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Article

A comparison of Vasicek and Cox-Ingersol-Ross models in determination of reserves for a term assurance policy

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Abstract: Interest rates influence the calculation of premiums, reserves and benefits in the long-term. Theoretically, calculation of such actuarial values is based on the assumption of constant interest rates although interest rates constantly move over time. To obtain a more realistic assessment in valuation, it is beneficial if stochastic interest rates are used. Accurate calculation of reserves ensures that the insurance company can pay claims. A Reserve is a sum of money held by a financial institution such as a life office or a pension fund to cover for the difference between present value of future liabilities including expenses and present value of future premiums. A term insurance contract is an insurance policy that pays the sum assured to the beneficiaries if the policyholder dies within the duration of the policy. The purpose of this study was to compare the reserves that would be needed for a term insurance policy using the Vasicek and the Cox-Ingersol-Ross models. The models were chosen and are widely used because they are tractable and their ease of implementation. The stages of this research activity started by estimating the parameters for the models using Maximum Likelihood Estimation Method. Interest rates were then simulated for the models under study. Next, the reserve value of term insurance policy was determined using the simulated interest rates for the models using the Prospective Method for four randomly generated people of different ages. At the final stage, the results of the reserve values for the models were interpreted and compared. Kenya's Life Table 2001–2003 was used as the reference in determination of mortality assumptions in reserve calculation. The goodness of fit of the models were done using Likelihood Ratio Test and CIR model was a better fit for the data. Interest rates were highly volatile, a feature replicated better by CIR model. Reserve values were also high for CIR model. Reserve values were higher for male insureds due to a higher mortality rate for men than women while the benefit reserves for the younger age were lower as compared to the older ages.

Keywords: reserve; stochastic; term insurance

1. Introduction

The reserve or policy value of a contract is money set aside by the insurer, to pay policyholders' benefits and, where appropriate, future expenses. Reserves are required for several purposes including; paying surrender values or transfer values in a pension fund, working out the revised premium or sum assured in a case where the policy is altered or is converted to another type, for inclusion in the statutory returns by supervisory bodies like Insurance Regulatory of Kenya (IRA) for purposes of demonstrating solvency in companies and for office calculations in deciding the bonus rates of participating (with-profit) contracts [1].

The reserve fund is one of the regulatory requirements for establishing a new insurance company. The regulation is introduced to prevent non-availment of claims due to incorrect technical reserve calculations which could lead to situations where insurance companies finances were inadequate [2]. The reserve is a benefit that you are able to access if any of the unexpected events happen like unexpected claim. Insurers suffer losses in a case where the value of the claim launched by the insured is higher than the anticipated claim [3]. These can be prevented if there are sufficient reserves.

Effectively managing uncertainty and complexity in the setting of reserves poses great challenges for decision-makers and policymakers [4]. Therefore, determining an efficient benefit reserve is a crucial matter in carrying out liabilities for insurance companies. The methods used in determination of benefit reserves are prospective and retrospective methods. The prospective method is based on the reserve value of future cashflows. Retrospective method is based on past premiums. Gross premium reserves have an allowance for expenses. Net premium reserves assumes that premiums and reserve bases agree and does not give an allowance for expenses.

The use of stochastic interest rate models in determining reserves has gained increased significance in the actuarial field. The models help to account for the uncertainities in interest rate movements which is essential in accurate pricing and reserving in pension and insurance companies [5,6].

Muthee carried out earlier studies on insurance reserves [7]. Muthee sought to determine the best estimate of outstanding reserves between deterministic and stochastic models. He demonstrated the advantages of stochastic models over traditional deterministic models. His research showed that models that incorporate stochastic interest rates like the Vasicek and CIR provided more accurate estimates of reserves compared to deterministic models. His conclusion was that the models resulted in better alignment between the actual liabilities of a company and the reserves therefore improving a company's solvency.

