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Evaluating Markowitz-Based Risk Measurement Approaches for Making Profitable Investment Decisions

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ABSTRACT

Risk is one of the most important factors in making desired profitable investment decisions. It is the main reason that why numerous financial researchers have interested in risk assessment, from the past to the present. The literature of risk measurement approaches for financial decision making has dramatically expanded since the early work of Markowitz. Konno, Cai and Teo risk assessment approaches are the most well-known and widely-used methods, developed based on the basic concepts of the Markowitz model. Although, all of these approaches have different advantages for measuring the risks and making profitable investment decisions, none of them are not universal method, which can be applied in all circumstances with desired performance. On the other hand, each of these methods may yield a high or low performance rate which varies in diverse situations and data sets. In this regard, and due to the lack of suitable research in order to compare risk measurement methods, the aim of this paper is to evaluate four aforementioned Markowitz-based risk measurement methods in different investment decision-making situations. Empirical results of using these methods in stock market indicate that Markowitz, Konno, Cai and Teo techniques can achieve, %-6.77, %-6.49, %2.85 and %-6.89 rate of return, respectively. Therefore, the Cai technique may be more appropriate risk measurement approach among aforementioned methods for making investment decisions in stock markets compared to the other three

***Keywords*:** Financial decision Making; Risk measurement; Markowitz-based methods; Investment decisions; Rate of return; Stock market

1. Introduction

One of the basic components that make up profitable investment decisions is risk. In other words, each investor, when making a decision, is required to know the risk measurement of investment and if the measured risk is more accurate, it will lead to more favorable decisions. Therefore, due to the importance of the accuracy of the measured risk in making investment decisions, proposed methods for measuring risk in the form of investment decisions have been evaluated repeatedly in literature. Some of these studies include: Frajtova-Michalikova *et al*.[1] have compared nonparametric methods for estimating the level of risk in finance. Alexander and Baptista[2] have analyzed the investment portfolio implications arising from imposing a value-at-risk constraint on the mean-variance model, and compared them with those arising from the imposition of a conditional value-at-risk constraint. Also in another paper, some different risk measures regarding performance of optimal portfolio strategies has been compared by Righi and Borensteina[3]. Spuchlakova *et al*.[4] have investigated different strategies in investment decision making and determined the risks and returns of each strategy in order to allow comparisons. Liu and Gao[5] have proposed a method to solve the portfolio selection problem based on the Konno Risk Measurement Approach and then compared the use of Konno and Markowitz risk measurement method in determining investment strategies.

Oloko[6] has used various methods to estimate optimal portfolios to investigate the different risks of the stock market. also Jin *et al*.[7] have explored the various risk assessment methods in the portfolio selection problems. Egozcue *et al*.[8] have ranked and compared different investment strategies in the case of risk-averse and risk-inclined investors. Also two different methods have been tested to measure the investment risk in the China's stock market by Jin[9] and results showed that there were significant differences between these two methods.

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Despite the studies on the evaluation and comparison of risk measurement methods, in the literature the fundamental methods of risk measurement and their impact on investment decisions has rarely been compared. While, over the years, many methods have been proposed to measure risk, each of which may, have better performance than the other methods under certain conditions. One of the most fundamental efforts to measure risk has been noted in Harry Markowitz's[10] study where he has considered risk as a concept of variability and expressed that risk was related to the dispersion of a random variable. Therefore, in terms of Markowitz, the dispersion indices, including variance and standard deviation, represent measure of risk. Equation (1) expresses the basic form of Markowitz risk measurement for each unique stock[10].

$$\mathrm{Risk}_{\mathrm{Markowitz}}(x) = \mathrm{Var}(x) = \mathrm{E}(x - \mathrm{E}(x))^2 \quad (1)$$

After presenting the Markowitz model for risk measurement, many researchers tried to improve this model from both theoretical and computational aspects. One of these researchers was Konno[11,12], who introduced the L1 risk function, based on expected absolute deviation instead of the L2 risk function (deviation) in determining the numerical index of risk measurement. Kano's goal was to create a model that, beside eliminating the computational problems of the classic Markowitz model, would be able to preserve its desirable characteristics. Konno risked measurement method for an unique stock, according to equation (2)[11].

$$\mathrm{Risk}_{\mathrm{Konno}}(x) = \frac{1}{n}\sum_{t=1}^{n} |x_t - \mathrm{E}(x)| \quad (2)$$

