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# Super-resolution by converting evanescent waves in microsphere to propagating waves and light transmitted from its surface to nano-jet

Y. Ben-Aryeh

Physics Department, Technion-Israel-Institute of Technology, Haifa 32000, Israel; [phr65yb@physics.technion.ac.il](mailto:phr65yb@physics.technion.ac.il)

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**Abstract:** The electro-magnetic (EM) waves transmitted through a thin object with fine structures are observed by a microsphere located above the thin object. The EM radiation transmitted through the object produces both evanescent waves, which include information on the fine structures of the object (smaller than a wavelength), and propagating waves, which include the large image of the object (with dimensions larger than a wavelength). The super-resolutions are calculated by using the Helmholtz equation. According to this equation, evanescent waves have an imaginary component of the wavevector in the z direction, leading the components of the wavevector in the transversal directions to become very large so that the fine structures of the object can be observed. Due to the decay of the evanescent waves, only a small region near the contact point between the thin object and the microsphere is effective for producing the super resolution effects. The image with super-resolution can be increased by a movement of the microsphere over the object or by using arrays of microspheres. Both propagating and evanescent waves arrive at the inner surface of the microsphere. A coupling between the transmitted EM waves and resonances produced in the dielectric sphere, possibly obtained by the Mie method, leads to a product of the EM distribution function with the transfer function. While this transfer function might be calculated by the Mie method, it is also possible to use it as an experimental function. By Fourier transform of the above product, we get convolution between the EM spatial modes and those of the transfer function arriving at the nano-jet, which leads the evanescent waves to become propagating waves with effective very small wavelengths and thus increase the resolution.

**Keywords:** microsphere; super-resolution; evanescent waves; nano-jet; transfer function; Mie method

## 1. Introduction

Any microscopic image can be magnified by using a microscope. But observing sub-wavelength structures is difficult because of the Abbe diffraction limit [1], by which light with wavelength  $\lambda$  travelling in medium with refractive index  $n$  and aperture angle  $\theta$  will make a spot with radius.

$$d = \frac{2(n \sin \theta)}{\lambda} \quad (1)$$

The term  $n \sin \theta$  is called the numerical aperture (NA) and the Abbe limit is of order  $\lambda/2$ .

I analyze in the present article the mechanism by which evanescent waves incident on a microsphere are converted into propagating waves, and by such conversion high resolution is obtained in the image by the microsphere, which is much beyond the Abbe limit. There is much interest in the optical properties of the microsphere system. Many studies on the super-resolution obtained by the microsphere system were described in recent articles [2–10]. The mechanisms by

which nan-jets are produced in the microsphere system were analyzed [11–16]. These works and many previous ones led to controversy about the origin of the observed super-resolution effects. In certain works, the super-resolution was related to evanescent waves emitted from the object, which include the fine structures of the object. In these works, the production of the nan-jet was considered a separate effect, not related to super resolution. On the other hand, it was claimed in other works that the super-resolution effects are produced by the same mechanisms that produce the nano-jet, while the evanescent waves might have only minor effect. In the present work, I show that the super-resolution effects are produced in two steps, where in the first step the information on the object's fine structures is obtained by the evanescent waves. In the second step, there is a convolution between the spatial modes of the evanescent waves and the spatial modes of the microsphere, which transforms the evanescent waves into propagating waves with very small effective wavelengths [17].