Kamila did a study in determining term insurance benefit reserves by comparing benefit reserves using the Vasicek and Cox-Ingersol-Ross models [8]. They used the Ordinary Least Squares method to estimate parameters for the models with data from the Bank of Indonesia from 2009 to 2023 and Indonesia's mortality table as the reference mortality table. They used the premium sufficiency method in determining reserves. The results showed that CIR model provided a better estimate than Vasicek model in predicting the future behavior of interest rates and therefore provided more stable estimates of reserves over longer periods. Their finding was attributed to CIR's ability to capture high volatility interest rates and prevent negative rates. Their research concluded that while both interest rate models were valuable, CIR model provided a more realistic representation of the financial environment especially during economic uncertainities.

Norberg did a study on the use of Vasicek model in pricing and reserving for pension and life insurance plans [9]. They demonstrated that the model provided a good estimate of interest rates in Germany and was useful in the calculation of reserves. The model's mean reversion feature made it appropriate in modelling long duration products like the term assurance.

Martellini et al. [10] did a comparative study on of Vasicek and CIR models in determining reserves for long-term policies of up to 30 years in Chinese life insurance companies. They used data from China's Central bank. Both models gave satisfactory results but CIR model performed better than Vasicek model in accounting for higher volatility.

Dufresne also did a comparative study of Vasicek and CIR models in United States to calculate reserves for life insurance [11]. They analyzed interest rate data from the United States Federal Reserve from the period 1990 to 2010. They concluded that CIR model performed better than Vasicek during periods of financial crisis due to its ability to accommodate high volatility.

Hussain et al did a study on the use of stochastic models in life insurance markets in Pakistani using data from the State Bank of Pakistani [12]. The study examined the use of CIR and Vasicek models in estimating reserves. The study concluded that the stochastic interest rate models provided better accuracy in reserve calculations and better long-term forecasting as compared to deterministic models which are used by insurance companies.

Kamila noted that there is an existing gap regarding the practical applications of stochastic interest rate models, particularly in the emerging markets like Kenya [8]. These models have been widely studied in the context of developed economies, where interest rates tend to follow well-established patterns, and the data is often more stable. While much of the research has focused on developed economies, there is limited research applying these models to African financial markets. This is particularly relevant given that the interest rate environment in developing countries is characterized by high volatility, data availability which differs significantly and frequent changes in monetary policy, which could affect the accuracy and suitability of models for reserve estimation. Ochieng noted that interest rate behavior in African markets differ significantly from developed economies due to factors such as political instability, inflation and changes in bank policies [13]. They also pointed out that while the Central Bank of Kenya (CBK) provides data on treasury bill rates which act as the reference interest rates, the prevailing financial market requires different approaches for accurate reserve estimation.

While Muthee [7] demonstrated the effectiveness of stochastic models in Kenya's insurance market, the use of these models has not been extensively studied with respect to term life insurance policies. These studies suggest that the existing body of knowledge is limited when it comes to understanding how models like Vasicek and Cox-Ingersoll-Ross perform in regions with more unstable financial environments. Consequently, there is a need for more localized studies to understand how the models behave in such contexts and whether they can be used effectively for actuarial purposes.

This paper seeks to fill this gap by applying both the Vasicek and the CIR models to Kenya's data. This study will examine the performance of the models in the context of determining term life insurance reserves and highlight the implications on the use of the models in a developing economy setup. This study will discuss how the choice between the models affects the determination of reserves and policy design considering interest rate fluctuations in Kenya.

The application of stochastic interest rate models to Kenya's data is important in providing a more accurate and sustainable approach to managing reserves, therefore ensuring the long-term solvency of insurance companies. Furthermore, findings from the study will provide insights to actuarial practitioners in African countries where similar economic and financial challenges exist.

