
66. Kalla D, Smith N, Samaah F, & Kuraku S. Study and analysis of chat GPT and its impact on different fields of study. International journal of innovative science and research technology. 2023; 8(3).
67. Kamaruddin R I. ChatGPT and the Future of Management Consulting: Opportunities and Challenges Ahead [D]. Massachusetts Institute of Technology, 2023.
68. Rane N. Role and challenges of ChatGPT and similar generative artificial intelligence in business management [J]. Available at SSRN 4603227, 2023
69. Bravo de Dios, Y. Identification of texts generated by AI (ChatGPT). 2023. https://diposit.ub.edu/dspace/handle/2445/212020

70. Petrin M. The impact of AI and new technologies on corporate governance and regulation [J]. Singapore Journal of Legal Studies, 2024: 90-106.
71. Bharat S. Youth Organisation Appoints ChatGPT AI Bot As Its CEO, Calls It 'Groundbreaking', India Times. Available online: www.indiatimes.com/technology/news/ organisation-appoints-chatgpt-bot-as-its-ceo-592620.html (accessed on 28 August 2024).
72. Lusa. Portuguese entrepreneur creates viral company. The Portugal News. Available online: www.theportugalnews.com/news/2023-04-28/portuguese-entrepreneur-creates-viral-company/77089 (accessed on 28 August 2024).
73. Shiller RJ. Narrative economics: How stories go viral and drive major economic events. Princeton University Press; 2020.
74. Gentzkow M, Kelly B, Taddy M. Text as Data. Journal of Economic Literature. 2019; 57(3): 535-574. doi: 10.1257/jel.20181020
75. Li D, Shen W. Can Corporate Digitalization Promote Green Innovation? The Moderating Roles of Internal Control and Institutional Ownership. Sustainability. 2021; 13(24): 13983. doi: 10.3390/su132413983
76. Yermack D. Corporate governance and blockchains[J]. Review of finance, 2017, 21(1): 7-31.
77. Goldstein I, Spatt CS, Ye M. Big Data in Finance. The Review of Financial Studies. 2021; 34(7): 3213-3225. doi: 10.1093/rfs/hhab038
78. Yu J, Wang J, Moon T. Influence of digital transformation capability on operational performance[J]. Sustainability, 2022, 14(13): 7909.
79. Liu, Ran, et al. "Impacts of the digital transformation on the environment and sustainability." Issue Paper under Task 3; 2019.
80. Hermalin BE, Weisbach MS. Information Disclosure and Corporate Governance. The Journal of Finance. 2012; 67(1): 195-233. doi: 10.1111/j.1540-6261.2011.01710.
81. Liu X, Zhang C. Corporate governance, social responsibility information disclosure, and enterprise value in China. Journal of Cleaner Production. 2017; 142: 1075-1084. doi: 10.1016/j.jclepro.2016.09.102
82. Wang HJ, Feng Q. The Evolution of Internet Finance: Academic Debate and Current Practice. Reform. 2015; (09):142-150.
83. Bloom N, Garicano L, Sadun R, et al. The Distinct Effects of Information Technology and Communication Technology on Firm Organization. Management Science. 2014; 60(12): 2859-2885. doi: 10.1287/mnsc.2014.2013
84. Lateef A, Omotayo FO. Information audit as an important tool in organizational management: A review of literature. Business Information Review. 2019; 36(1): 15-22. doi: 10.1177/0266382119831458
85. Jia W, Redigolo G, Shu S, et al. Can social media distort price discovery? Evidence from merger rumors. Journal of Accounting and Economics. 2020; 70(1): 101334. doi: 10.1016/j.jacceco.2020.101334
86. Zhu C. Big Data as a Governance Mechanism. The Review of Financial Studies. 2019; 32(5): 2021-2061. doi: 10.1093/rfs/hhy081
87. Brooks JM, Carroll JS, Beard JW. Dueling Stakeholders and Dual-Hatted Systems Engineers: Engineering Challenges, Capabilities, and Skills in Government Infrastructure Technology Projects. IEEE Transactions on Engineering Management. 2011; 58(3): 589-601. doi: 10.1109/tem.2010.2058858
88. Bridoux FM, Vishwanathan P. When Do Powerful Stakeholders Give Managers the Latitude to Balance All Stakeholders' Interests? Business & Society. 2018; 59(2): 232-262. doi: 10.1177/0007650318775077
89. Li ZH, Tian WG, Gao P. Stock Market Liberalization, Corporate Governance and Market Fairness. Finance & Economics. 2020; 07: 13-26.
90. Sun K, Wang D, Xiao X. Internet scrutiny and corporate governance effect of social media[J]. J. Manag. World, 2020, 36: 106-132.
91. Cheng Q, Lo K. Insider Trading and Voluntary Disclosures. Journal of Accounting Research. 2006; 44(5): 815-848. doi: 10.1111/j.1475-679x.2006.00222.
92. Wang T, Chen C, Jia X. Can digital transformation improve the quality of corporate information disclosure? SSRN; 2022.
93. Shu W, Cao J,Cao J, Wang FJ. Can enterprise informatization investment restrain surplus management?--empirical evidence from Chinese A-share listed firms. Journal of Soochow University (Philosophy & Social Science Edition). 2021; 42(05): 115-127.
94. Li C, Liu J, Liu Y, et al. Can digitalization empowerment improve the efficiency of corporate capital allocation? —Evidence from China. Economic Analysis and Policy. 2023; 80: 1794-1810. doi: 10.1016/j.eap.2023.11.014
95. Luo LB, Zhang B. Research on Internal Control in the Process of Enterprise Informatization. Accounting Research. 2008;