In the following of Markowitz and Konno, Cai[13,14] has presented a different method for risk measurement in his studies. Referring to the fact that previous studies did not adequately model the concerns of risk averse investors, he has introduced the L∞ function as the risk aversion measure. In fact, the method of Cai *et al*. Relates to a situation that an investor is conservative. In this method, the risk of each unique asset is equal to the maximum deviation from expected returns. Therefore, in this method of risk measurement, the object is to control the highest standard deviation and An investor who does not want to face high risk can use this risk averse approach. Equation (3) shows how to calculate the risk of each unique asset by using the Cai method[13].

$$\mathrm{Risk}_{\mathrm{Cai}}(x) = \mathrm{Max}_{1\le t\le n}|x_t - \mathrm{E}(x)| \quad (3)$$

After Cai, Teo[15] provided another risk measurement. In this method, the risk is equivalent to the average of the maximum deviation from expected returns over several periods of time. In this method, the data is divided into several periods. In each period, the absolute deviation is calculated. Then, the risk is equivalent to the average of the maximum of these absolute deviations. In equation (4), the risk of each unique asset is calculated by the Teo method. In this method, the data are divided into P periods of time, then in each period, the maximum deviation from the expected rate of return is obtained and, finally, the average of these maximums shows the risk. In fact, Teo and Cai risk measurement methods are very similar but Teo risk measurement is more balanced than the Cai risk measurement.

$$\mathrm{Risk}_{\mathrm{Teo}}(x) = \frac{1}{p}\sum_{n=1}^{P} \mathrm{Max}_{n(p-1)/P\le t\le np/P}|x_t - \mathrm{E}(x)| \quad (4)$$

All of these methods have had a Markowitz theoretical basis and determined the index of dispersion around the expected returns as risk. But, in each method, the researchers have identified an index as a risk measure, which, in their view, was more important than other indices in Risk measurement. Therefore, Markowitz and Konno have considered average dispersion around the expected return as the risk index and believed that any asset that was generally more stable would be less risky. However, by contrast, Cai has introduced the most deviations from expected returns as risk measures. In the other word Cai has argued that an asset which has experienced a significant deviation from expected returns would be more likely to face these conditions again, so it would be more risky. Finally, Teo has combined the ideas of Markowitz, Konno, and Cai by adding time periods to the Cai model and then calculating the average. Therefore, in these methods different ideas have been considered in determining the risk index. Therefore, in these methods different ideas have been considered in determining the risk index, each of which may have different performance in data sets with different characteristics. Accordingly, the goal of this study is to evaluate and compare these methods regarding to the risk of different data sets. For this purpose, in the first step, the risk of 30 selected stocks from Tehran Stock Exchange was calculated using all four methods. Then, given that the net measured risk was not

comparable, in the second step, in order to make comparison possible, the performance of each method was determined on investment decision making. The rest of this paper is organized as follows: In section II, the characteristics and method of collecting data on the shares of companies that have more trading volume in the Tehran Stock Exchange are briefly described. In section III, four Markowitz-based risk measurement methods is implemented in order to make investment decisions. In Section IV, the results of risk measurement methods are compared. Section V explains the conclusions.

2. Description the studied data

In order to evaluate and compare four Markowitz-based risk measurement methods as well as to test them versus each other, data from Tehran Stock Exchange was used. For this purpose, stocks were selected that in the five-year period between 2012 and 2017 had higher financial transaction than the other stocks in the main hall and the first market of Tehran Stock Exchange. The returns of these stocks were extracted and refined on a weekly basis. These data included 259 weeks, of which 233 weeks were used for training and modeling and the remaining 26 weeks were used to test the performance of under study methods. **Table 1** introduces these stocks[16]:

First, the risk of each data set was calculated using four fundamental Markowitz-based methods of risk measurement, including Markowitz, Konno, Cai and Teo approaches. Then, the performance of each of these methods on investment decision making have been evaluated and compared. For this purpose, first, the appropriate strategies, including: buying or selling stocks, based on measured risk were made. Then the performance of adopted strategies were compared to each other. This comparison is based on the fact that if a risk measurement method is more accurate, it will have a higher investment performance. Therefore, a risk measurement method will lead to more favorable investment decision that provides a better risk measurement.