2. Modelling interest rates

2.1. Calibration of data

This article exhibits calibrations of both Vasicek and CIR models. Data used is the 91-day treasury bill rate as a proxy for interest rate data from the period 2014 to 2024. The data used is obtained from the Central Bank of Kenya website. The data consists of 522 data sets which are the weekly data for the ten-year period. For real-world calibration, we use the Maximum Likelihood Estimation (MLE) method in this study [14]. In a statistical context, like the likelihood methods, MLE attempts to discover parameters that optimize some likelihood function for producing our observed data from an assumed distribution. MLE was chosen due to computational simplicity. Computation of MLE is straight forward when the likelihood function is well-defined and can be derived explicitly, like in the case of these models under study. The likelihood function can also be maximized efficiently which makes it ideal for large datasets like the one in this study. Also, the estimators of MLE are asymptotically unbiased and efficient. As the size of the sample increases, MLE estimators converge to true parameter values, thereby decreasing estimated standard errors. Given the relatively large sample size of our data, MLE would be an appropriate method to achieve reliable parameter estimates.

Alternative parameter estimation methods could have been used such as the Generalized method of moments (GMM) or Bayesian methods [15]. However, although GMM is useful in certain contexts, it requires specification of moment conditions which might not always be straightforward. Additionally, Bayesian methods often requires extensive computation, especially when incorporating priors and also performing Markov Chain Monte Carlo sampling which can be computationally expensive especially in application to large datasets.

2.2. Parameter estimation

To find the parameters for Vasicek and CIR models using the Maximum Likelihood Estimation (MLE) method:

a) Find the likelihood function for the observed data as:

$$L(\alpha, \mu, \sigma | r_i = \prod_{i=1}^{n} f(r_i | r_{i-1}, \alpha, \mu, \sigma) \tag{1}$$

where $f(r_i | r_{i-1}, \alpha, \mu, \sigma)$ is the transition density function. The transition density for Vasicek model is given by:

$$f(r_i | r_{i-1}, \alpha, \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2(1 - e^{-2\alpha\Delta t})}} e^{-(\frac{(r_i - \bar{r})^2}{2\sigma^2(1 - e^{-2\alpha\Delta t})})} \tag{2}$$

Given that r is the mean of r_i.

The transition density of CIR model is given by:

$$f(r_i|r_{i-1},\alpha,\mu,\sigma)=\frac{2\alpha e^{\mu\alpha}}{\sigma^2(1-e^{-\alpha\Delta t})}(\frac{r_{i-1}e^{-\alpha\Delta t}}{r_i})^{\frac{\mu\alpha}{\sigma^2}-1}(e^{-\left(\frac{2\alpha e^{\mu\alpha}}{\sigma^2(1-e^{-\alpha\Delta t})}\right)r_{i-1}e^{-\alpha\Delta t}-\frac{\mu\alpha(1-e^{-\alpha\Delta t})}{r_i\sigma^2}} \tag{3}$$

b) Find the log-likelihood function for the models

The log likelihood of Vasicek model is given by:

$$l(\alpha,\mu,\sigma|r_i)=-\frac{n}{2}log(2\pi)-\frac{n}{2}log\left(\sigma^2\left(1-e^{-2\alpha\Delta t}\right)\right)-\frac{1}{2\sigma^2(1-e^{-2\alpha\Delta t})}\sum_{i=1}^{n}\left(r_i-r_{i-1}e^{-\alpha\Delta t}+\mu(1-e^{-\alpha\Delta t}\right)^2 \tag{4}$$

The log likelihood of CIR model is given by:

$$l(\alpha,\mu,\sigma|r_i)=-\frac{n}{2}log(2\pi)-\frac{n}{2}log\left(\sigma^2\left(1-e^{-2\alpha\Delta t}\right)\right)-\frac{1}{2\sigma^2(1-e^{-2\alpha\Delta t})}\sum_{i=1}^{n}\left(r_i-r_{i-1}e^{-\alpha\Delta t}+\mu(1-e^{-\alpha\Delta t}\right)^2 \tag{5}$$

where α is the speed of mean reversion, μ refers to the long-term mean value, σ is the volatility interest rate, r_i is the interest rate, *n* is the amount of data used, and Δ is the interval of time.

c) Set the initial guesses for the parameters then find the values of these parameters that maximize the log likelihood function using R statistical package.