The super-resolution obtained by the microsphere is like the field of scanning near-field optical microscopy (SNOM), where the resonant effect with the tip detector enhances the super-resolution. We follow in the present analysis the idea that the evanescent wavevectors are coupled to the microsphere by resonances produced by Mie theory analysis [18,19]. Such Mie theory is also used for a good description of the nano-jet [19]. The conversion of evanescent waves to propagating waves, after the transmission through the microsphere surface, is obtained by convolution between the evanescent spatial modes and the transfer function of the microsphere modes [17]. The idea of using such convolution was suggested already in a previous article [20]. My approach to microsphere high resolution will be developed in the next sections by using two steps: 1) In the first step, we use the Helmholtz equation, by which large wave vectors are produced above the object, producing high resolutions. 2) The EM fields of both evanescent waves and propagating waves arriving parallel to the microsphere surface are preserved during transmission through the microsphere surface due to boundary conditions. The enhancement of the conversion of evanescent waves to propagating waves is due to coupling between evanescent waves and resonances, produced in the microsphere, for example, by Mie theory [19]. In the second stage we do not use the Helmholtz equation, and the above coupling is described as a convolution between the spatial modes of the evanescent waves and those of the microsphere modes, described by the transfer function [17]. Although the propagating waves are stronger than those of the evanescent waves, the modulation of the total intensity by the evanescent waves is the source of the high resolutions obtained by the microsphere system.

## 2. Methods

The use of evanescent waves to increase the resolution beyond the Abbe limit can be related to Helmholtz equation [17]. In homogenous medium this equation is given by

$$(nk_0)^2 = k_x^2 + k_y^2 + k_z^2 \quad (2)$$

where  $k_0 = 2\pi/\lambda_0$ ,  $\lambda_0$  is the wavelength in vacuum,  $k_x, k_y, k_z$  are the wavevector components. The evanescent waves satisfying the relation:

$$k_x^2 + k_y^2 > (nk_0)^2 \quad (3)$$

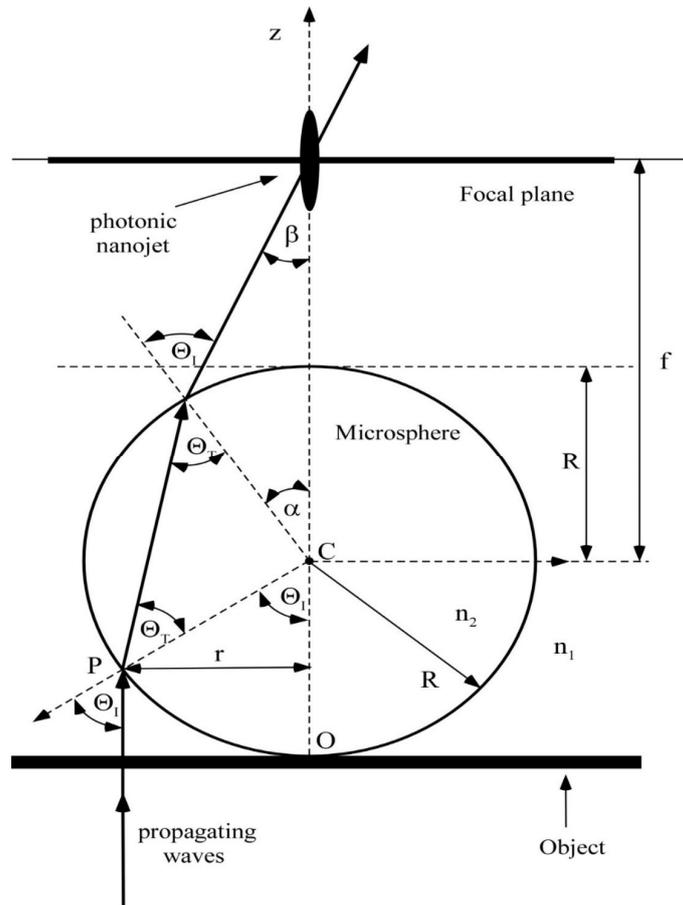
are arriving at the microsphere with imaginary  $k_z$ . The increase of the components of the wavevector  $\vec{k}$  in the plane  $x, y$  decreases the “effective” wavelength in this plane, and thus increases the resolution.

