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Article

Autoregressive moving average approaches for estimating continuous non-negative time series

Gisele de Oliveira Maia[1,*], Ana Julia Alves Camara[2]

[1] Institute of Pure and Applied Sciences, Federal University of Itajubá, Itajubá 35903-087, Brazil
[2] Department of Statistics, Federal University of Espírito Santo, Vitória 29075-910, Brazil
* **Corresponding author:** Gisele de Oliveira Maia, giseleemaia07@gmail.com

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Abstract: This study performs a comparative analysis of autoregressive moving average models for non-negative time series within a financial context, aiming to identify the model that offers the best fit and forecasting accuracy. The analysis is applied to two financial datasets: the stock trading volume of Banco Bradesco and the insurance volume of Porto Seguro. Four models are fitted in the process: Autoregressive Moving Average (ARMA), Rayleigh Autoregressive Moving Average (RARMA), Generalized Autoregressive Moving Average (GARMA), and Generalized Linear Autoregressive Moving Average (GLARMA). Model performance is evaluated through fit comparison metrics and forecasting accuracy measures to determine the most effective model.

Keywords: non-negative time series; stock volume; GARMA; GLARMA

1. Introduction

Statistical methods have assumed that data follow a normal distribution for many years. However, data often deviate from this pattern, showing asymmetry, kurtosis, heteroscedasticity, and non-linear trends. In finance, variables such as salaries, financial returns, and stock trade volumes often display these asymmetric behaviors, which can challenge traditional analysis methods. Forcing data into a normal distribution can distort its true structure, leading to biased estimates, incorrect conclusions, and unreliable predictions. Most financial and macroeconomic data are time series, where observations are collected over time and are not independent of each other. This lack of independence can further complicate the analysis. Therefore, it is important to evaluate the data's distribution carefully and before applying statistical methods, ensuring that the analyses are accurate and appropriately represent the nature of the data.

Linear models were frequently employed to describe random phenomena, even when the data exhibited autocorrelated observations. However, alternative methods designed for data with time-dependent structures began to emerge over time. Among these, the autoregressive moving average (ARMA) model, introduced by Box and Jenkins [1], is particularly significant. This method assumes that the data follows a normal distribution. This model has been widely used to analyze linear characteristics, such as autocorrelation, in financial time series and is considered a benchmark model. Choudhury and Jones [2] used the ARMA model to assess crop yield estimates for insurance purposes in Ghana, Tang [3] adjusted the model to predict prices of Apple Inc from 2018 to the end of 2019, Ibrahim et al. [4] used the methodology to predict price movement's direction of Bitcoin for the next 5-minute time frame, among others. Despite the extensive use of the ARMA methodology, advances in computational methods have enabled the development of new approaches to explaining various phenomena in practical situations.

However, it is crucial to use models that capture the specific characteristics of the variable of interest, such as asymmetry. New models have been developed to address this need that extend autoregressive moving average (ARMA) models to non-Gaussian time series. In this context, we can mention the Rayleigh autoregressive moving average (RARMA) model, originally proposed by Bayer et al. [5] for signal and image processing, offers an alternative for modeling continuous, asymmetric, and non-negative processes. It models the mean of Rayleigh-distributed discrete-time signals using a dynamic structure incorporating autoregressive (AR) and moving average (MA) terms, a set of regressors, and a link function. De Araújo et al. [6] considered this methodology to model stock trading volumes.

Zeger and Qaqish [7] introduced a quasi-likelihood approach to time series regression, which Benjamin et al. [8] later generalized into the generalized autoregressive moving average (GARMA) models. GARMA extends the ARMA model to non-Gaussian processes where the conditional mean (given past information in a time series setting) belongs to the exponential family. Specifically, GARMA-GAMMA is used for non-negative time series. Recently, Alcoforado et al. [9] applied the GARMA model to predict an index related to cattle spot and future prices, while de Araújo et al. [6] used the GARMA model to analyze and forecast the trading volume of Banco Bradesco S.A. (BBD) stocks. Although is an interesting alternative Zheng et al. [10] argues that unless an identity link function is used, the model's error sequence does not form a martingale difference sequence, complicating the study of the series' probabilistic properties and the asymptotic behavior of its estimators. On the other hand, for non-negative time series, GARMA models with an identity link function do not accommodate negative autocorrelation, which is quite common in real-world applications. Albarracin et al. [11] states that the structure of the GARMA model can lead to multicollinearity issues.

In parallel, Davis et al. [12] proposed the generalized linear autoregressive moving average (GLARMA) model as an extension of the generalized linear model (GLM) to handle time-dependent data, originally designed for count and discrete time series. Maia [13] later introduced the GLARMA for positive continuous processes, named GLARMA-GAMMA and GLARMA-IG, and derived some properties. In this work, the GLARMA-GAMMA and GLARMA-IG models are applied to estimate the trading volume of a Brazilian insurance company and Banco Bradesco. Although theoretical properties of the class of GLARMA models have only been rigorously established for a very limited case, Davis et al. [14] in a recent literature review highlighted that, despite this complexity, this family remains one of the most flexible and easily applicable methods.