2.3. Simulation of future interest rates

a) Calculate future interest rates for Vasicek as:

$$r_{t+1}=r_t+\alpha(\mu-r_t)dt+\sigma\sqrt{\Delta t}\varepsilon_t \tag{6}$$

where $\varepsilon_t \sim N(0,1)$ is the standard normal random variables generated using Monte Carlo simulations [16,17].

b) Calculate future interest rates for CIR as:

$$r_{t+1}=r_t+\alpha(\mu-r_t)\Delta t+\sigma\sqrt{r_t}\sqrt{\Delta t}\varepsilon_t \tag{7}$$

c) Compare and analyze the results of interest rates for Vasicek and the Cox-Ingersoll-Ross interest rate model [18,19].

2.4. Testing the Vasicek and CIR models

To evaluate the accuracy of the models, we first analyze the summary statistics of the resulting rates. Thereafter, the Likelihood ratio test will be used to evaluate the goodness of fit of the models. The likelihood ratio test (LRT) is a statistical method that is used to compare two nested models where one is a null model (restricted) and another alternative model (unrestricted). LRT assesses whether additional parameters in the alternative model significantly improve the fit of the model to the data.

Steps:

1) Write the hypothesis as:

a. Null hypothesis H_0: The simpler model, Vasicek, fits the data well. The Vasicek model assumes constant volatility independent of r_t.

b. Alternative hypothesis H_1: The more complex model, CIR, provides a significantly better fit for the data. For the CIR model, volatility is dependent on $\sqrt{r_t}$.

2) Estimate parameters of the models using MLE to obtain the log-likelihood of the models. The log-likelihood for Vasicek model under the null hypothesis assumes a Gaussian transition density. The log-likelihood for CIR model under the alternative hypothesis assumes a non-central chi-square transition density.

3) Under the null hypothesis, the test statistic follows a chi-squared distribution with degrees of freedom equal to the difference in the number of parameters between null (Vasicek) and alternative (CIR) models. CIR model adds an extra degree of freedom (because of dependence of σ on r_t compared to Vasicek model. Therefore, the degrees of freedom are 1.

4) Compute the likelihood ratio test statistic as:

$$\Lambda = -2(l_0 - l_1) \tag{8}$$

where Λ is the test statistic, l_0 is the log-likelihood of Vasicek and l_1 is the log likelihood of CIR model.

5) Compare the test statistic to the critical value of the chi-squared distribution at a chosen significance level to decide whether to reject the null hypothesis. For this study, the significance level to be used is 5%. If the test statistic is greater than the critical value from the chi-square distribution with 1 degree of freedom, then reject the null hypothesis and conclude that H_1 provides a significantly better fit for the data.

2.5. Determining term life insurance reserves

The mortality, interest and expense assumptions used to evaluate reserves are known as the reserving basis. For this study, Kenya's mortality table 2001–2003 was used with expenses assumed to be 5% of annual premiums from the first premium [20]. The interest rate is assumed to follow Vasicek and CIR models. Weekly data from Central Bank of Kenya [21] was used for interest rate modeling. However, for this study, a yearly average of the data will be used to get the annual interest rates which shall be used in determining annual reserves for the participants. The steps taken are as follows:

1) A table of randomly generated participants will be used. **Table 1** below shows the participant's data.

Table 1. Participants data.

Person	Gender	Age	Registration year
1	Male	26	2015
2	Female	24	2015
3	Female	46	2015
4	Male	46	2015

Where the policy duration is 10 years and the sum assured is Kshs.100,000.

2) Determine the benefit reserves as below:

$$\text{Prospective reserve} = E(L)$$

$${}_tV = A^{1}_{x+t:\overline{n-t}|} - P(A^{1}_{x:\overline{n}|})\ddot{a}_{x+t:\overline{n-t}|} \tag{9}$$

$$A^1_{x:\overline{n}|} = \frac{M_x - M_{x+n}}{D_x} \tag{10}$$

$$\ddot{a}_{x:\overline{n}|} = \frac{N_x - N_{x+n}}{D_x} \tag{11}$$

where;

$D_x = v^x l_x$,

$A^1_{x:\overline{n}|}$ is the present value of term assurance policy benefit payable at death for a life aged x.$\ddot{a}_{x:\overline{n}|}$ is the present value of annuity payable annually in advance.