As described in **Figure 1**, a dielectric microsphere with radius  $R$  and a refractive index  $n_2$  is located above a thin object at a contact point  $O$ . The medium between the object and the microsphere has a refractive index  $n_1$ . Parallel EM waves are transmitted in a direction perpendicular to a thin planar object, which may be transmitted as “propagation” waves and as “evanescent” waves. But the increase of resolution by the microsphere is related to the conversion of evanescent waves to propagating waves.

We analyze the conversion of evanescent waves to propagating waves at a point  $P$ , which is on the microsphere surface. The incident and transmitted angles for the EM waves transmitted into the microsphere at point  $P$  are given as  $\theta_I$  and  $\theta_T$ , respectively. At this point, EM waves with wavevectors  $k_x, k_y$  are arriving at the microsphere, where  $k_z$  is imaginary. The increase in the component of the wavevector  $\vec{k}$  in the plane  $x, y$  decreases the “effective” wavelength in this plane, and thus increases the resolution. But the evanescent waves decay in the perpendicular  $z$  direction, so that to “capture” the fine structure that is available in the evanescent waves, we need that the point  $P$  will be near the contact point  $O$ , so that its perpendicular distance to the object will be of a wavelength order. According to geometric optics, the microsphere has a spherical symmetry under rotation around the  $z$  axis, which connects the center of the microsphere at point  $C$  with the contact point  $O$ . The line connecting the point  $C$  with point  $P$  has an angle  $\theta_I$  with the symmetric  $z$  axis.

As shown in **Figure 1**, the EM waves transmitted through the microsphere are converging into photonic-jet (PJ), where its role in producing the high resolution is controversial. The original work in obtaining high resolution in microsphere imaging was made by Zengo Wang et al. [21]. That analysis was made with a virtual image as follows from the geometric optics description. Since this time, a very large number of papers were published on various effects in the microsphere system. In **Figure 1**, we describe a real image that is produced by using a high-refractive index microsphere (e.g., [22,23]).

The focusing of light in the microsphere system is concentrated in the nano-jet sub-diffraction region, which does not obey the classical laws of geometrical optics. but might be explained by diffraction effects, known as photonic-jet (PJ) (e.g., [24,25]). Exact solutions for non-diffraction beams might be related to the central part of the photonic nano-jet [26,27].



**Figure 1.** Microsphere and nano-jet.

Propagating EM waves are transmitted through a thin object with a certain structure, whose image is produced by evanescent waves, arriving at the microsphere surface at point P, for example. The EM transmitted through the microsphere is converging into a non-diffraction region known as a photonic jet (PJ).

Rigorous Mie theory predicts the interaction of light with spherical particles, and this theory was used to describe various properties of the PJ's produced by the microsphere system [19]. The exact use of the Mie theory is usually done by numerical calculations, as it is obtained by the sum of many terms that do not give an analytical result. Using Mie theory, optical resonances in microsphere photonic nano-jets were observed [18]. We analyzed the properties of evanescent waves, produced by plane EM waves transmitted through nano-corrugated-metallic thin film, which includes information on its fine structures [28]. A microsphere located above the metallic surface collects the evanescent waves, which are converted to propagating waves. The magnification of the nano-structure images is explained by a geometric optics description, but the high resolution is related to the evanescent wave analysis. Such an approach for explaining the high resolutions obtained by microspheres was developed by using complex Snell's law [29]. Very high resolutions by microspheres were reported also in other works (e.g., [30,31]).

Maslov and Astratov [32,33] studied the origin of super-resolution in microsphere-assisted imaging. The imaging of the nano-jet cannot give a good explanation for the high resolutions obtained in the microsphere system. The