This work aims to compare the prediction capability of the ARMA, RARMA, GARMA, and GLARMA models. To the best of our knowledge, comparisons involving the cited methods in the financial context have not yet been explored, and this work aims to fill that gap. The comparison is based on real-world data from two applications. The first focuses on forecasting the trading volume of Banco Bradesco for the period from 14 February 2022 to 10 February 2023. The second application involves forecasting the trading volume of the insurance company Porto Seguro from January 2005 to April 2023, incorporating an explanatory variable related to stock volume.

Section 2 outlines the methodologies under comparison. Section 3 presents and discusses the results of the real data applications for stock volume. Finally, Section 4 concludes the study, summarizing the main findings.

2. Materials and methods

In this section, we detail the procedures adopted to conduct this study, including the tools used and the methodology applied. Initially, we present the models employed. Next, we describe the data processing and analysis techniques, highlighting the parameters used and the criteria for model selection. Finally, we address the tools and software used to implement the analyses and validate the results.

2.1. ARMA models

The autoregressive moving average (ARMA(p, q)) model is a methodology for analyzing stationary time series based on four steps: identification, estimation, validation, and forecasting. The order p represents the autoregressive component, which captures the relationships between a value and its previous values. The order q corresponds to the moving average component, which reflects how forecast errors are incorporated into future predictions. For non-stationary time series, where the mean and variance change over time, the ARIMA(p, d, q) model is used. Here, the "I" denotes the integrated component, accounting for trends or other systematic changes in the data.

Let $\{Y_t\}_{t\in\mathbb{N}}$ be a stationary process, the ARMA model is defined as

$$\tilde{Y}_t = \phi_1\tilde{Y}_{t-1} + ... + \phi_p\tilde{Y}_{t-p} + a_t - \theta_1 a_{t-1} - ... - \theta_q a_{t-q} \quad (1)$$

where $\tilde{Y}_t = Y_t - \mu$ and a_t is an white noise. The orders p and q are identified using the autocorrelation function (ACF) and the partial autocorrelation function (PACF). The PACF measures the correlation between two observations, excluding the influence of any intermediate observations. Once the orders p and q are determined, the parameters $\boldsymbol{\phi} = (\phi_1, \phi_2, ..., \phi_p)^\top$ and $\boldsymbol{\theta} = (\theta_1, \theta_2, ..., \theta_q)^\top$ are estimated. The maximum likelihood method is widely used for parameter estimation in the model. The Akaike Information Criterion (AIC) proposed by Akaike [15]), the Bayesian Information Criterion (BIC), and the Hannan-Quinn information criteria (HQ) are used to determine the appropriate number of parameters to include in the model.

For validation, the adequacy of the estimated models can be assessed through residual analysis. According to Box et al. [16] the residuals in ARMA(p,q) are defined as

$$\hat{a}_t = \tilde{Y}_t - \hat{\phi}_1\tilde{Y}_{t-1} - ... - \hat{\phi}_p\tilde{Y}_{t-p} + \hat{\theta}_1 a_{t-1} + ... + \hat{\theta}_q a_{t-q}$$

The assumption that a_t, $t = 1, 2, .., n$ are independent must be satisfied.

2.2. RARMA models

Let $\{Y_t\}_{t\in\mathbb{N}}$ be a discrete-time signal and $\mathcal{F}_{t-1} = \sigma\{Y_s, s \leq t-1\}$ is the past of the observations up to time t. It is assumed that each Y_t conditioned to $\mathcal{F}_{t-1}$ is a Rayleigh with conditional mean μ_t, and the conditional distribution density of Y_t is given by

$$f(y_t|\mathcal{F}_{t-1}) = \frac{\pi y_t}{2{\mu_t}^2}\exp\left(-\frac{\pi {y_t}^2}{4{\mu_t}^2}\right) \quad (2)$$

where $y_t > 0$ and $\mu_t > 0$

A strictly monotonic and twice differentiable link function $g(\cdot)$ maps μ_t into a linear predictor (η_t). Thus, the structure of the RARMA model is given by

$$g(\mu_t) = \eta_t = \boldsymbol{X}_t^\top\boldsymbol{\beta} + Z_t \quad (3)$$

where $\boldsymbol{X}_t = (1, X_{1,t}, \ldots, X_{k,t})^\top$ is a k-dimensional vector of regressors observed for $t = 1, ..., n$, and $\boldsymbol{\beta} = (\beta_0, \beta_1, \ldots, \beta_k)^\top$ are the regression coefficients. The component $Z_t = \sum_{i=1}^{p}\phi_i g(y_{t-i}) + \sum_{j=1}^{q}\theta_j r_{t-j}$, where $r_{t-j} = g(y_t) - g(\mu_t)$, adds the autoregressive and moving average terms to the linear predictor, where $\boldsymbol{\phi} = (\phi_1, \phi_2, ..., \phi_p)^\top$ and $\boldsymbol{\theta} = (\theta_1, \theta_2, ..., \theta_q)^\top$ are respective parameters to be estimated. The estimators are obtained upon maximizing the conditional log-likelihood function. For more details, see Bayer et al. [5].