No.	Stock		Average of stock prices	Maximum of stock prices	Minimum Of stock prices	Mod of stock prices	Average of stock prices			
							Train	Test	Train	Test
1-	Metals and Mines	1071	3377	10633	1010	1399	950	121	3600	1369
2-	Ind. and Mine Investment	1188	1634	3537	867	1088	1066	122	1643	1557
3-	Kerman Cement	1094	5746	13059	2385	3848	974	120	6058	2942
4-	Sepahan Cement	1131	1508	3837	843	1149	1014	117	1541	1207
5-	Calcimine	1105	4585	11465	1606	1978	1002	103	4717	3397
6-	Karafrin Bank	1121	2523	3451	1270	2776	1002	119	2471	2992
7-	Shazand	1043	9136	29674	2513	7578	922	121	9844	2763
8-	Ghadir Investment	1128	3837	8054	1299	7971	1015	113	4079	1651
9-	North Diriling	1068	4213	9078	1614	3570	962	106	4294	3488
10-	Motogen	1007	5911	13906	2618	5299	912	95	6002	5098
11-	Chadormalu	1096	5818	13832	1593	2710	975	121	6260	1844
12-	Sepah Investment	1176	1771	3410	1045	1405	1055	121	1804	1470
13-	Sobhan Pharmcy	1120	6169	10530	2548	8090	1005	115	6082	6955
14-	Ansar Bank	1168	2489	3648	1698	2715	1047	121	2547	1963
15-	Gol-e-gohar	1064	6137	14527	1689	3041	956	108	6579	2155
16-	Iran Industrial Development	1182	1398	2816	759	1300	1065	117	1425	1155
17-	Mobarake Steel	1119	2749	5337	1001	1845	998	121	2908	1316
18-	Iran Const Investment	1139	3046	7332	1489	3446	1046	93	3061	2911

No.	Stock		Average of stock prices	Maximum of stock prices	Minimum Of stock prices	Mod of stock prices	Average of stock prices			
							Train	Test	Train	Test
19-	Tuka Investment	1081	2636	8386	1066	1372	1010	71	2766	1464
20-	Bahman Investment	1153	1370	2260	666	1560	1033	120	1379	1288
21-	Rayan Saipa	1127	2194	5597	907	5529	1015	112	2231	1863
22-	Pars Toushe Investment	1156	2911	5621	1484	2731	1035	121	2840	3544
23-	Bahman Grop	1146	1984	3341	1056	1481	1039	107	1965	2161
24-	Parsian Oil and Gas	1069	6019	15283	1703	8960	968	101	6463	2022
25-	Jaber Hayan Phamcy	1159	7658	13640	2238	3520	1040	119	7094	12734
26-	Housing Investment	1156	1576	3102	844	1114	1035	121	1644	962
27-	Pars oil	1001	12792	50017	3876	21722	890	111	13622	5331
28-	Alborz Investment	1128	4083	8962	2288	3493	1011	117	4088	4041
29-	Khouzestan Steel	1046	12809	47223	1867	3429	940	106	13894	3053
30-	Iran Telecom	1090	2555	4003	1899	2341	992	98	2585	2281

Table 1. The under study stocks

3. Implementing Markowitz-based risk measurement methods and make investment decisions

This section describes how the under study methods were used to make investment decisions. This investment approach was implemented for all training and testing data and ultimately the average of investment performance was selected as the investment rate of return. Therefore, assuming that the transaction cost is ignored, the decision on how to invest in each stock and in each of the 26 tests is summarized in six steps as follows:

1. The risk of each stock is measured using each of under study methods.
2. Measured risks are normalized.
3. As the ultimate goal of measuring risk and determining the expected return is using these indicators in decision making. Expected returns will determine buying or selling strategy, then the measured risk will complete the process so investment decisions are made in relation to each stock on 9 different risk levels which were arranged from 0.1 to 0.9 at intervals of 0.1.
4. The performance of each investment decision is determined as follows: if the correct decision is made, the value of the actual return with the positive sign is considered as the decision performance. But, where the decision making does not lead to the correct decision, the actual return with the negative sign is recorded as the decision performance.
5. The average of these performances at different risk levels indicate the performance of investment decisions at each under study method.
6. The average of method performance in all 26 tests indicates the overall performance of each method on each stock.