(05): 69-75.
96. Adner R, Puranam P, Zhu F. What Is Different About Digital Strategy? From Quantitative to Qualitative Change. Strategy Science. 2019; 4(4): 253-261. doi: 10.1287/stsc.2019.0099
97. Chen HW, Wang JM, Yang DG. Audit committee transparency and executive perquisites_a quasi-natural experimental research based on the mandatory disclosure requirements of the Shanghai Stock Exchange. Auditing Research. 2020; (05): 57-66.
98. Peng MZ, Jin QL. Corporate Earnings Pressure and Supply Chain Real Earnings Management. Journal of Finance and Economics. 2021; 47(10): 156-169.
99. Luo JB, Xiong Y. Minority Shareholders "Online Voice" and Accounting Conservatism: Evidence from the Snowball Forum. Journal of Finance and Economics. 2021; 47(12): 150-165.
100. Du Y, Ma WL. Institutional Joint Ownership and Enterprise Total Factor Productivity. Journal of Shanghai University of Finance and Economics. 2021; 23(05): 81-95.
101. Yin HY, Xu XM, Quan XF, Zhang R. Pressure or monitor: institutional investor' cooperate site visits and surplus management. Journal of Soochow University(Philosophy & Social Science Edition). 2021; 42(05): 102-114.
102. Ye KT, Zang WJ. External Supervision and the Manipulation of Enterprise Expense Classification. Journal of Management World. 2016; (01): 121-128,138.
103. Loughran T, Mcdonald B. Measuring Readability in Financial Disclosures. The Journal of Finance. 2014; 69(4): 1643-1671. doi: 10.1111/jofi.12162
104. Li CT, Liu BB, & Zhou P. Short selling and corporate information disclosure: evidence from a quasi-nature experiment on margin trading in China. Financial Research. 2017; 9: 130-145.
105. Yu Jun, Sui Geng. How digital transformation affects sustainable business performance-findings from fsQCA. Soft Science. 2023; 1-17.
106. Guo B, Hu P, Lin J. The effect of digital infrastructure development on enterprise green transformation. International Review of Financial Analysis. 2024; 92: 103085. doi: 10.1016/j.irfa.2024.103085
107. Wang C, Guo J, Xu W, et al. The impact of digital transformation on corporate green governance under carbon peaking and neutrality goals: Evidence from China. PLOS ONE. 2024; 19(6): e0302432. doi: 10.1371/journal.pone.0302432
108. Liu X, Liu F, Ren X. Firms' digitalization in manufacturing and the structure and direction of green innovation. Journal of Environmental Management. 2023; 335: 117525. doi: 10.1016/j.jenvman.2023.117525
109. Yang Y, Han J. Digital transformation, financing constraints, and corporate environmental, social, and governance performance. Corporate Social Responsibility and Environmental Management. 2023; 30(6): 3189-3202. doi: 10.1002/csr.2546
110. May G, Stahl B, Taisch M, et al. Energy management in manufacturing: From literature review to a conceptual framework. Journal of Cleaner Production. 2017; 167: 1464-1489. doi: 10.1016/j.jclepro.2016.10.191

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