2.3. Gamma-GARMA models

Let $\{Y_t\}_{t\in\mathbb{N}}$ be the observations and $\mathcal{F}_{t-1} = \sigma\{Y_s, s \leq t-1; X_{i,s}, 1 \leq i \leq k, s \leq t\}$, where Y_s is the past of the observed positive process and $X_{i,s}$ is the past and present of the regressor variables. The density function of $Y_t|\mathcal{F}_{t-1}$ follows a Gamma(μ_t, ν) distribution, which can be written in exponential family as

$$f(y_t|\mathcal{F}_{t-1}) = exp[\nu(-y_t\mu_t) - log\mu_t - log\Gamma(\nu) + \nu log(\nu y_t) - logy_t] \tag{4}$$

where $y_t > 0, \nu > 0, \mu_t = E(Y_t|\mathcal{F}_{t-1})$ is positive and $\Gamma(\nu) = \int_0^\infty t^{\nu-1}e^{-t}dt$ is the gamma function.

The linear prediction expression is given by

$$g(\mu_t) = \eta_t = \boldsymbol{X}_t^\top \boldsymbol{\beta} + Z_t \tag{5}$$

where $\boldsymbol{X}_t = (1, X_{1,t}, \ldots, X_{k,t})^\top$ is a k-dimensional vector of regressors observed for $t = 1, ..., n$, and $\boldsymbol{\beta} = (\beta_0, \beta_1, \ldots, \beta_k)^\top$ are the regression coefficients. The link function used is the logarithm. The term $Z_t = \sum_{i=1}^p \phi_j \left\{g(y_{t-j}) - \boldsymbol{X}_t^T\boldsymbol{\beta}\right\} + \sum_{j=1}^q \theta_j \left\{g(y_{t-j}) - \eta_{t-j}\right\}$ captures the autoregressive and moving average components, where $\boldsymbol{\phi} = (\phi_1, \phi_2, ..., \phi_p)^\top$ and $\boldsymbol{\theta} = (\theta_1, \theta_2, ..., \theta_q)^\top$ are the corresponding parameters. Equations (4) and (5) define the GARMA-GAMMA(p, q) model. The parameters $\boldsymbol{\beta}, \boldsymbol{\phi}, \boldsymbol{\theta}$ and ν are estimated via maximum likelihood. For further details, refer to Benjamin et al. [8].

2.4. GLARMA-GAMMA and GLARMA-IG models

Let $\{Y_t\}_{t\in\mathbb{N}}$ be the positive continuous time series and $\mathcal{F}_{t-1} = \sigma\{Y_s, s \leq t-1; X_{i,s}, 1 \leq i \leq k, s \leq t\}$, denote the past information available on the response series and the past and present information on the regressors. The distribution of Y_t conditioned on $\mathcal{F}_{t-1}$ is assumed to be a Gamma(μ_t, ν) or Inverse Gaussian(μ_t, ν), where $\mu_t = E(Y_t|\mathcal{F}_{t-1})$ is positive and $\nu > 0$ is the shape parameter of the conditional distribution. The conditional density of the GLARMA-GAMMA model is defined in Equation (4), while the conditional density of the GLARMA-IG model is given by

$$f(y_t|\mathcal{F}_{t-1}) = \exp\left\{\nu\left[\frac{y_t}{2\mu_t^2} + \frac{1}{\mu_t}\right] + \frac{1}{2}\log\nu - \frac{1}{2}\log(2\pi y_t^3) + \frac{\nu}{2y_t}\right\}$$

The linear prediction expression is given by

$$g(\mu_t) = \eta_t = \boldsymbol{X}_t^\top \boldsymbol{\beta} + Z_t$$

where the logarithmic function is used as the link function, $\boldsymbol{X}_t = (1, X_{1,t}, \ldots, X_{k,t})^\top$ is a k-dimensional vector of regressors observed for $t = 1, ..., n$, and $\boldsymbol{\beta} = (\beta_0, \beta_1, \ldots, \beta_k)^\top$ denotes the regression coefficients. The term $Z_t = \sum_{i=1}^\infty \zeta_i e_{t-i}$ introduces the time-dependent structure into the model, with ζ_i as the parameters and e_t as the error term. After some mathematical manipulations, the term Z_t can be rewritten as

$$Z_t = \phi_1(Z_{t-1} + e_{t-1}) + \cdots + \phi_p(Z_{t-p} + e_{t-p}) + \theta_1 e_{t-1} + \cdots + \theta_q e_{t-q}$$

where $\boldsymbol{\phi} = (\phi_1, \phi_2, ..., \phi_p)^\top$ and $\boldsymbol{\theta} = (\theta_1, \theta_2, ..., \theta_q)^\top$ are the autoregressive and moving average parameters, and the error terms are

$$e_t = \frac{Y_t - \mu_t}{\mu_t} \quad \text{and} \quad e_t = \frac{Y_t - \mu_t}{\sqrt{\mu_t^3}}$$

for the Gamma and Inverse Gaussian distribution, respectively. The parameters $\boldsymbol{\beta}$, $\boldsymbol{\phi}$, $\boldsymbol{\theta}$ and ν are estimated using maximum likelihood. Refer to Davis et al. [12] and Maia [13] for more information.