$PA^1_{x:\overline{n}|}$ is the premium paid for a term assurance policy for a life aged x for n years.

v^x is the discounting factor. Interest rate is from the simulated rates for the models.

3) Compare and analyze benefit reserves for the Vasicek and the Cox-Ingersoll-Ross interest rate models.

3. Results and discussion

3.1. Parameter estimation

Table 2 shows the results of estimating the parameters of the Vasicek and Cox-Ingersoll-Ross (CIR) interest rate models.

Table 2. Parameter estimation values.

Parameters	CIR	Vasicek
Alpha	0.064	0.089
Mu	8.95	8.81
Sigma	2.95	1.16

From **Table 2** above, the results show that both models have almost similar values for the long-term mean and the speed of mean reversion. However, the values of standard deviation vary widely. The value of sigma for CIR model is larger which makes the variance of the rates larger. Also, the value of alpha (mean reversion speed) for Vasicek is larger than that of CIR which means that it takes longer for the rates to go back to the long-term mean for CIR than the Vasicek model. To calculate the time it takes for the rates to revert to the long-term mean, we do a reciprocal of the mean reversion speed α. It takes 15 weeks (the data is weekly) for CIR model rates to move back to their long-term mean. It takes 11 weeks for Vasicek model rates to move back to their long-term mean.

3.2. Analysis of interest rate estimation results

Figure 1 below shows the results of the estimated interest rate using the Vasicek interest rate model and the CIR interest rate model.