evanescent wave source cannot explain the high resolutions obtained by the microsphere system since its effect is too weak. They suggested using a direct approach to Maxwell equations, including optical principles such as point-spread functions, in microspherical imaging. I claim, however, that there are strong arguments that evanescent waves should be included in the theories about high resolutions: a) The propagating distance between the object and the microscope must be small enough to reveal the nanometric features. To generate super-resolution over large areas of the sample, we need to attach the microsphere to a frame, which is scanned on the sample in step-by-step fashion [34] or by using microspheres arrays [6]. b) There is a dependence of the super-resolution on the radius of the microsphere and its index of refraction, which is in good correspondence with the evanescent wave properties [29]. c) The super-resolution is related to information theory, where the information on the fine structures of the object area is available in the evanescent waves. Magnification of the image and optical transforms cannot introduce high resolution if the information on the object's fine structures is not available in such transforms. I find, however, that there is an important mechanism that enhances the transformation of evanescent waves into propagating waves after the microsphere surface, which is beyond classical geometric optics (beyond the use of complex Snell's law [29]. To explain this new mechanism, it will be helpful to compare the high resolution obtained by the microsphere with the high resolutions obtained in metallic grating [35,36]. It was shown in these works that a thin film of metallic grating with arrays of subwavelength holes can transmit light at certain frequencies, which is order of magnitude larger than the light intensity incident on the area of these holes. Most investigators agree that these experiments, conducted originally by Ebessen et al. [35,36], are related to coupling the light with surface plasmons. I explained these phenomena as transmission enhancement by converting evanescent waves, entering the small holes to propagating waves, due to convolution of the evanescent spatial modes with the plasmon spatial modes, producing high spatial wavevectors with very small "effective" wavelengths [37]. Such effects were described in a similar way by relating the 'tunnelling' of evanescent waves to propagating waves due to the convolution of the high spatial frequencies of the source with those of the detector [38]. One should consider that in the dielectric sphere we don't have plasmons, but we have other coupling mechanisms between the evanescent waves and the microsphere EM modes.

### **3. An analysis for the super resolution, in the microsphere system, obtained in many experiments**

In experimental studies on microspheres by other authors [2–23,30,31,34], super resolution effects were observed. These works raised the question: Is the super-resolution related to evanescent waves? or is it related to the existence of the nano-jet? The question led to controversy between the various works. I show in the present theoretical analysis that a combination of two effects produces the high resolution: 1) The evanescent waves produced by the object are incident on the microsphere surface, preserving high resolutions. 2) The same mechanism which produces the nano-jet leads to conversion of the evanescent waves to propagating waves, but with very small wave lengths. The reduction of the wavelength was explained also as a quantum effect

[39,40] where  $n$  entangled photons lead to effective wavelength  $\lambda/n$ , and there is a distribution of such effective wavelengths.

We develop the analysis for the microsphere system into two parts: a) In the first part, we describe the propagation of evanescent waves, produced on thin planar objects with fine structures, to the microsphere surface. In this stage, high resolutions of the image are obtained related to the use of the Helmholtz equation. b) In the second part, we consider the propagation of both evanescent and propagating waves produced on the inner surface of the microsphere to the nano-jet. This propagation is described by the convolution of these EM fields with the microsphere modes described by a transfer function [17], which is related to Mie theory, but it is more convenient to use it as an experimental function.

### 3.1. The use of Helmholtz equation for getting high resolutions by evanescent waves

Let us assume that the planar surface of an object is given by  $z = 0$ , and the EM field in this plane is given by the Fourier transform

$$U(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(k_x, k_y) \exp[-i(k_x x + k_y y)] dk_x dk_y \quad (4)$$

where  $u(k_x, k_y)$  is the distribution of the EM spatial modes in the  $x, y$  plane. Then, the EM waves propagating from the planar surface of the object into homogenous medium in the space  $z > 0$  with a refractive index  $n_1$  is given by:

$$U(x, y, z > 0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(k_x, k_y) \exp[-i(k_x x + k_y y + k_z z)] dk_x dk_y \quad (5)$$