3. Results and discussion

All analyses were conducted using the R software ([17]), a platform widely used for statistical analysis and time series modeling. R provides a wide range of specialized packages that were employed to fit the models. The adjustments of the RARMA and GARMA models were carried out with the help of the R-package *PTSR* (for more details, see Prass et al. [18]), while for the ARMA model, we used the R-package *stats*. In the case of the GLARMA model, we utilized custom codes developed by the authors, which are available upon request.

The selection and comparison of the models were based on the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Hannan-Quinn Information Criterion (HQ). These three measures are defined as follows respectively,

$$\begin{aligned} AIC &= -2l(\hat{\delta}) + 2r, \\ BIC &= -2l(\hat{\delta}) + r\log(n), \\ HQ &= 2l(\hat{\delta})\left(\frac{n}{n-m}\right) + r\log[\log(n)] \end{aligned} \tag{6}$$

where $l(\cdot)$ is the log-likelihood function, δ is the parameter vector estimate, n is the sample size of the time series and r is the number of parameters in the model. The models were fitted with different combinations of p and q ($\max(p,q) = 4$), with selection based on the AIC, BIC, HQ criteria, combined with the residual analysis. That is, the final selection of p and q values involves choosing the smallest values according to the criteria mentioned, ensuring that the model can capture the characteristics of the series and results in residuals that behave like white noise. Additionally, in the comparison of forecast values between the models, we used the following measures: Mean Square Error (MSE), Mean Absolute Percentage Error (MAPE), and Mean Absolute Scaled Error (MASE). The MSE, MAPE, and MASE measures can be expressed as

$$\begin{aligned} MSE &= \frac{1}{h_0}\sum_{h=1}^{h_0}(y_h - \hat{y}_h)^2, \\ MAPE &= \frac{1}{h_0}\sum_{h=1}^{h_0}\frac{|y_h - \hat{y}_h|}{|y_h|}, \\ MASE &= \frac{1}{h_0}\sum_{h=1}^{h_0}\frac{|y_h - \hat{y}_h|}{\frac{1}{h-1}\sum_{h=2}^{h_0}|y_h - y_{h-1}|} \end{aligned} \tag{7}$$

respectively, where y_h are the observed values, and $\hat{y}_h$ are the predicted values for the forecast horizon ($h = 1, \ldots, h_0$). The forecast uses a one-step approach, where the parameters are re-estimated in each step. This methodology was also adopted in the works of Agosto et al. [19], Maia et al. [20] and Mendes et al. [21].

3.1. Trading volume of Banco Bradesco

The first data set refers to the trading volume of Banco Bradesco (U.S. Dollars (US$)). The time series is available at *Yahoo Finance* website [22]. This positive continuous time series was observed from 14 February 2022 to 10 February 2023, totaling 250 observations. Since trading volumes are usually large-scale, we divided the series by one hundred million. The last thirty observations of the series were removed to obtain accuracy measures for choosing the best model. This dataset has been analyzed by de Araújo et al. [6] under the BXII autoregressive moving average (BXII-ARMA) time series model.

Table 1 presents descriptive statistics of the dataset, while **Figure 1** displays the time series of Banco Bradesco's trading volume, along with its corresponding Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF). The ACF plot indicates a positive correlation, with most observations falling outside the confidence interval (CI). In contrast, the PACF plot shows that nearly all observations remain within the CI.

Table 1. Descriptive statistics for the trading volume of Banco Bradesco.

Minimum	Median	Mean	Maximum	Variance
US$ 0.126	US$ 0.332	US$ 0.356	US$ 1.485	US$ 0.025