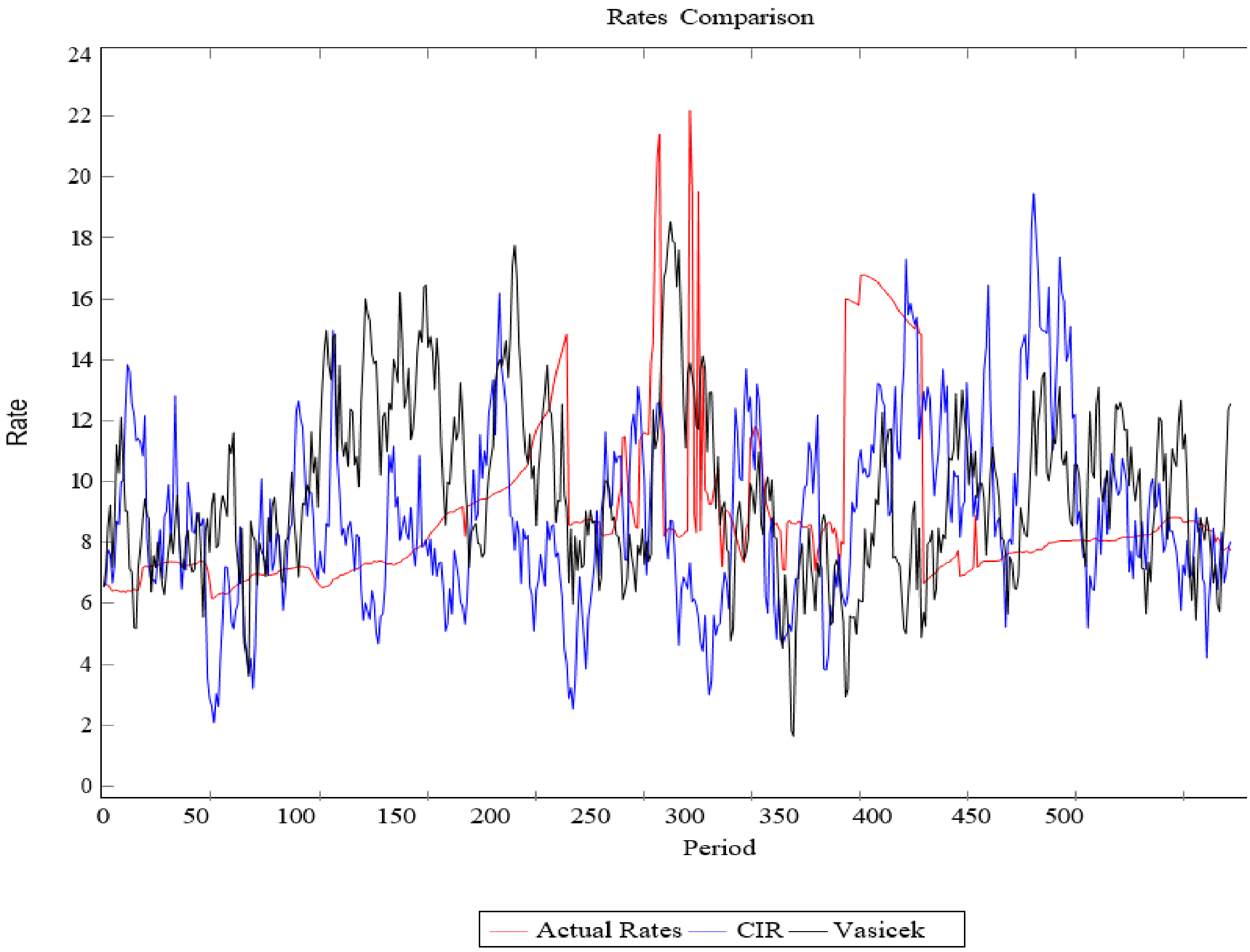

Figure 1. Comparison of rates over time.

Where period is the time from week 1 of the duration of data to the last week at week 522.

Based on **Figure 1** above, the interest rate of the CIR model has almost a similar trend to that of the original data.

While Maximum Likelihood Estimation (MLE) has several advantages, there were challenges encountered during the calibration of the models. The first is that the models were sensitive to the choice of initial parameter values. This is because the models involved multiple parameters that had to be estimated simultaneously. Initial parameter estimates were obtained from exploratory data analysis of historical data.

Another challenge was convergence to a global maximum. The likelihood functions of the models in this study are non-linear and therefore have multiple local maxima. Therefore, the optimization algorithm could converge to suboptimal values if improper initial values are chosen. A range of initial values were tested for optimization, and a comparison of optimization algorithms was done to choose the one that results in a better fit of the models.

Another challenge faced during calibration was model fit and diagnostics. A test of fit was done after calibration using the likelihood ratio test to assess the goodness of fit of the models. Adjustments were made in the cases where the model did not fit the data well, and alternative parameters were tested to ensure a good fit.

3.3. Testing the models

Table 3 shows the results of the summary statistics of the rates obtained using the models.

Table 3. Interest rate summary statistics.

Rates	minimum	1st quarter	median	mean	3rd quarter	Maximum
Actual Rates	6.256	7.284	8.079	8.852	9.185	17.274
CIR Rates	3.436	7.086	8.380	8.778	10.368	17.658
Vasicek Rates	4.082	7.610	9.171	9.652	11.307	17.444

From **Table 3** above, the summary statistics of the CIR model replicate the summary statistics of actual rates better.

The results for likelihood ratio test are as follows: The log-likelihood for CIR model was -318.1772; log-likelihood for Vasicek model was -333.1205. Likelihood ratio test statistic is 29.88653 p-value: 3.841 (critical value at $\alpha = 0.05$).

Since 29.88653 > 3.841, we reject the null hypothesis. The fact that we reject the null hypothesis allows us to state that the CIR model is relatively good at predicting future interest rates. By comparing the values of the summary statistics above and also the results on the likelihood ratio test of Vasicek and CIR, it can be affirmed that the CIR model is more efficient than the Vasicek model in terms of forecasting the future movement of the Kenyan rates.

3.4. Benefit reserves results

The term insurance product used in this study is 10-year term insurance with a premium payment period of 10 years. Four participants were randomly generated for this study. **Table 4** below shows the results for reserve calculations.

Table 4. Reserves data.