Substituting Eq. (5) into the Helmholtz equation we get:

$$(\Delta^2 + k^2)U(\vec{r}) = 0 \quad (6)$$

obtaining the result:

$$[k^2 - (k_x^2 + k_y^2 + k_z^2)]U(x, y, z > 0) = 0 \quad (7)$$

Under the condition  $k^2 < k_x^2 + k_y^2$ ,  $k_z$  is imaginary, and for such case we get the evanescent wave solution:

$$U(x, y, z > 0) = U(x, y, z = 0) \exp(-\gamma z) \quad ; \gamma = \sqrt{k_x^2 + k_y^2 - k^2} \quad (8)$$

A dielectric microsphere with refractive index  $n_2$  and radius  $R$  is located above a thin film of the object at a contact point  $O$ , where the medium between the object and the microsphere has a refraction index  $n_1$ . A plane EM wave is transmitted through the thin film of the object in the perpendicular direction and incident on the microsphere at point  $P$ . The evanescent waves are incident on the microsphere surface near the contact point  $O$ .

For evanescent waves, there is a decay of the wave in the  $z$  direction. The resolution obtained by the evanescent waves is limited by the lateral component  $k_T$  of the wavelength given by:

$$\lambda_T = \frac{2\pi}{k_T} = \frac{2\pi}{\sqrt{k_x^2 + k_y^2}} \quad (9)$$

As the evanescent waves satisfy the equation  $k^2 = k_x^2 + k_y^2 - \gamma^2$ , then Eq. (9) can be written as

$$\lambda_T(\text{evan.}) = \frac{2\pi}{\sqrt{k^2 + \gamma^2}} \quad (10)$$

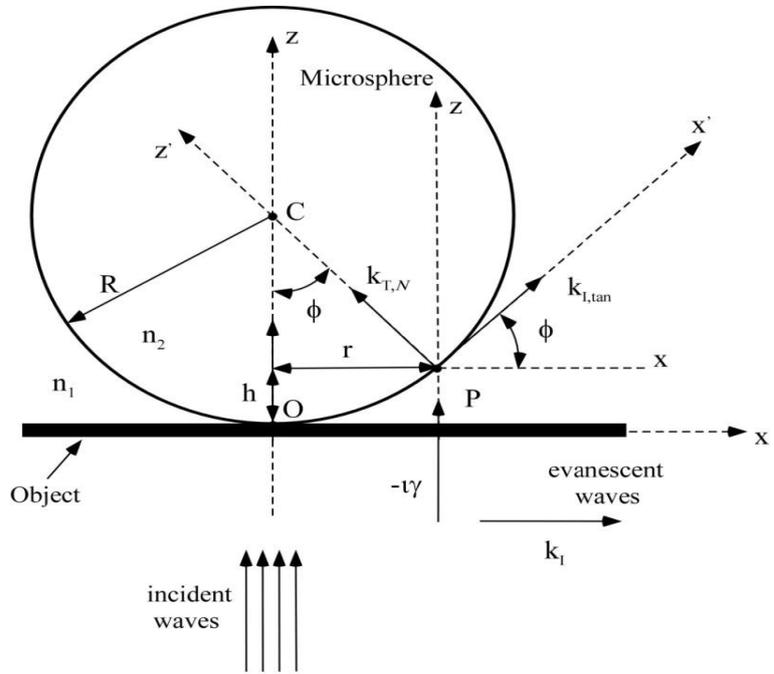
The minimal value of  $\lambda_T$  for propagating waves is given by:

$$\lambda(\text{prop.}) \frac{2\pi}{k_{T,\min}} \quad (11)$$

since the minimum is obtained when  $\vec{k}$  is in the x, y plane.