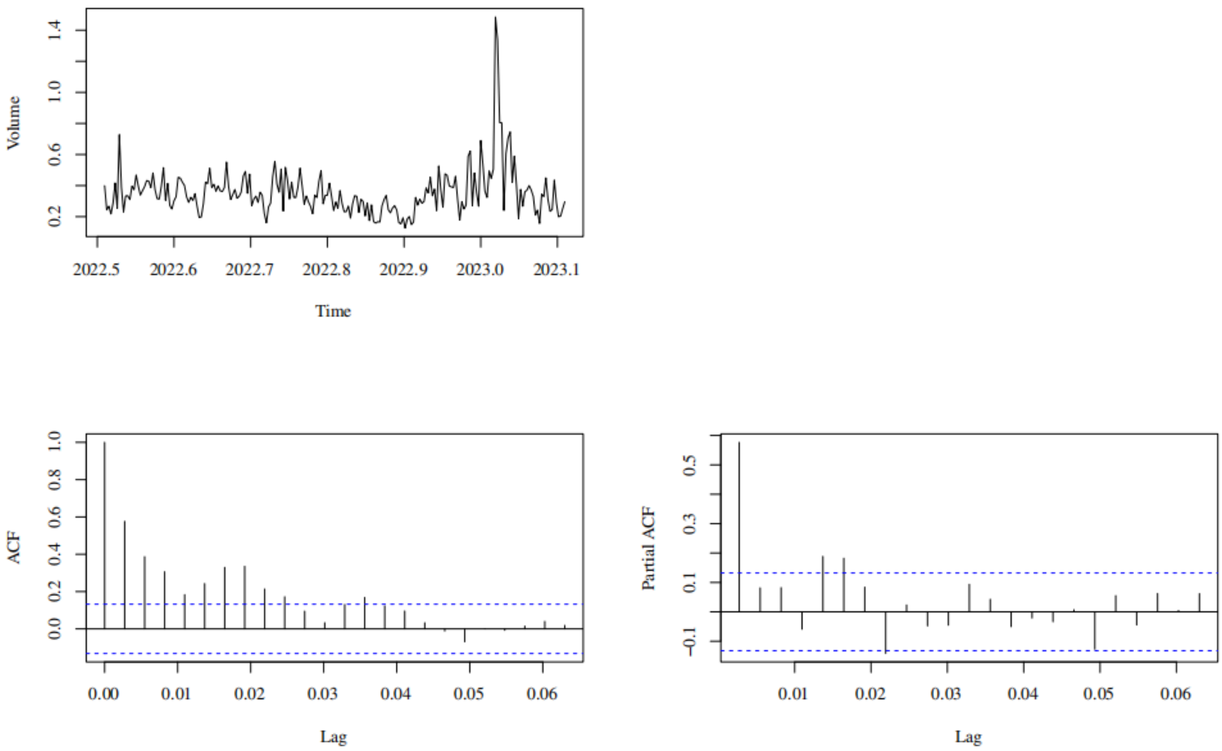

Figure 1. Plots of the trading volume of Banco Bradesco from 14 February 2022 to 10 February 2023 (top) and its associated ACF (bottom to the left) and PACF (bottom to the right).

We performed the Phillips-Perron (PP) test to verify the trading volume series's stationarity. The results confirmed that the series is stationary, with a p-value of 0.01. Subsequently, the ARMA(3,1), RARMA(0,3), Gamma-GARMA(4,4), and GLARMA-GAMMA models, with autoregressive components at lags 1, 5, and 6, were fitted to the time series. In **Table 2**, we present the parameter estimates, the standard errors (SE), and the p-value for each estimate. We observed that only the parameter ϕ_2 is not significant at the 5% level. For the Gamma-GARMA(4,4) model, only the parameters ϕ_4, θ_1, and θ_4 were significant at the 5% level. Mean-while, for the RARMA and GLARMA models, all parameters were significant at the 5% level. **Table 3** presents the AIC, BIC, and HQ measures used to evaluate the fitted models, highlighting the GLARMA model, which exhibited the lowest values. However, it is observed that the measures for the GARMA model are quite close to those of the GLARMA model.

Table 2. Estimates, standard errors, and p values of the parameters for the ARMA, RARMA, Gamma-GARMA and GLARMA-GAMMA adjustments in the Banco Bradesco's trading volume time series.

Coef.	**Estimate**	**SE**	***p* value**	**Coef.**	**Estimate**	**SE**	***p*-value**
ARMA(3,1)				RARMA(0,3)			
Int.	0.3548	0.0225	0.0000	α	-1.2183	0.0744	0.0000
ϕ_1	-0.4062	0.0868	0.0000	θ_1	1.4985	0.1649	0.0000
ϕ_2	0.5398	0.0777	0.0589	θ_2	1.1429	0.3237	0.0004
ϕ_3	0.1370	0.0725	0.0000	θ_3	1.1465	0.2201	0.0000
θ_1	0.9574	0.0623	0.0000	-	-	-	-
Gamma-GARMA(4,4)				GLARMA-GAMMA(1,5,6)			
α	-0.3834	0.2041	0.0603	Int.	-1.0613	0.0735	0.0000
ϕ_1	0.0103	0.1038	0.9210	ϕ_1	0.4481	0.0567	0.0000
ϕ_2	0.1596	0.1195	0.1816	ϕ_5	0.1297	0.0550	0.0184
ϕ_3	0.1625	0.0874	0.0630	ϕ_6	0.1740	0.0557	0.0018
ϕ_4	0.2823	0.0813	0.0005	ν	11.7766	1.1074	0.0000
θ_1	1.1676	0.2224	0.0000	-	-	-	-
θ_2	0.5008	0.4396	0.2546	-	-	-	-
θ_3	0.4505	0.3387	0.1834	-	-	-	-
θ_4	-0.8959	0.2311	0.0001	-	-	-	-
φ	12.2942	1.1567	0.0000	-	-	-	-

Table 3. Information criteria for the best fits in each Banco Bradesco time series model class.