Time	CIR_1	Vas_1	CIR_2	Vas_2	CIR_3	Vas_3	CIR_4	Vas_4
1	−138	−958	−1372	−2192	−324	−1603	−124	−143
2	671	1905	563	671	8000	9131	9234	10,591
3	5055	4944	3821	3710	11,821	12,170	13,055	13,630
4	9511	8278	8277	7044	16,277	15,505	17,512	16,964
5	6576	6574	5341	5340	13,341	13,800	14,576	15,260
6	4369	4545	3134	3311	11,134	11,771	12,369	13,231
7	28,232	29,234	26,997	28,000	34,997	36,460	36,232	37,920
8	27,091	26,893	25,857	25,659	33,857	34,120	35,091	35,579
9	31,155	31,921	29,921	30,687	37,921	39,147	39,155	40,607
10	0	0	0	0	0	0	0	0

Where Vas_1 is the value of the reserve for participant 1 where the discount factor follows Vasicek model and CIR_1 is the value of the reserve for participant 1 where the discount factor follows CIR model.

From **Table 4** above, the benefit reserves for all participants are negative at time 1 when the participants enroll for insurance. This is because premiums paid by policyholders for the first year are used to finance company operations such as paying commissions to agents, medical examinations for prospective insureds, and policy administration [22]. The reserve from year 2 to 9 for both models is positive. This is because the probability of death and thereafter paying a claim increases with time. Therefore, the amount required for reserves also increases with time. The benefit reserve for both models in year 10 is 0. This is because for a term insurance policy, if the insurance coverage period ends before the life dies, then no payment will be made to the life assured.

The calculation of the reserves also shows that the values are higher for male insureds due to the higher mortality rate for men than women. Also, the values for benefit reserves for the younger age are lower as compared to the older ages. This is because the probability of death is higher as age increases and therefore the reserve needed is also high. It is also clear that just as the interest rate deviates more for the CIR model than Vasicek model, the same also happens for benefit reserves. The Vasicek model provides more consistent results in a stable economic environment where interest rates are predictable and unlikely to exhibit high volatility. A limitation of Vasicek model emerges during extreme market conditions where interest rate volatility is higher. However, its feature to be able to predict negative rates is advantageous during times of financial distress when interest rates fall below zero.

On the other hand, the CIR model is more responsive to large fluctuations in interest rates. The model is able to capture interest rates with high volatility, making it ideal for insurers operating in emerging markets with unpredictable interest rate behavior or during times of financial turbulence. The CIR model assumes the square root mean reversion behavior, which prevents it from forecasting negative rates, which can discourage investors.

The practical implication of these findings is that actuaries should consider both the features of the interest rate data and the economic environment when selecting an interest rate model. In a stable environment where interest rate changes are consistent and predictable, the Vasicek model would suffice. However, in uncertain or volatile market conditions, the CIR model provides a more realistic projection of interest rates and, consequently, more reliable reserve estimates [23].

4. Discussion and conclusion

Pricing and reserving are at the heart of actuarial decision-making. The choice of an interest rate model significantly influences the present value of future liabilities and, therefore, the reserves to meet policyholders' claims. This has profound implications in risk management and a company's solvency. Therefore, effectively managing uncertainty in setting aside reserves would be a great achievement. Therefore, choosing the right interest rate model is of essence in calculating reserves. The research focused on two commonly used single-factor models, that is, CIR and Vasicek models, to assess their suitability in modeling interest rate dynamics with a focus on how the models influence the financial stability and decision-making process in the insurance industry.

The objectives of this research were to estimate parameters for the specified models, simulate future interest rate values and assess the performance of the two models in predicting interest rate values. These models' parameters were estimated using the Maximum Likelihood Estimation Method. The discrete versions of the models were then used in predicting future interest rates. The likelihood ratio test and the summary statistics were used to assess the suitability of the models. The results shows that the CIR model is more suitable as compared to Vasicek model in predicting interest rates. From the results on the determination of reserves, it can be observed that the values are higher for male insureds due to the higher mortality rate for men than women. Also, the values for benefit reserves for the younger age are lower as compared to the older ages. It is also clear that just as the interest rate deviates more for the CIR model as compared to Vasicek model, the same also happens for benefit reserves. Ultimately, the choice of model should align with the practical needs and characteristics of the interest rates under consideration.