As by the Abbe limit the resolution is of order  $\lambda/2$ , the increase of resolution  $F$  by using evanescent waves is given by:

$$F = \lambda_T(\text{prop.})/\lambda_T(\text{evan.}) = \sqrt{\frac{k^2 + \gamma^2}{k^2}} = \sqrt{1 + \frac{\gamma^2}{n_1^2 k_0^2}} \quad (12)$$



**Figure 2.** Evanescent waves on microsphere surface.

The distance of the point P from the planar object is given by  $h = R(1 - \cos \phi)$  (see **Figure 2**). For decay constant  $\gamma$  of the evanescent waves, represented in unit  $\gamma/n_1 k_0$ , the decay of the evanescent wave at point at point P, after transversing the distance  $h$ , is given by

$$\exp(-\gamma h) = \exp\left[-\frac{\gamma}{n_1 k_0} n_1 k_0 R(1 - \cos \phi)\right] = \exp\left[-\frac{\gamma}{n_1 k_0} 2\pi(1 - \cos \phi) \frac{R}{\lambda}\right] \quad (13)$$

We find that this decay increases very much by increasing  $\phi$ , so that only a small region around the contact point O is efficient in obtaining the high resolution by evanescent waves.

### 3.2. Microsphere imaging by a transfer function from the microsphere surface to the nano-jet

Let us assume that the EM field  $E(x, y)_{tan}$ , which is tangent to the microsphere surface at the point  $x, y$  (of evanescent wave in a small region around the contact point O, plus a propagating wave in a larger region), is given by:

$$E \cos \phi_{tan} \quad (14)$$

where  $E(x, y)$  is the EM field before the microsphere surface (see **Figure 2**, and [29]). The EM field  $E_{tan}$  is preserved during transmission through the microsphere surface due to boundary conditions, and it includes both the evanescent waves and the propagating waves. The EM field propagating after the microsphere surface is given as

$$E'(x, y) = E_{tan}(x, y)G(x, y) \quad (15)$$

where the 'transfer function'  $G(x, y)$  can be related by Fourier transform [17] to  $G(k_x, k_y)$ :

$$G(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(x, y) \exp[-i(k_x x + k_y y)] dx dy \quad (16)$$

One should notice that  $G(k_x, k_y)$  should include both evanescent waves for which  $k_x^2 + k_y^2 > k^2$ , and propagating waves for which  $k_x^2 + k_y^2 < k^2$ .

Due to the small distance between the points  $x, y$  and the contact point O, the evanescent field  $E_{tan,eva}(x, y)$  is approximately parallel to the  $x, y$  plane. This EM field is smaller from the evanescent EM field on the thin object, by the factor given approximately by Eq. (13).

The Fourier amplitude  $A(k_x, k_y)$  is described by the Fourier inverse of  $E_{tan}$ , which is given by:

$$A(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp[-i(k_x x + k_y y)] E(x, y)_{tan} \quad (17)$$

where  $k_x, k_y$  are the wavevectors in the  $x$  and  $y$  directions. The Fourier transform of  $E'(x, y)$ , (given by the product of Eq. (14)) is obtained by the convolution of the spatial wavevectors  $G(k_x, k_y)$  and  $A(k_x, k_y)$  [17]:

$$E'(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(k'_x, k'_y) G(k_x - k'_x, k_y - k'_y) dk'_x dk'_y \quad (18)$$

The convolution given by Equation (18) includes spatial wavevectors  $k_x - k'_x$  and  $k_y - k'_y$ , which might be very large, so that the corresponding wavelengths can be reduced to very low values. For example, if approximately,  $k_x - k'_x = kn$  where  $n$  is a large integer, then the effective wavelength will be reduced to  $\lambda/n$  (see the analysis in [39,40]). We use the convolution effect described by Equation (18), to obtain enhancement of the conversion of evanescent waves to propagating waves by reducing the effective wavelength. Although the modulation of the EM waves by evanescent waves is small relative to the total intensity of the EM field, such modulation is effective in producing the high resolution. The spreading of the spatial modes by convolution is like the point spread function used in [32,33], but one should not ignore

the super resolution obtained for the object in the first stage of the imaging process, which is related to the use of Helmholtz equation.

The above reduction of the effective wavelength by convolution, was also described as a quantum effect where entanglement between the  $n$  photons reduces the effective wavelength to  $\lambda/n$  [39,40]. Such entanglement remains true also after the transformation to propagating waves.