Model	**AIC**	**BIC**	**HQ**
ARMA(3,1)	-277.1876	-256.8258	-280.7615
RARMA(0,3)	-263.387	-249.8125	-264.6461
Gamma-GARMA(4,4)	-388.0148	-354.0785	-391.1626
GLARMA-GAMMMA(1,5,6)	-388.2893	-371.3211	-389.8632

The last 30 observations of Banco Bradesco's trading volume series were used for forecasting. **Table 4** presents the comparison metrics between the forecast values and the real observations. The results indicate that the RARMA model showed the lowest forecast accuracy measures. However, since the differences in measures between the models are small, a more in-depth analysis is needed to determine whether these differences are statistically significant. For this, first, the nonparametric Friedman test was conducted to assess whether there were significant differences between the predictions of each model, which was confirmed with a p-value of 0.022. Next, we conducted more detailed analyses, still using the Friedman test, all considering a significance level of 5%, obtaining the results: the ARMA model has forecast similar to the RARMA, GARMA, and GLARMA models, as there was no significant difference between them. The RARMA model differs from GARMA and GLARMA, indicating that the predictions of these two models are significantly different from RARMA. The GARMA model is similar to GLARMA, as there was no significant difference between them. Therefore, the main conclusion is that, although ARMA is similar to the other three models in terms of forecast, RARMA stands out with predictions different from those of GARMA and GLARMA, while GARMA and GLARMA are similar to each other. Finally, **Figure 2** compares the actual and fitted values of Banco Bradesco's trading volume, showing the close fit of the models.

Table 4. Forecasting performance comparison among different fitted models in each Banco Bradesco time series class.

Model	MSE	MAPE	MASE
ARMA(3,1)	0.159	1.138	1.882
RARMA(0,3)	0.159	1.101	1.807
Gamma-GARMA(4,4)	0.167	1.153	1.897
GLARMA-GAMMMA(1,5,6)	0.167	1.178	1.957

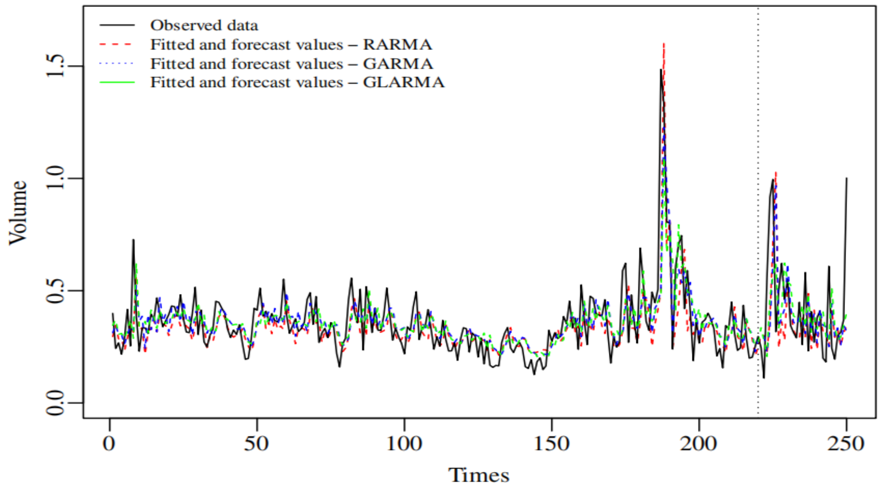

Figure 2. Observed and adjusted values of the RARMA, GLARMA and GARMA models for the Banco Bradesco time series.

3.2. Trading volume of porto seguro

We now analyze the monthly trading volume of Porto Seguro, a Brazilian insurance company, for the period from January 2005 to April 2023, comprising a total of 220 observations. The last 10 observations are reserved for forecasting, leaving $n = 210$ observations for model fitting. To manage the large scale of the data, the series was divided by one hundred million. The time series can be obtained in *Yahoo Finance* website [22]. Furthermore, in this application we use the Dollar Price covariate, which was sourced from *Investing.com* website [23].

Table 5 provides the descriptive statistics of the series, while **Figure 3** presents the time series of monthly trading volume for the analyzed period, along with the corresponding autocorrelation and partial autocorrelation functions. The ACF plot shows a positive correlation with most points outside the CI, while the PACF plot has nearly all points within the CI.

Table 5. Descriptive statistics for the trading volume of Porto Seguro.