The choice of interest rate model can significantly impact the calculation of reserves and the pricing of insurance products. The CIR model provides a good fit for the data and also offers an advantage by preventing negative interest rates, which may be a more realistic assumption for investors. Insurers should carefully consider the characteristics of the interest rate environment when selecting a model. However, the Vasicek model would be more appropriate in environments where interest rates are consistent and predictable. Vasicek model is also suitable in a market with stable interest rates as it is mean-reverting and tends to smooth out large fluctuations. On the other hand, the CIR model provides better performance in capturing high volatility rates and extreme economic conditions, making it more suitable for emerging markets like Kenya.

This study contributes to the actuarial literature by providing insights into the practical implications of using these models for reserve determination. The selection of a suitable model influences reserve calculations significantly. In a stable interest rate environment, the Vasicek model leads to more predictable and lower reserves which would be easier to manage from both a regulatory and operational standpoint. The CIR model, in contrast, suggests higher reserves in response to sudden interest rate fluctuations, offering a more conservative approach to risk management. By capturing interest rate volatility, it allows insurers to prepare well for adverse market movements, leading to higher reserves in periods of economic stress and thus ensuring greater solvency and risk management.

The study demonstrates that the choice of model is not merely an academic exercise but has practical, real-world consequences. By selecting an appropriate model, actuaries in the insurance companies would better align their reserve strategies with economic conditions and regulatory requirements.

The study also contributes to the actuarial community by shedding light on the strengths and limitations of two commonly used interest rate models and offering guidance on the selection of an appropriate model for calculation of life insurance reserves based on market conditions. The results are not only relevant to actuaries in Kenya but also for actuaries in other emerging markets where interest rates are highly unpredictable.

Through providing a transparent evaluation on calibration of the models and highlighting the challenges faced during calibration, this research advances existing knowledge on interest rates. This paper discusses the effect of initial values on model convergence and also provides insights into how optimization methods can affect the reliability of model outputs. The transparency helps researchers and actuaries to better understand real-world challenges in the application of the models in practice, therefore ensuring accurate calibration of real-world data.

While both the Vasicek and Cox-Ingersoll-Ross models provide strong statistical fits for interest rate data, their impact on actuarial decision-making extends far beyond just fitting the data. The choice of model influences crucial areas such as pricing, reserving, capital management, and risk management. The Cox-Ingersoll-Ross model may offer greater robustness in volatile and uncertain environments, making it suitable for markets with high interest rate risk, while the Vasicek model provides a simpler, more stable framework suited for stable economic conditions. Actuaries must carefully consider the economic environment, regulatory requirements, and the insurer's risk profile when selecting the appropriate model, ensuring that the chosen model supports long-term financial stability and solvency. The statistical fit of both models is crucial, but it alone cannot guide an actuary in selecting a model. It is essential to assess how the model's assumptions align with the real-world conditions of the insurance environment, and how each model influences decisions around pricing, reserving, and risk management.

The data used in calibration was limited to the 91-day Treasury bill rates from the Central Bank of Kenya website between 2014 and 2024. While the period would be a representative sample for the Kenyan market, it may not fully capture extreme interest rate conditions or other regional factors that might affect interest rates over a long time period. Future studies would benefit from incorporating a broader set of data, such as long-term bond yields, to improve robustness in findings. Both the Vasicek and CIR models are based on assumptions that might not fully align with real-world data such as constant volatility for the case of Vasicek model. Future work should explore more complex models that would allow for time-varying volatility or other dynamics that would better fit the data well. Calibration of the models using Maximum Likelihood Estimation presented some challenges like sensitivity of the models to initial values and convergence in certain data sets. This underscores accurate calibration and testing to ensure stability of model outputs. Future work should explore other parameter estimation methods. Future research should also focus on improving stress-testing methodologies using stochastic models. By simulating extreme interest rate behavior such as a sharp increase in interest rates, future research should examine the adequacy of the models in predicting capital shortfalls and helping insurers prepare for worst-case scenarios therefore improving solvency.

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