#### 4. Discussions and results

The high resolution obtained in microspheres is due to the following two factors: a) The EM radiation transmitted through the object produces both evanescent waves, which include information on the fine structures of the object (smaller than a wavelength), and propagating waves, which include the large image of the object (with dimensions larger than a wavelength). Evanescent waves lead to a super-resolution, which is better than the Abbe limit, by the factor  $F = \sqrt{(k_x^2 + k_y^2)/k^2} = \sqrt{(k^2 + \gamma^2)/k^2}$  where  $k^2 = k_x^2 + k_y^2 - k_z^2$  is the wavevector of the evanescent EM field,  $k_z = -i\gamma$  is imaginary, and  $k_x, k_y$  are the transversal components. The evanescent waves arrive at the microsphere surface after a decay in the  $z$  direction by the factor  $\exp(-\gamma h)$ . where  $h$  is the distance from the object to the microsphere surface, and  $\gamma$  is a certain decay constant., so that only the evanescent waves which are near the contact point  $O$  of the object are efficient in conserving the fine structure imaging. But one can attach the microsphere to a frame which moves on the object and scans a large image, or to use arrays of microspheres. b) By using boundary conditions, we get on the inner microsphere surface both evanescent waves and propagating waves. The convolution between the EM waves, on the inner surface of the microsphere, and the transfer function, representing the coupling with microsphere EM modes, enables propagation with small effective wavelengths (smaller than a wavelength and correspondingly large wave vectors). This effect is due to the spread of the wavevectors due to the above convolution. The transfer function also includes the wavevectors of the evanescent waves and thus enables them to be transferred to the image without evanescent wave decay. The analytical description of this convolution is given in the article by using Equations (14–18). It is possible to use Mie theory for calculating the transfer function, but such calculations are very complicated and usually give only numerical results. We suggest, therefore, to use the transfer function as an empirical function, which can also be used for a description of the nano-jet.

**Figure 1** gives only a geometric optics picture. The incident and transmitted angles  $\theta_I$  and  $\theta_T$  are derived by Snell's law, respectively. There are different features in the microsphere system which can be explained by the geometric optics picture. For example, for a small microsphere index of refraction, we get a diverging beam with a virtual image, while if this index of refraction is large, then the beam is converging with a real image, like that in **Figure 1**. The angle  $\beta$  between the beam converging to the nano-jet and the symmetric axis, and the distance  $r$  from the point P to the symmetric axis, can be obtained by simple geometric calculations. But the microsphere super-resolutions can be calculated, only by using an analysis, similar to

that presented in the present article. Some features of the nano-jet may be described by using Bessel beams [26,27]. **Figure 2** describes the transmittance of both evanescent waves and propagating waves from the object to the inner surface of the microsphere, where the fine structures of the object are included in the evanescent waves.

## 5. Conclusion

By using the Helmholtz equation, it was shown how the fine structures of the object are transferred by the evanescent EM radiation to the inner surface of the microsphere. The information on the fine structures of the object is included in the evanescent waves, and this information is transferred from the inner microsphere surface to the nano-jet by using the coupling between the EM fields and the EM modes of the microsphere, which might be calculated by Mie theory (e.g., [19]). This coupling leads to transmittance of both the evanescent waves and propagating waves to the nano-jet by using ‘transfer functions’ (see Equations (14–17) and reference [17]), which lead to a very small ‘effective wavelength’ for the evanescent waves, although they are not decaying any more. While the propagating waves are stronger than the evanescent waves, the modulation of the total EM radiation intensity in the nano-jet by the evanescent waves is the source of the super-resolutions of the microsphere system. The present analysis solves the controversy about whether the nano-jet or the evanescent waves are the source of the super-resolution effects, as the source of the super-resolution is composed of these two different mechanisms in two different parts of the microsphere system.

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