Minimum	**Median**	**Mean**	**Maximum**	**Variance**
US$ 0.826	US$ 2.518	US$ 2.864	US$ 9.537	US$ 2.097

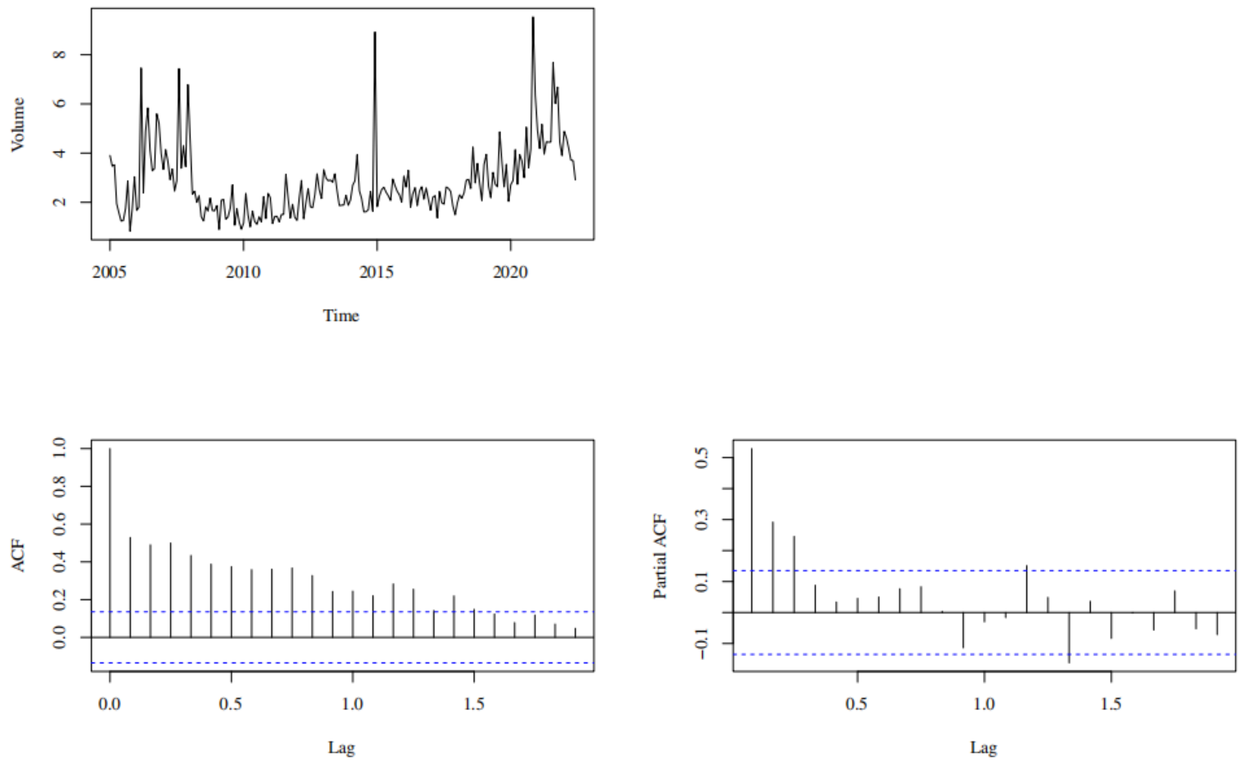

Figure 3. Plots of the monthly trading volume of Porto Alegre from January 2005 to June 2022 (top to the left) and its associated ACF (bottom to the left) and PACF (bottom to the right).

First, the PP test was conducted to check the stationarity of the series, resulting in a p-value of 0.01, indicating that the insurer's trading volume series is stationary. The following covariates were considered: the dollar opening price and a dummy variable representing the peak around August 2014 to February 2015. This peak can be explained by a 32.9% increase in net income reported by the insurer compared to the same period in the previous year. After a preliminary analysis, the following models were fitted to the data: ARMA(1,1,2), RARMA(3,0), Gamma-GARMA(3,0), and GLARMA-IG, with autoregressive components at lags 1 and 3.

Table 6 presents the parameter estimates, with respective standard errors and p-values for each model. Considering a 5% significance level, the parameter β_2 is not significant in the ARMA and Gamma-GARMA model fits, while the parameter β_1 is not significant in the RARMA model fit. In **Table 7**, we compare the fit quality of the ARMA, RARMA, GARMA and GLARMA models, noting that both, GARMA and GLARMA models, show superior performance based on lower AIC, BIC, and HQ criteria.

Finally, forecasts were made for the last 10 observations removed from the series. To evaluate the accuracy of these predictions for each model, we calculated the MSE, MAPE, and MASE measures, with the results presented in **Table 8**. **Figure 4** compares the actual trading volume observations of Porto Seguro with the predicted values. Again, the Friedman test was performed to verify the presence of significant differences between the forecast of all models, which was confirmed with a p-value of 5.283×10^{-5}. We then compared the predictions of each model using the Friedman test, with a significance level of 5%. We obtained: the ARMA model has predictions significantly different from the RARMA and GARMA models, but is similar to GLARMA. The RARMA model differs from both GARMA and GLARMA. The GARMA model is different from GLARMA. Therefore, we conclude that the predictions of the ARMA and GLARMA models show greater accuracy. The performance metrics for the RARMA model indicate the best results; however, when analyzing **Figure 4**, we observe the opposite.

Table 6. Estimates, standard errors, and p values of the parameters for the ARMA, RARMA, Gamma-GARMA and GLARMA-IG adjustments in the Porto Seguro's trading volume time series.