These steps have been done for each of the listed stocks, using four Markowitz-based risk measurement methods. So according to the first step, the risk of each of the under study stocks was calculated. **Table 2** illustrates the risk of each of the listed stocks which was measured using each of under study methods. According to **Table 2**, the calculated

risk by the Konno method have had the lowest value, and then the Teo method was located. But, from both the Cai and Markowitz methods, the sequence of risk have depended on under study data sets. Also the sequence of under study assets based on the risk measured using different methods is not necessarily the same.

In the following to compare the under study risk measurement methods, the average performance of each of these methods in making investment decisions related to 30 selected stocks at different levels of risk accepted by investors is shown in Table 3. Each cell in this table represents the average of rate of return for investment decisions which were made based on different risk measurement methods for each of the stocks at all risk levels and during the 26 tests.

No.	Risk Measurement Methods & Under Study Stocks	Markowitz Risk Measurement	Konno Risk Measurement	Cai Risk Measurement	Teo Risk Measurement
1-	Metals and Mines	46.74	4.18	58.62	12.84
2-	Industrial and Mine Investment	23.42	3.43	26.46	10.35
3-	Kerman Cement	30.84	3.37	41.47	12.54
4-	Sepahan Cement	26.86	3.91	17.72	10.24
5-	Calcimine	35.79	4.27	42.58	12.62
6-	Karafrin Bank	17.89	2.6	36/68	8.449
7-	Shazand	51.56	3.92	68.87	14.69
8-	Ghadir Investment	26.11	3.3	36.22	10.67
9-	North Diriling	21.56	3.43	18.85	10.19
10-	Motogen	21.29	2.96	30.46	9.591
11-	Chadormalu	26.46	3.28	39.57	11.55
12-	Sepah Investment	21.94	3.26	20.06	11.55
13-	Sobhan Pharmcy	21.75	3.09	20.88	10.02
14-	Ansar Bank	18.91	3.23	15.56	9.296
15-	Gol-e-gohar	31.77	3.4	38.37	12.59
16-	Iran Industrial Development	23.67	3.38	29.62	10.71
17-	Mobarake Steel	22.16	3.22	31.39	10.02
18-	Iran Const Investment	41.82	5.29	18.85	12.23
19-	Tuka Investment	43.02	4.78	38.41	11.97
20-	Bahman Investment	14.6	2.62	15.52	8.512
21-	Rayan Saipa	35.85	4.45	29.28	12.28
22-	Pars Toushe Investment	33.88	4.27	34.42	11.27
23-	Bahman Grop	30.42	4.07	20.58	11.38
24-	Parsian Oil and Gas	38.15	3.57	54.1	13.54
25-	Jaber Hayan Phamcy	20.53	3.13	19.09	9.308
26-	Housing Investment	17.86	2.91	14.93	8.643
27-	Pars oil	50.91	4.38	60.24	14.58
28-	Alborz Investment	23.16	2.94	24.17	10.63
29-	Khouzestan Steel	45.43	3.6	52.43	13.66
30-	Iran Telecom	15.42	2.56	14.68	8.644

Table 2. The risk of under study stocks

According to **Table 3,** the large number of zeroes in the investment decision making based on Cai method indicates risk aversion of this approach, so that only if there is high certainty about the absence of losses in the transaction, this

investment Will be offer. In fact, according to Cai, the expected returns of the studied stocks were heavily risky, which is why he has not invested in these stocks. While other methods have more risky way in making investment decisions. Of course, the performance of investment decision making based on Cai approach than other under study methods shows its superiority.