Coef.	**Estimate**	**SE**	***p* value**	**Coef.**	**Estimate**	**SE**	***p*-value**
ARMA(1,1,2)				RARMA(3,0)			
-	-	-	-	α	0.606	0.199	0.002
β_1	0.517	0.202	0.011	β_1	0.062	0.041	0.136
β_2	-0.386	0.596	0.517	β_2	-0.628	0.247	0.011
ϕ_1	0.919	0.047	0.0000	ϕ_1	0.075	0.035	0.031
θ_1	-1.684	0.087	0.000	ϕ_2	0.056	0.031	0.071
θ_2	0.684	0.086	0.000	ϕ_3	0.102	0.035	0.004
Gamma-GARMA(3,0)				GLARMA-IG(1,3)			
α	0.242	0.096	0.012	Int.	1.017	0.252	0.000
β_1	0.148	0.058	0.012	β_1	0.167	0.055	0.002
β_2	-0.131	0.225	0.560	β_2	-0.515	0.186	0.006
ϕ_1	0.261	0.072	0.000	ϕ_1	0.409	0.090	0.000
ϕ_2	0.158	0.070	0.024	-	-	-	-
ϕ_3	0.279	0.072	0.000	ϕ_3	0.331	0.095	0.000
ν	8.726	0.836	0.000	ν	20.595	2.010	0.000

Table 7. Information criteria for the best fits in each model class for the Porto Seguro time series.

Model	**AIC**	**BIC**	**HQ**
ARMA(1,1,2)	651.092	671.146	647.474
RARMA(3,0)	637.910	657.993	635.970
Gamma-GARMA(3,0)	549.150	572.580	546.886
GLARMA-IG(1,3)	551.822	571.905	549.882

Table 8. Forecasting performance comparison among different fitted models in each class for the Porto Seguro time series.

Model	**MSE**	**MAPE**	**MASE**
ARMA(1,1,2)	17.112	1.016	3.937
RARMA(3,0)	8.055	0.696	2.706
Gamma-GARMA(3,0)	18.336	1.052	4.082
GLARMA-IG(1,3)	16.840	1.005	3.907

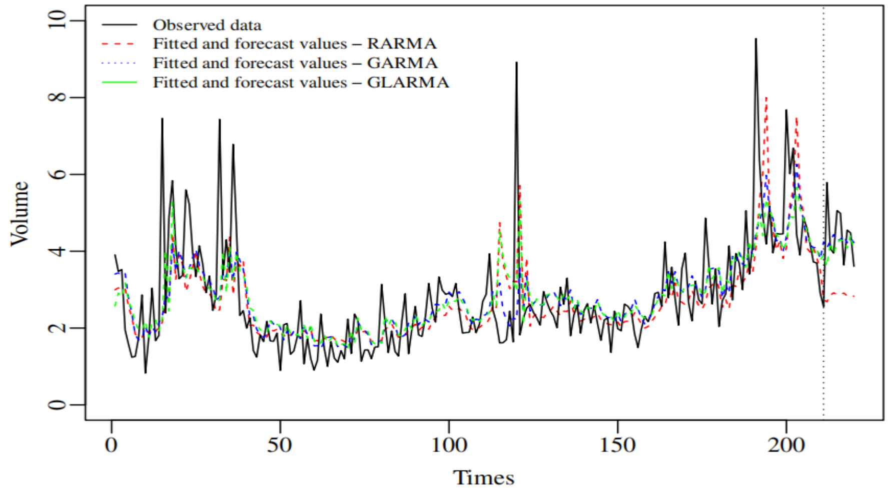

Figure 4. Observed and adjusted values of the RARMA, GARMA, and GLARMA model for the Porto Seguro time series.

4. Conclusion

The present study aimed to perform a comparative analysis of the ARMA, RARMA, GARMA, and GLARMA models applied to financial data to identify the model with the best performance in terms of fit and forecasting. Two financial time series were used, and the models were fitted and evaluated based on model selection criteria such as AIC, BIC, and HQ. Forecast accuracy measures included MSE, MAPE, and MASE.

In the first application, the GARMA and GLARMA models stood out for their goodness of fit compared to the ARMA and RARMA models, exhibiting lower values for the AIC, BIC, and HQ criteria. However, the ARMA and RARMA models presented more accurate results in forecasting future observations. In the second application, GARMA and GLARMA obtained the lowest values for the criteria, with the ARMA and GLARMA models standing out as the most accurate in forecasting future observations.

Although the GARMA and GLARMA models demonstrated a better fit to the data, as indicated by the lower values of the AIC, BIC, and HQ criteria, the ARMA and RARMA models were more effective in forecasting future observations in the trading volume of Banco Bradesco application. However, in the trading volume of Porto Seguro application, the GLARMA model was the most accurate in both fitting and forecasting, but obtaining results similar to the ARMA model in the forecast. This suggests that the choice of the ideal model depends on the objective (fitting or forecasting) and the specific context of the application.

Future research is recommended to apply these models in other financial areas and to consider additional models, such as BXII-ARMA ([6]) and CHARMA ([24]).

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