No.	Risk Measurement Methods & Under Study Stocks	Markowitz Risk Measurement	Konno Risk Measurement	Cai Risk Measurement	Teo Risk Measurement
1	Metals and Mines	-0.20095	-0.22313	0	-0.15728
2	Industrial and Mine Investment	-0.17153	-0.16785	0	-0.18989
3	Kerman Cement	0.045842	0.005608	0	0.048066
4	Sepahan Cement	0.056825	0	0	0.056825
5	Calcimine	-0.91725	-0.91725	0	-0.91725
6	Karafrin Bank	-0.37474	-0.42368	0	-0.37474
7	Shazand	0.255243	0.319054	0	0.255243
8	Ghadir Investment	1.053146	1.248657	0	0.183108
9	North Diriling	-0.08004	-0.16009	0	-0.09686
10	Motogen	-0.50584	-0.57754	0	-0.33723
11	Chadormalu	0.031442	0.062957	0	0.054146
12	Sepah Investment	0.2672	0.270191	0	0.164954
13	Sobhan Pharmcy	-0.32938	-0.29441	0	-0.01079
14	Ansar Bank	-0.01959	-0.01842	0	0.039555
15	Gol-e-gohar	-0.14083	-0.23471	0	-0.20014
16	Iran Industrial Development	-0.03199	-0.00424	0	-0.03328
17	Mobarake Steel	-0.06596	-0.0895	0	-0.09193
18	Iran Const Investment	-0.20797	-0.24319	0	0.029324
19	Tuka Investment	-0.09622	-0.17134	0	-0.07659
20	Bahman Investment	-0.04635	-0.23005	0	-0.09695
21	Rayan Saipa	0.24124	0.299829	0.854915	0.291194
22	Pars Toushe Investment	-0.16555	-0.16555	0	-0.10892
23	Bahman Grop	-0.38033	-0.50264	0	-0.3398
24	Parsian Oil and Gas	0.208278	0.346875	0	0.195921
25	Jaber Hayan Phamcy	0.044659	0.032797	0	0.048023
26	Housing Investment	0.238562	0.435315	0	0.080477
27	Pars oil	-0.14864	-0.21473	0	-0.13627
28	Alborz Investment	-0.34226	0	0	-0.22817
29	Khouzestan Steel	-0.24912	-0.3331	0	-0.13621
30	Iran Telecom	0.002438	0.002438	0	0.019121

Table 3. The performance of investment

4. Compare and analyse results

In order to compare the behavior of the studied methods, the performance of each method on the 30 studied stocks and at different risk levels has been investigated. **Table 4** shows the average of the performance of investment decisions on the under study stocks at each level of risk.

No.	Risk Measurement Methods & The level of risk	Markowitz Risk Measurement	Konno Risk Measurement	Cai Risk Measurement	Teo Risk Measurement
1-	0.1	0.007325	0.001569	0	0
2-	0.2	0.001569	0.033711	0	0
3-	0.3	0.020505	0.026941	0	-0.02061
4-	0.4	-0.00165	0.01809	0	-0.01705
5-	0.5	0.002023	-0.00066	0.051295	-0.00881
6-	0.6	0.034399	0.012248	0.051295	-0.0314
7-	0.7	-0.1701	-0.20714	0.051295	-0.13093
8-	0.8	-0.29797	-0.29528	0.051295	-0.24489
9-	0.9	-0.20499	-0.17379	0.051295	-0.16621
	Total	-0.06766	-0.06492	0.028497	-0.06888

Table 4. The average of the performance of investment decisions at levels of risk

Based on **Table 4**, among Markowitz-based risk measurement methods, the performance of Markowitz and Konno methods in lower risk levels is more desirable, but by increasing the risk level, the utility of these methods has decreased and at high risk levels the Cai method has been more efficient. In other words, Markowitz and Konno methods, due to their risk-taking nature, provide the possibility of investing in safer stocks even when the investor is risk averse and only accepts low risk levels. In contrast, the Cai approach, due to its risk-averse nature, presents a high risk for each stock and if high risk level is acceptable by investors it will offer stock transaction. so at low risk levels this method will not perform well. While the high risk level is accepted by the investor, risk-taking methods like Markowitz and Kano encourage the investor to high-risk transactions. These investments may not be profitable in general. But in the same situation, the Cai approach discourages risky investors from investing in high-risk assets. Therefore, a risk averse approach like Cai has been more suitable for risk-taking investors, vice versa, for risk-averse investors, the use of risk-taking methods, such as Markowitz and Konno, has been more appropriate.

5. Conclusion

Based on the importance of risk in investment topics, several methods have been developed to measure risk. Each of these methods has unique advantages in risk measurement and improving the quality of investment decisions. Also Each of these methods may perform better than other methods under certain situation. In this regard, the purpose of this paper has been evaluated and compared four fundamental and widely used risk measurement methods, including Markowitz, Konno, Cai and Teo. For this purpose, the risk of 30 selected stocks from Tehran Stock Exchange was calculated at 26 weeks based on the four under study approaches and then the calculated risk was the basis for making investment decisions. The results indicated that, although Cai approach generally have performed better than the other under study methods, but this method was not desirable when the investor was risk averse. Finally, according to the results of this study, it is concluded that there is no unique methods among the Markowitz-based risk measurement methods that always has had the most favorable performance in making investment decisions and the performance of each method is depended on data sets and also Risk-taking or risk aversion decision makers.

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